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ADM 2304 [P] ASSIGMENT 1

PERSONAL ETHICS STATEMENT

Individual Assignment:

By signing this Statement, I am attesting to the fact that I have reviewed the entirety of my attached work and that I have applied all the appropriate rules of quotation and referencing in use at the Telfer School of Management at the University of Ottawa, as well as adhered to the fraud policies outlined in the Academic Regulations in the University's Undergraduate Studies Calendar [Academic Fraud Webpage](#).

Y.S. Pandit

7th February 2021

Signature

Date

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Student Number

Question 1- Electronic Appliances Warranties

For the two groups (Last year's survey and current year's survey), we can conduct the z-test for difference between two proportions.

The hypotheses are as follows:

H₀: The proportion of would-be buyers is same for both the years

H_a: The proportion of would-be buyers in the last year's survey is greater than the proportion of would be buyers in the current year's survey (since we are testing whether the popularity of warranties for electronic components have decreased)

Let sample 1 be the last year's sample and

Sample 2 be current year's sample.

We write the above hypotheses as:

$$H_0: \hat{p}_1 - \hat{p}_2 = 0$$

$$H_a: \hat{p}_1 - \hat{p}_2 > 0$$

Where:

\hat{p}_1 = the proportion of consumers in last year's survey who have indicated that they would buy the warranty

\hat{p}_2 = the proportion of consumers in the current year's survey who have indicated that they would buy the warranty.

Finding the sample proportions:

We have $n_1 = 1100$ and $n_2 = 800$

$$\hat{p}_1 = 56\% = 0.56, \hat{p}_2 = 46\% = 0.46$$

Thus

$$\hat{q}_1 = 1 - \hat{p}_1 = 1 - 0.56 = 0.44$$

$$\hat{q}_2 = 1 - \hat{p}_2 = 1 - 0.46 = 0.54$$

We check that the normal approximation is met i.e. $n_1\hat{p}_1$ and $n_2\hat{p}_2$ are >5 (or 10)

This is a one-tailed z-test at the upper tail for the difference between two sample proportions and we should compare the observed result with the critical value of z at the upper tail at $\alpha=0.05$

$$\begin{aligned} Z &= [(\hat{p}_1 - \hat{p}_2)] / \sqrt{\hat{p}_1 \cdot \hat{q}_1 / n_1 + \hat{p}_2 \cdot \hat{q}_2 / n_2} \\ &= (0.56 - 0.46) / \sqrt{(0 \cdot 56)(0 \cdot 44) / 1100 + (0 \cdot 46)(0.54) / 800} \\ &= 4.32 \end{aligned}$$

We find that the critical value of z at $\alpha=0.05$ is 1.645

Since the observed z-statistic $4.32 > 1.645$ we reject the null hypothesis and conclude that we have sufficient evidence to say that the popularity of warranties for electronic components have decreased.

b) From the standard normal table, we find that the percentile of $z=4.32$ is more than 0.9998.

Hence the p-value = $1 - 0.9998 = 0.0002$ or less than that

Since p-value of 0.0002 or less $< \alpha = 0.05$ (less than 5% significance level) we reject the null hypothesis which confirms our action in part (a). It is consistent with the critical value approach.

c) Test whether popularity has decreased by more than 5% using 5% significance level

We can write the hypotheses as:

$$H_0: \hat{p}_1 - \hat{p}_2 = 0.05$$

$$H_a: \hat{p}_1 - \hat{p}_2 > 0.05$$

Calculating the test statistic z:

$$\begin{aligned} Z &= \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}_1 \cdot \hat{q}_1 / n_1 + \hat{p}_2 \cdot \hat{q}_2 / n_2}} \\ &= \frac{(0.56 - 0.46) - 0.05}{\sqrt{(0.56)(0.44) / 1100 + (0.46)(0.54) / 800}} \\ &= 2.16 \end{aligned}$$

Since the observed z-statistic $2.16 > 1.645$, we reject the null hypothesis and conclude that we have sufficient evidence to say that the popularity of warranties for electronic components have decreased by more than 5%.

