Université Concordia University

Midterm Examination

Course: PHYS 205-Section EC1
Date: November 01/2020
Exam time: 13:00 – 14:15
Instructor: Seyyed Nima Nateghi

Multiple Choice

1. **(3 marks)** We place a point charge Q at the center of a cube of side L and another charge Q (the same) at the center of a sphere of radius L. Which object (cube or sphere) will have a larger electric flux passing through it?
   a) Cube
   b) Sphere
   c) Same electric flux passes through both
   d) Cannot be determined

   $\Phi_E = \frac{q_{in}}{\epsilon_0}$ → same for cube and sphere

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2. **(2 marks)** What is the direction of the electric field <u>inside</u> a positively charged solid conductor that is in static equilibrium?
   a) The electric field points towards the positive charges sitting on the surface
   b) The electric field is zero
   c) The electric field is going out of the charges sitting on the surface

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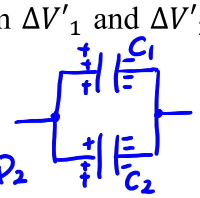
3. **(5 marks)** Two capacitors C_1 and C_2 are fully charged separately using batteries ΔV_1 and ΔV_2 ($\Delta V_1 > \Delta V_2$), having charges Q_1 and Q_2, respectively. We then disconnect them from their circuits and connect them together with the same polarity (positive plate to positive plate, and negative plate to negative plate). Note that in this new configuration we only have these two capacitors and no batteries. In this configuration, the capacitors will have charges Q'_1 and Q'_2 and potential differences across them $\Delta V'_1$ and $\Delta V'_2$. Which one is true comparing $Q_1 + Q_2$ to $Q'_1 + Q'_2$ and $\Delta V'_1$ to $\Delta V'_2$?
   a) $Q_1 + Q_2 > Q'_1 + Q'_2$ and $\Delta V'_1 > \Delta V'_2$
   b) $Q_1 + Q_2 < Q'_1 + Q'_2$ and $\Delta V'_1 < \Delta V'_2$
   c) $Q_1 + Q_2 > Q'_1 + Q'_2$ and $\Delta V'_1 < \Delta V'_2$
   d) $Q_1 + Q_2 = Q'_1 + Q'_2$ and $\Delta V'_1 = \Delta V'_2$
   e) $Q_1 + Q_2 = Q'_1 + Q'_2$ and $\Delta V'_1 > \Delta V'_2$

   $Q'_1 + Q'_2 = Q_1 + Q_2$ Conservation of charge



   $\Delta V'_1 = \Delta V'_2$ plates are connected + → + - → -

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4. **(5 marks)** What happens to the charge and the potential difference across a fully charged capacitor which is disconnected from the battery, if we place a dielectric with $k > 1$ in between its plates?

   a) Q increases and ΔV increases
   b) Q increases and ΔV decreases
   c) Q remains the same and ΔV increases
   d) Q decreases and ΔV decreases
   e) Q remains the same and ΔV decreases

   Q doesn't change ← cons. of charge
   ΔV decreases ← $\vec{E}$ does work to turn dipole moments → energy decreases

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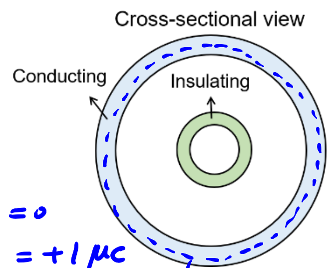
5. **(5 marks)** An insulating spherical shell of charge $q = -1\ \mu C$ (uniform volume charge distribution ρ) is placed inside a conducting spherical shell with charge $Q = +3\ \mu C$. What is the charge on the outer surface of the conducting shell?



   a) $+2\ \mu C$
   b) $+3\ \mu C$
   c) $-1\ \mu C$
   d) $+1\ \mu C$
   e) 0

   $\vec{E}$ inside conductor = 0
   $\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \rightarrow q_{in} = 0 \rightarrow q + Q_{inner} = 0 \rightarrow Q_{inner} = +1\ \mu C$
   $Q_{inner} + Q_{outer} = +3\ \mu C \rightarrow Q_{outer} = +2\ \mu C$
   Gaussian surface

