SASS

Table of Points for marking purposes. *Do not write in this table.*

	multiple-choice	Q4	Q5	Q6	Q7	Q8	TOTAL
max points possible	8 pts	4 pts	5 pts	5 pts	5 pts	6 pts	33 points
points obtained							

MULTIPLE-CHOICE QUESTIONS.

Write your choices in the answer boxes. No justification is needed.

Q1. Here is the truth table for 4 *mystery* compound propositions P_1, P_2, P_3, and C each consisting of atomic variables x, y, and z:

x	y	z	P_1	P_2	P_3	C	$\neg z$	$P_1 \oplus P_2$	$P_3 \oplus C$	$(P_1 \oplus P_2) \land (P_3 \oplus C)$
T	**T**	**T**	**F**	**F**	**F**	**F**	F	F	F	F
T	**T**	**F**	**T**	**T**	**F**	**T**	T	F	T	F
T	**F**	**T**	**F**	**F**	**T**	**F**	F	F	T	F
T	**F**	**F**	**T**	**T**	**F**	**F**	T	F	F	F
F	**T**	**T**	**F**	**T**	**T**	**F**	F	T	T	T
F	**T**	**F**	**T**	**T**	**F**	**T**	T	F	T	F
F	**F**	**T**	**F**	**T**	**F**	**T**	F	T	T	T
F	**F**	**F**	**T**	**T**	**F**	**F**	T	F	F	F

Which 4 of the following statements are true? *Only 4 statements are true.*

A. The set $\{P_1, P_2, P_3\}$ is **consistent**. False

B. The set $\{P_1, P_2\}$ is **consistent**. True

C. The argument $(P_1 \land P_2 \land P_3) \rightarrow C$ is **valid**. True

D. The argument $(P_1 \land P_2) \rightarrow C$ is **valid**. False

E. $(P_1 \oplus P_2) \land (P_3 \oplus C)$ is a **tautology**. False

F. $(x \lor \neg y \lor z) \land (\neg x \lor y \lor z)$ is a **disjunctive normal form (DNF)** for P_3. False

G. $(P_1 \oplus P_2) \land (P_3 \oplus C)$ is a **contradiction**. False

H. $(P_1 \oplus P_2) \land (P_3 \oplus C)$ is a **contingency**. False

I. $\neg z$ is a **disjunctive normal form (DNF)** for P_1. True

Answers: $\boxed{B}$ $\boxed{C}$ $\boxed{H}$ $\boxed{I}$ [4 points]

Q2. Consider the following propositional variables:

a: "Mark writes a deferred exam."
b: "Mark gets an A+ in this course."
c: "Mark missed the final exam accidently."

Now consider the following compound proposition:

" Mark does not miss the final exam accidently is a necessary condition for Mark to get A+ in this course unless Mark writes a deferred exam."

Which of the following is a correct translation of the above compound proposition?

A. $(b \to \neg c) \lor a$ **B.** $(b \to \neg c) \land a$ **C.** $a \to (b \to \neg c)$

D. $a \to (\neg c \to b)$ **E.** $(\neg c \to b) \lor a$ **F.** $(\neg c \to b) \land a$

G. none of the above

Answer: $\boxed{A}$ **[2 points]**

Q3. Let p, q and r be propositional variables. Which one the following is logically equivalent to propositional logical formula $(p \lor r) \to (q \leftrightarrow r)$ circle the best response. **[2 points]**

A. $(\neg p \land r) \lor (q \land r) \lor (\neg q \land \neg r)$.

B. $(p \land r) \lor (q \land r) \lor (\neg q \land \neg r)$.

C. $(\neg p \land r) \lor (\neg q \land r) \lor (q \land \neg r)$.

D. $(\neg p \land \neg r) \lor (q \land \neg r) \lor (\neg q \land r)$.

E. $(p \land \neg r) \lor (q \land r) \lor (\neg q \land \neg r)$.

F. $(\neg p \land \neg r) \lor (q \land r) \lor (\neg q \land \neg r)$.

$(p \lor r) \to (q \leftrightarrow r)$
$\equiv \neg(p \lor r) \lor (q \leftrightarrow r)$
$\equiv (\neg p \land \neg r) \lor (q \land r) \lor (\neg q \land \neg r)$