The corresponding p-value = $1 - 0.9846$ (from the table for $z = 2.16$) = 0.0154 which is less than $\alpha = 0.05$. Thus the p-value method confirms the rejection of the null hypothesis by the critical value method.

d) 95% confidence interval:

$$\text{The point estimate} = (\hat{p}_1 - \hat{p}_2) = 0.56 - 0.46 = 0.10$$

$$\begin{aligned} \text{Margin of error} &= \sqrt{(0.56)(0.44) / 1100 + (0.46)(0.54) / 800} \\ &= 0.0231 \end{aligned}$$

$$\text{Upper limit} = 0.10 + (1.64)(0.0231) = 0.1379$$

Hence, 95% one sided upper CI is $(-\infty, 0.1379)$

$$\text{One-sided lower CI is } ((\hat{p}_1 - \hat{p}_2) \pm z\alpha \sqrt{\hat{P}(1 - \hat{P}) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)})$$

$$\text{Hence, lower limit} = 0.10 - (1.64)(0.0231) = 0.0621$$

Hence, one sided 95% CI is $(0.0621, \infty)$

The interval is consistent with conclusion in part a and b because the hypothesized value $p_1 - p_2$ is not included in the confidence interval. Both limits are positive. Hence there is sufficient evidence to conclude that percentage has decreased in current year.

Question 2:

a) Sample target market is 320

Out of this 125 people voted for “disability insurance with 2% discount”

$$\hat{p} = 125/320 = 0.39$$

b) Standard Error of Target Market = $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.39 \cdot 0.61}{320}} = 0.02726$

c) Null hypothesis: $p_0 > 0.35$

Alternative Hypothesis: $p_0 \leq 0.35$

$$\text{Prob}(p > 0.35) = \text{prob}(z > (0.391 - 0.35) / \sqrt{(0.35)(1 - 0.35) / 320})$$

$$= \text{prob}(z > 0.041 / 0.0267)$$

$$= \text{prob}(z > 1.54) = 0.0618$$

Hence, P-value = 0.0618

Alpha is 0.1 since here we are considering only one side of the distribution.

P value is 0.0618 which is smaller than the significance level we will reject the null hypothesis.

d) Sample size for 98% confidence:

$Z_{\alpha} = 2.325$ at 0.02 level of significance

$$n = \frac{z^2 \cdot p \cdot (1-p)}{S.E.^2} = 20.28 \approx 20$$

e) One sided approximate 90% confidence interval for the target market is

$$p \pm z_{\alpha} S.E.$$

$$0.39 \pm 1.282(0.02727)$$

$$0.355 \text{ or } 0.425$$

Confidence interval is (0.355, 0.425)

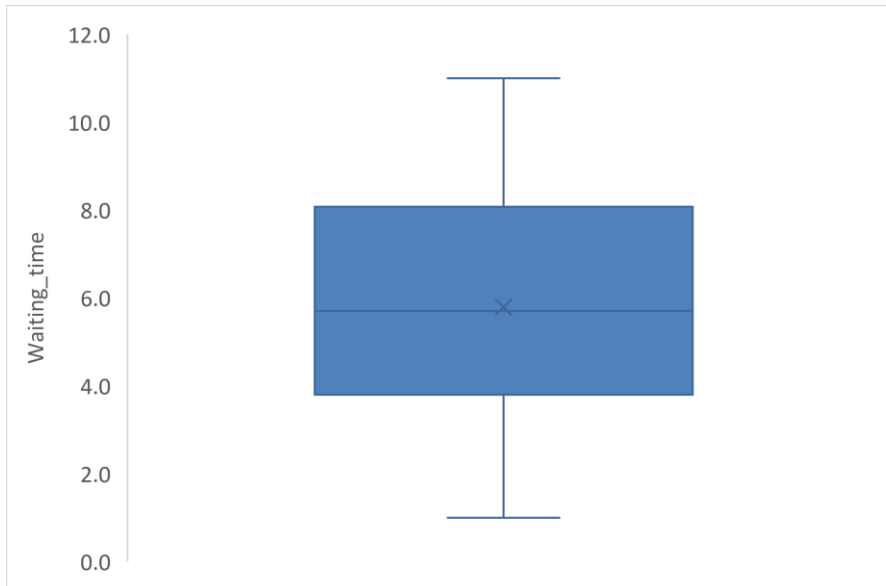
Hence, yes this interval is consistent with part c as upper limit is greater than 35% and is at least 42.5%

Question 3:

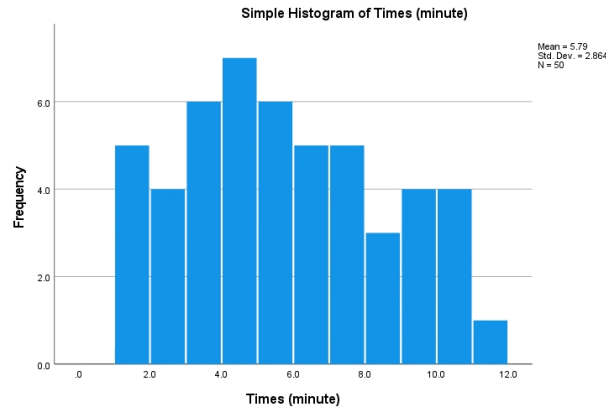
Given:

A manager of a large grocery store wants to determine the time spent by customers waiting in line. She collected a random sample of the waiting times from 50 customers.

a) Box plot and histogram of Time variable:



Since the median line is approximately dividing the box into equal parts, it can be said that the data collected for the waiting time in a line at a grocery store, is approximately symmetric or normally distributed.



Since the histogram does not have wither a longer left tail or a right tail and maximum data is collected in the middle of the chart, it can be said that the data collected for the waiting time in a line at a grocery store is approximately symmetric or normally distributed.

b) Sample mean and standard deviation is as follows:

$$\begin{aligned}\bar{x} &= \frac{1}{n} \sum x \\ &= 1/50 * 289.5 \\ &= 5.79\end{aligned}$$

$$\begin{aligned}s &= \sqrt{\frac{1}{n-1} \sum (x - \bar{x})^2} \\ &= \sqrt{\frac{1}{n-1} (\sum x^2 - n\bar{x}^2)} \\ &= \sqrt{\left(\frac{1}{50-1} \times 2078.13\right) - \left(\frac{50 \times (5.79)^2}{50-1}\right)} \\ &= \sqrt{8.202551}\end{aligned}$$

~ 2.86

$$\text{Standard error} = \text{Standard Deviation} / \sqrt{n} = 2.8640 / \sqrt{50} = 0.40503$$

Thus, the sample mean and sample standard deviation are 5.79 minutes and 2.86 minutes respectively.

c) A hypothesis test is to be conducted to determine whether there is enough evidence suggesting that the mean waiting time for the grocery store's customers is less than 6 minutes.

The hypothesis is:

H_0 : The mean waiting time for the grocery store's customers is not less than 6 minutes i.e. $\mu \geq 6$

H_a : The mean waiting time for the grocery store's customers is less than 6 minutes i.e. $\mu < 6$

The significance level of the test is $\alpha = 0.10$

As the population standard deviation is not known we will use a t-test for single mean.

T-Test

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Times (minute)	50	5.790	2.8640	.4050

One-Sample Test

Test Value = 6

	t	df	Sig. (2-tailed)	Mean Difference	90% Confidence Interval of the Difference	
					Lower	Upper
Times (minute)	-.518	49	.606	-.2100	-.889	.469

The test statistic value is $t = -0.518$

The p-value for the 2-tailed test will be, $p = 0.606/2 = 0.303$

Decision rule:

If the p-value of the test is less than the significance level then the null hypothesis will be rejected and vice-versa

$$p\text{-value} = 0.306 > \alpha = 0.10$$

The null hypothesis will not be rejected at 10% level of significance.

There is not enough evidence suggesting that the mean waiting time for the grocery store's customers is less than 6 minutes.

d) The one sided 90% confidence interval for the true mean is:

Computing the 90% confidence interval for the true mean waiting time for the grocery store's customers as follows:

$$CI = 5.79 - [1.645 * 0.405] = 5.12$$

The 90% confidence interval for the true mean waiting time for the grocery store's customers is 5.12 which is the lower bound which is less than 6 minutes.

The interval is consistent with part(c) result

e) The margin of error of a confidence interval for mean (when the population standard deviation is known) is:

$$MOE = z_{\alpha/2} * \frac{\sigma}{\sqrt{n}}$$

The critical value of z for 90% confidence interval is $z = 1.645$

Compute the required sample size as follows:

$$MOE = z_{\alpha/2} * \frac{\sigma}{\sqrt{n}}$$

$$N = \left[\frac{z_{\alpha/2} \sigma}{MOE} \right]^2$$

$$= \left[\frac{1.645 \times 2.86}{0.34} \right]^2$$

$$= 191.47 \approx 192$$

Thus the minimum required sample size is 192.