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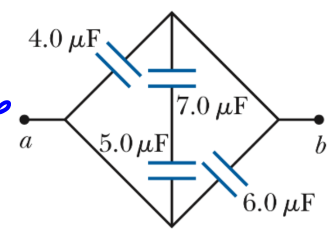
6. **(5 marks)** If the potential difference between points a and b is 120 V, what is the charge stored on the 4 μF capacitor?



   a) 120 μC
   b) 480 μC
   c) 30 μC
   d) 1550 μC

   ΔV across 4 μF capacitor = ΔV_{ab}
   $Q = C\Delta V = (4\ \mu F)(120\ V) = 480\ \mu C$

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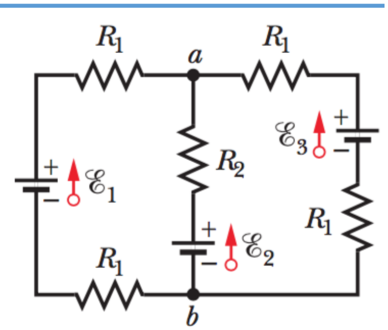
7. **(5 marks)** In the figure, $R_1 = 1\ \Omega$ and $R_2 = 2\ \Omega$, and the ideal batteries have emfs of $\varepsilon_1 = 2\ V$ and $\varepsilon_2 = \varepsilon_3 = 4\ V$. What is the current passing through ε_3? Note that the red arrows do not necessary show the correct direction of the currents in the batteries.



   Kirchhoff's rule

   a) 0.66 A
   b) 1 A
   c) 1.2 A
   d) 0.33 A

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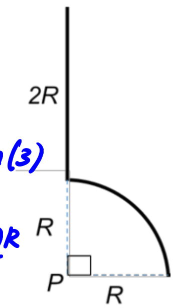
8. **(5 marks)** In the figure below, the rod is uniformly charged with linear charge density $+\lambda$. What is the electric potential at point *P*?

a) $k_e \frac{\lambda}{3R}$

b) $k_e\lambda(2\ln(3)+\pi)$

c) $k_e\lambda\left(\ln(3)+\frac{\pi}{2}\right)$

d) 0



Rod: $dV = k_e \frac{dq}{r} = k_e \frac{\lambda dx}{3R-x}$

$V_{Rod} = \int_0^{2R} k_e\lambda \frac{dx}{3R-x} = k_e\lambda \ln(3)$

$\frac{1}{4}$ of ring: $dV = k_e \frac{dq}{R} \rightarrow V_{Rod} = \frac{k_e}{R}\int dq = k_e \frac{Q}{R} = k_e \frac{\lambda(\pi/2)R}{R}$

$V_{Rod} = k_e\lambda\left(\frac{\pi}{2}\right)$

Long Answer

Instruction:

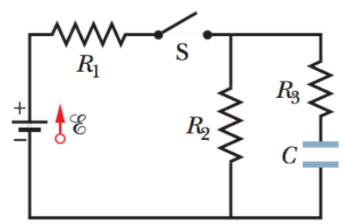
You MUST submit your detailed solution, written on a white sheet of paper, through COLE.

9. **(10 marks)** In the circuit, ε = 1.2 kV, C = 6.5 mF, R_1=R_2=R_3=1 MΩ. With C completely uncharged, switch S is suddenly closed (at t=0). Determine:

(a) at t=0, the current I_1 in resistor 1, current I_2 in resistor 2, and current I_3 in resistor 3?

(b) As $t \rightarrow \infty$ (when the capacitor is fully charged), what are I_1, I_2, and I_3? Describe your answer.

(c) If we open switch S when the capacitor is fully charged, how long will it take for the current in resistors 2 and 3 to reach half its maximum value.



10. **(10 marks)** A particle of mass m and charge q is dropped from the height of h towards a point charge Q, which lies on the ground. What will be the minimum distance between the charges? Treat the charges as particles (zero dimension).
<u>**Note:**</u> Provide your answer symbolically (in terms of m, q, Q, h, g, k_e).

