CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

| Course | Number | Section |
|---|---|---|
| Mathematics | 203 | CA |
| **Examination** | **Date** | **Duration** |
| Midterm Test | July 23rd, 2013 | 1 h 30 min |
| **Special Instructions:** | Only approved calculators are allowed<br>**Show all your work** | |

# Part A.

**1.** (6 marks) [Choose and solve any two equations]

Solve for $x$ (find the exact values, do not approximate).

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**(a)** $e^{2x} - 3e^x + 2 = 0$

**(b)** $\log_5 x + \log_5(x-1) = \log_5(4x)$

**(c)** $2e^{3x-5} = 7$

**2.** (6 marks) [Choose and calculate any two limits]

Evaluate the following limits.

**(a)** $\lim\limits_{x\to\infty} \dfrac{x^2+4x+6}{3x^2+1}$

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**(b)** $\lim\limits_{x\to\infty} \dfrac{\ln(2x)}{\ln x}$

**(c)** $\lim\limits_{x\to\infty} (\sqrt{x^2+2x} - x)$

**3.** (9 marks) [You have to differentialte all three functions]

Find the derivatives of the following functions (You do not have to simplify!).

**(a)** $f(x) = (x^2+4)^7 \sin(2x)$

**(b)** $f(x) = \dfrac{x^{7/6} + 5x^{1/5} - 15}{x^{1/5}}$

**(c)** $f(x) = e^{\ln 2} + e^x \sin x$

## Part B.

[You have to choose and solve any five questions]

**4.** (4 marks) Sketch the graph of the function $f(x) = |(x-2)^2 - 3|$ starting from the graph $f(x) = x^2$ and using appropriate transformations.

**5.** (4 marks) Find all horizontal and vertical asymptotes of the graph of the function $f(x) = \dfrac{2x+3}{\sqrt{x^2-2x-3}}$.

**6.** (4 marks) Find parameters $a$ and $b$ such that the function

$$f(x) = \begin{cases} \cos x, & \text{if } x < 0 \\ ax + b, & \text{if } 0 \le x < 1 \\ 0, & \text{if } x \ge 1 \end{cases}$$

will be continuous at every point. Sketch the graph of this function.

**7.** (4 marks) Given the function $f(x) = 1 + \ln(3+x)$, find the inverse function $f^{-1}$ and the range of $f^{-1}$.

**8.** (4 marks) Using the Intermediate Value Theorem show that the equation $\ln x - 1 = 0$ has a root between 2 and 3.

**9.** (4 marks) Let $f(x) = \sqrt{4-x}$ and $g(x) = \dfrac{5}{x^2+2}$. Find the composite functions $f \circ f$ and $g \circ f$, and determine their domains.

**10.** (4 marks) Consider the curve given by the equation $x^2 + y^2 = 2y + x$. Verify that the point $(1,\ 2)$ belongs to this curve and find the equation of the tangent line to the graph of this curve at that point.

**11.** (4 marks) Given the function $f(x) = x^2 + \dfrac{1}{x}$. Use the definition of derivative to find the derivative of the function and then use the appropriate differentiation rule(s) to verify your answer.