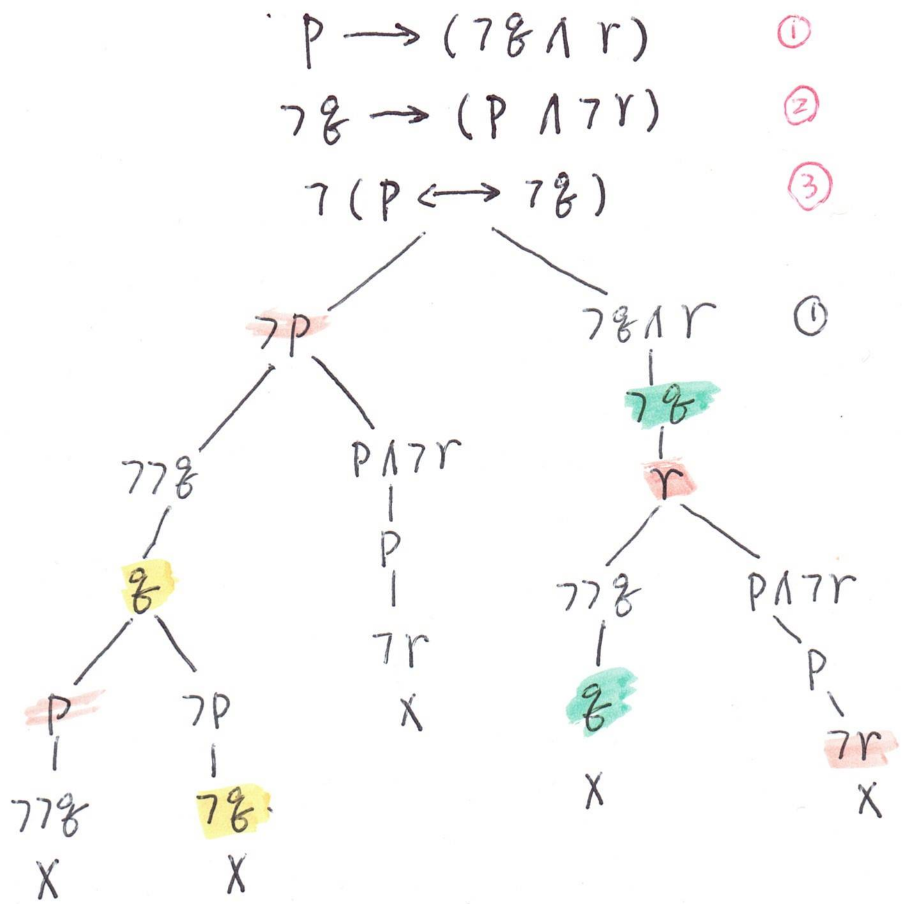
Answer: $\boxed{F}$ **[2 points]**

Long-Answer Questions. Detailed justifications are required.

Q4. Using a truth tree to determine whether the following set of three compound , **[4 points]** $\{p \to (\neg q \wedge r), \neg q \to (p \wedge \neg r), \neg(p \leftrightarrow \neg q)\}$ is a consistent or inconsistent.

Using Branching Rules for Truth Trees.

Points will be deducted for combining more than one splitting rules in one step.



All Branches are dead. So it is inconsistent

Q5. For this question, you will prove logical equivalence in two ways. **[5 points]**

5a. Using the laws from the Table of Equivalences given on page 9, prove that:

$$(\neg z \to x) \wedge (\neg z \to y) \equiv \neg(x \wedge y) \to z$$

You must use **one and only one law per step**, and name the law you are using at each step.

$(\neg z \to x) \wedge (\neg z \to y) \equiv (\neg\neg z \vee x) \wedge (\neg\neg z \vee y)$ Implication Law

$\equiv (z \vee x) \wedge (z \vee y)$ Double negation law

$\equiv (x \wedge y) \vee z$ Distribution law

$\equiv \neg(\neg(x \wedge y)) \vee z$ Double negation law

$\equiv \neg(x \wedge y) \to z$ implication law

5b. Now, prove that $(\neg z \to x) \wedge (\neg z \to y)$ is **logically equivalent to** $\neg(x \wedge y) \to z$ using a truth **table** and a brief explanation.

x	y	z	$\neg z$	① $\neg z \to x$	② $\neg z \to y$	① $\wedge$ ②	$\neg(x \wedge y)$	$\neg(x \wedge y) \to z$
T	T	T	F	T	T	T	F	T
T	T	F	T	T	T	T	F	T
T	F	T	F	T	T	T	T	T
T	F	F	T	T	F	F	T	F
F	T	T	F	T	T	T	T	T
F	T	F	T	F	T	F	T	F
F	F	T	F	T	T	T	T	T
F	F	F	T	F	F	F	T	F