Formula you may need

$e = 1.6 \times 10^{-19}\ C$ | $1\ m\ (mili) = 10^{-3}$ | $1\ \mu = 10^{-6}$

$\vec{F} = k_e \frac{qQ}{r^2}\hat{r}$ | $\vec{E} = \frac{\vec{F}}{q}$ | $k_e = 9 \times 10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$ | $K = \frac{1}{2}mv^2$ | $\Delta V = -\int \vec{E}.d\vec{s}$

$V = \sum k_e \frac{Q}{r}$ | $\Delta V = \frac{\Delta U}{q}$ | $\phi_E = \int \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_0}$

$C = \frac{Q}{\Delta V}$ | $C = k\epsilon_0 \frac{A}{d}$ | $U_E = \frac{Q^2}{2C}$

$\Delta V = IR$ | $R = \rho\ \frac{l}{A}$ | $P = \Delta VI$ (P = power)

$U_g = mgy$ $\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

Parallel configuration: $C_{eq} = C_1 + C_2 + \cdots$ $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$

Series configuration: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$ $R_{eq} = R_1 + R_2 + \cdots$

Current in RC circuit: $I(t) = I_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$, $\tau = RC$

$I(t) = I_{max} e^{-\frac{t}{\tau}}$

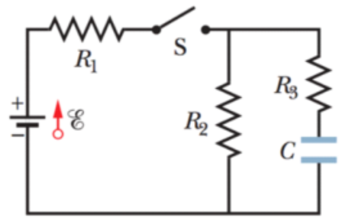
$U_C = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V)^2$ $R = \frac{\Delta V}{I}$ $P = RI^2 = \frac{(\Delta V)^2}{R}$

9. **(10 marks)** In the circuit, $\varepsilon = 1.2$ kV, $C = 6.5$ mF, $R_1 = R_2 = R_3 = 1$ MΩ. With C completely uncharged, switch S is suddenly closed(at t=0). Determine:

(a) at t=0, the current I_1 in resistor 1, current I_2 in resistor 2,and current I_3 in resistor 3?

(b) As $t \to \infty$ (when the capacitor is fully charged), what are I_1, I_2, and I_3? Describe your answer.

(c) If we open switch S when the capacitor is fully charged, how long will it take for the current in resistors 2 and 3 to reach half its maximum value.



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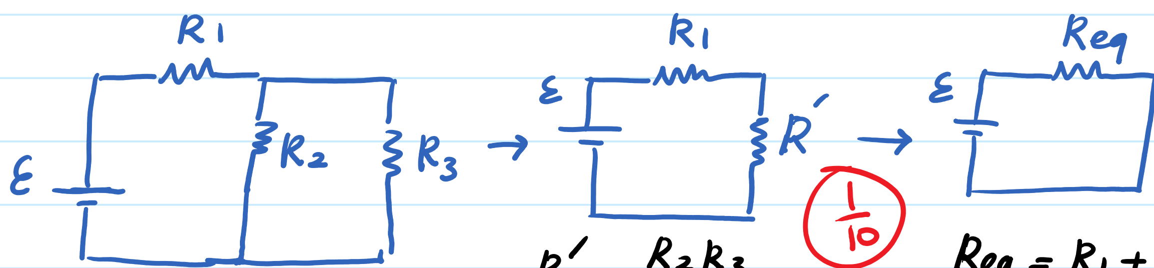
a) Immediately after the switch is closed, the capacitor is empty of charge and acts as a wire. 1/10



1/10

$$R' = \frac{R_2 R_3}{R_2 + R_3}$$

$$R' = 0.5 \text{ M}\Omega$$

$$R_{eq} = R_1 + R'$$

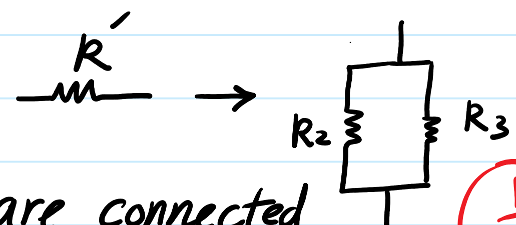
$$R_{eq} = 1.5 \text{ M}\Omega$$

$$\varepsilon = I R_{eq} \rightarrow I = \frac{\varepsilon}{R_{eq}} = \frac{1.2 \times 10^3}{1.5 \times 10^6} = 8 \times 10^{-4} \text{ A}$$

1/10

This is the total current, which passes through R_1 & R'

$\rightarrow I_1 = 8 \times 10^{-4}$ A



Since R_2 and R_3 are connected in parallel ($\Delta V_2 = \Delta V_3$) and $R_2 = R_3$:

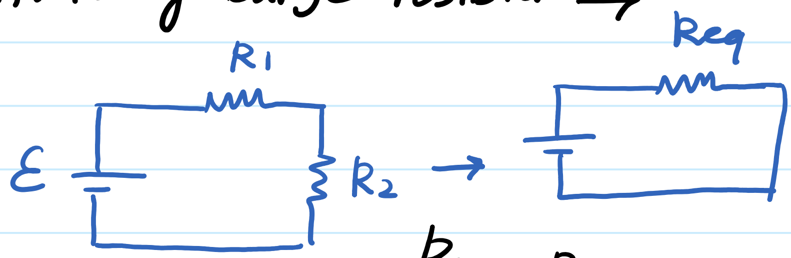
1/10

$$I_2 = I_3 = \frac{1}{2}(8 \times 10^{-4} \text{ A}) = 4 \times 10^{-4} \text{ A}$$

b) As $t \to \infty$, the capacitor fully charges and acts as an infinitely large resistor →

3/10

$I_3 = 0$ (1/10) →



$R_{eq} = R_1 + R_2 = 2\times10^{6}\,\Omega$ (1/10)

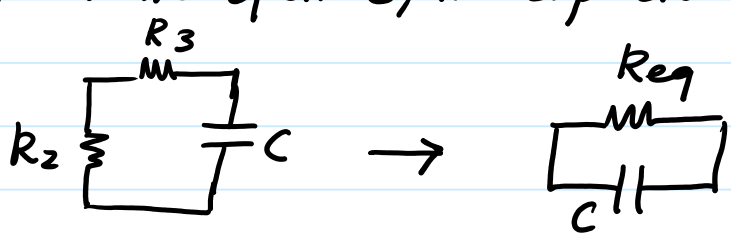
$$I = \frac{\varepsilon}{R_{eq}} = \frac{1.2\times10^{3}}{2\times10^{6}} = 6\times10^{-4}\,A$$

since R_1 and R_2 are connected in series:

$$\boxed{I_1 = I_2 = 6\times10^{-4}\,A}$$ (1/10)

c) When we open S, The capacitor discharges:

3/10



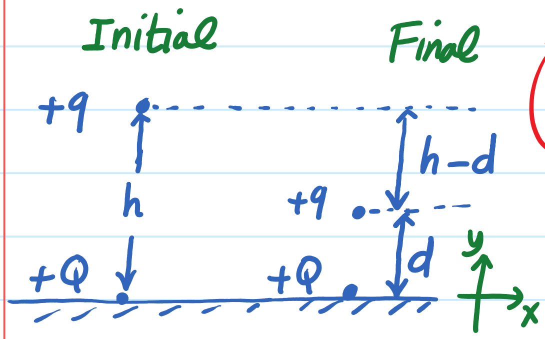
(1/10) $R_{eq} = R_2 + R_3 = 2\times10^{6}\,\Omega$

$$I(t) = \frac{1}{2} I_{max} \to \frac{1}{2} I_{max} = I_{max}\, e^{-t/\tau}, \quad \tau = RC$$ (1/10)

(1/10) $\to -t/\tau = \ln\left(\frac{1}{2}\right) = -\ln(2) \to t = \tau \ln(2) = RC\ln(2)$

$$t = (2\times10^{6})(6.5\times10^{-3})(0.69) \approx \boxed{9\times10^{3}\,s}$$

10. **(10 marks)** A particle of mass m and charge q is dropped from the height of h towards a point charge Q, which lies on the ground. What will be the minimum distance between the charges? Treat the charges as particles (zero dimension).
Note: Provide your answer symbolically (in terms of m, q, Q, h, g, k_e).



3/10

We use conservation of energy

Initial: $E_i = K_i + U_{gi} + U_i$ ($K_i \to 0$)

$E_i = mgh + k_e \frac{qQ}{h}$

Final: $E_f = K_f + U_{gf} + U_f$ ($K_f \to 0$)

$E_f = mgd + k_e \frac{qQ}{d}$

6/10

Cons. of energy:

$E_f = E_i$

$mgd + k_e \frac{qQ}{d} = mgh + k_e \frac{qQ}{h}$

$\rightarrow mg(h-d) = k_e qQ \left(\frac{1}{d} - \frac{1}{h}\right)$

$\rightarrow mg(h-d) = k_e qQ \left(\frac{h-d}{dh}\right) \rightarrow \boxed{d = \frac{k_e qQ}{mgh}}$

1/10