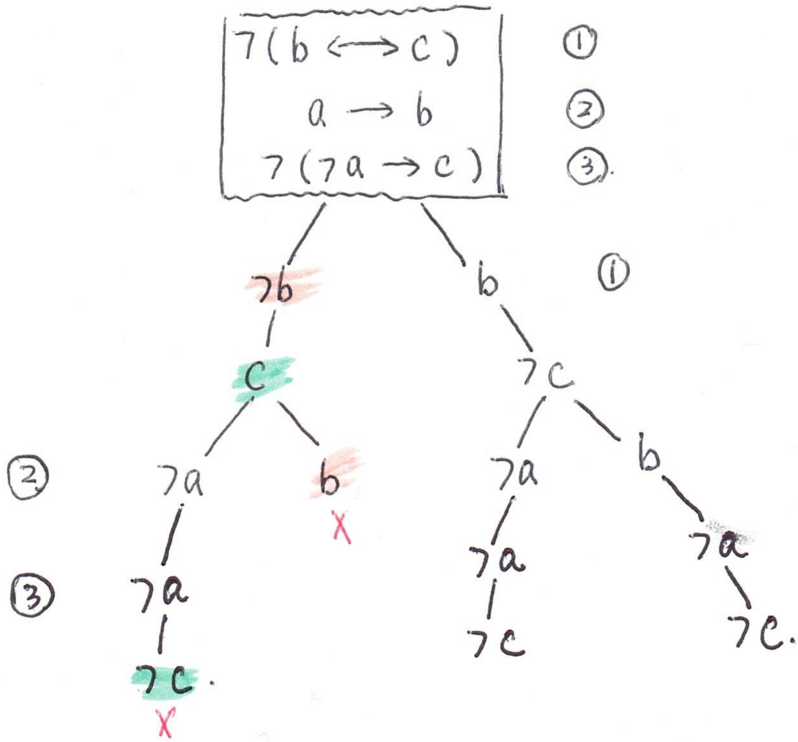
① $\wedge$ ② and $\neg(x \wedge y) \to z$ have exactly Truth Value, so is logically equivalent.

Q6. **[5 points]**

Use the method of your choice to determine whether or not the following argument is **valid**. Show your work in the space below.

Argument:

$$\neg(b \leftrightarrow c)$$
$$a \rightarrow b$$
$$\therefore\ \neg a \rightarrow c$$



$\neg a \rightarrow c$
$\equiv \neg\neg a \vee c$
$= a \vee c.$

Since there exists at least one complete active path, it means the root is not a contradiction. the root is equivalent to the argument's negation, so the argument itself is not a tautology. ∴ this argument is invalid.

Is the argument valid?	*Circle:*	**YES**	**NO**

If you circle **YES**, briefly explain, referring to your work above.

If you circle **NO**, give **all counterexamples** and briefly explain.

counter example.

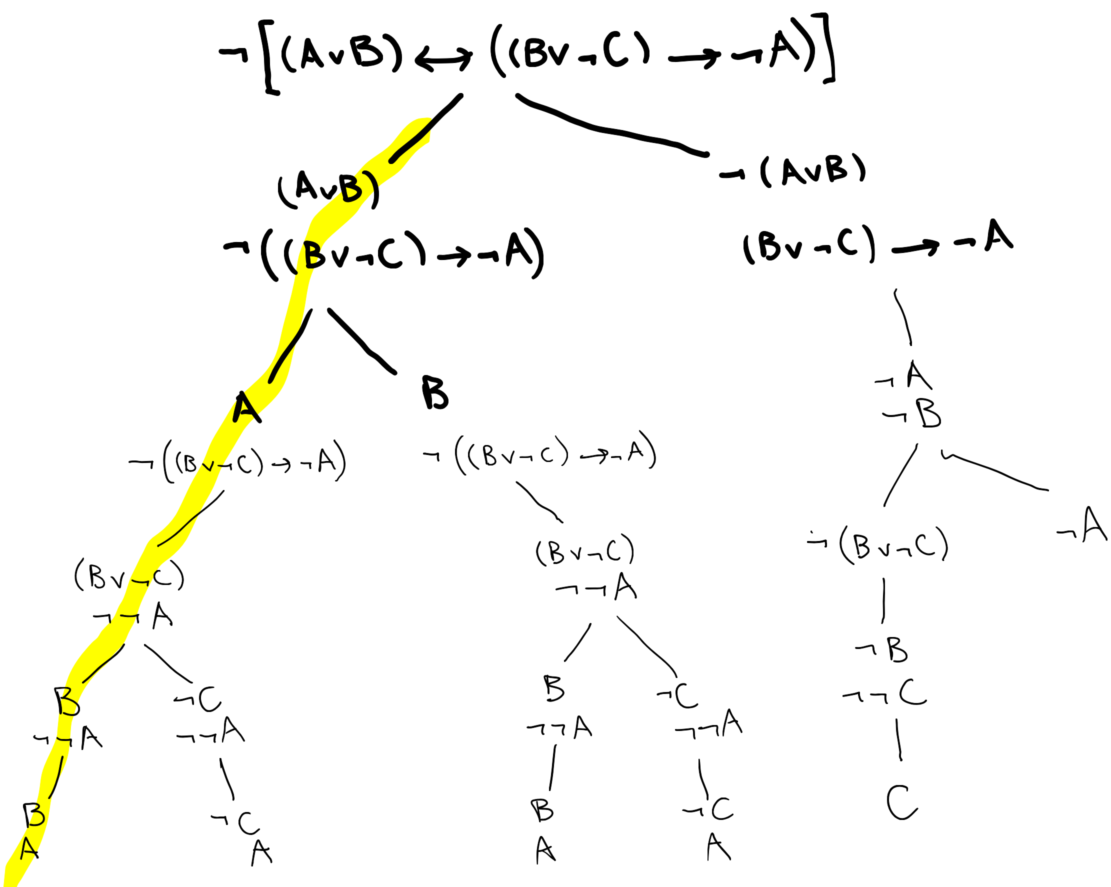
when a = F, b = T, c = F, all premises are T, but the conclusion is F.

Q7. Consider the compound proposition X, defined as follows: **[5 points]**

$$X : \qquad \Big[\Big(A \vee B\Big) \leftrightarrow \Big((B \vee \neg C) \to \neg A\Big)\Big]$$

Use an appropriate **truth tree** to determine whether X is a **tautology.** Grow a complete truth tree. Make sure you apply the *official* branching rules to the propositions as they are written (*i.e. do not use logical equivalences to change the propositions in your tree – stick to the official branching rules*).

Complete truth tree: What is the root of your tree? *Circle:* X $\neg X$



Is X a tautology ? *Circle:* **YES** **NO**

Explain your answer making reference to your tree, its root, and any relevant paths in the tree, and give all counterexamples if applicable.

Not a tautology since $\neg X$ is not a contradiction. Indeed, $\neg X$ is not a contradiction since it has at least one active path.

Q8. If a is a rational number and r is an irrational number and consider the following statement: **[6 points]**

If a is a rational and r is an irrational number then $x = a + r$ is an irrational number .

Recall: A rational number is a real number that can be expressed as an integer divided by an integer. In other words, x is a rational number if $x = \frac{p}{q}$, where p and q are integers. A real number is irrational if it is not a rational number. In other words, y is an irrational number then y can not be expressed as a fraction $y = \frac{p}{q}$, where p and q are integers.

(a) Write the **converse** of this statement (in English):

If $x = a + r$ is an irrational number, then a is a rational and r is an irrational number.

(b) Write the **contrapositive** of this statement (in English):

If $x = a + r$ is not an irrational number, then a is not a rational or r is not an irrational number.

(c) Give an **Indirect Proof** of the following theorem:

Theorem 1. Let a is a rational and r is an irrational number.

If a is a rational and r is an irrational number then $x = a + r$ is an irrational number .

IMPORTANT! *In each step of your proof, you must clearly indicate whether you are assuming something, or whether what you wrote is something that follows from a definition or a previous step of your proof. If any variables appear in your proof, make sure you clearly write what they represent.*

Indirect proof of Theorem 1. We use contrapositive:

Let $a = \frac{p}{q}$, where p and q are integers.

Assume $\neg Q$ is True. $x = a + r$ is a rational number.

then x can be expressed as a fraction $x = \frac{m}{n}$.

now $x = a + r = \frac{p}{q} + r = \frac{m}{n}$. Hence

$r = \frac{m}{n} - \frac{p}{q} = \frac{mq - np}{nq}$. Since $mq - np$ and nq are integers, r is a rational number, $\neg P$ is True.. this is a contradiction. $\therefore P \to Q$ is True.