

**MAT1348 Assignment 1**

due: Thursday, January 28 before 11:30 pm

## INSTRUCTIONS

- You must upload your assignment on Brightspace on or before **Thursday, January 28 by 11:30 pm. Late assignments will not be accepted.** Make sure you submit your whole assignment without missing any part.
- Please print this document, including the cover page, and write your answers in the space provided. You may write on the backs of pages or insert extra pages if necessary, so long as the assignment is clearly organized, with solutions in the same order as the questions, and **scan whole including the cover page. Be sure without missing any part.**
- You must hand in a legible, organized and properly stapled assignment. If it is too difficult to read your solutions, then you may get zero.
- Solutions must include all relevant steps and justifications where appropriate. If you only write the final answer without explanation, then you may not receive full marks.
- The maximum points possible = 20 points.
- **Please just upload a PDF file including your whole work, otherwise your assignment may not be marked completely.**

1.

FAMILY NAME:

STUDENT NUMBER:

FIRST NAME:

†SIGNATURE:

**Q1-a** Let A, H, J, N, R, S, U denote the following propositional variables: [6 points]

A : “ The shark attacks Nancy”                      H: “ The shark is hungry ”  
J: “ Jack is a Knight.”                                      N: “ Nancy is a Knight.”  
R: “ Jack rescues Nancy.”                                S: “ Nancy goes swimming around the island”  
U: “ Swimming around the island is unsafe.”

Translate each of the following sentences into compound propositions using the above propositional variables, logical connectives, and parentheses when appropriate. *Parentheses have been added to some sentences for clarification; otherwise, follow the conventions of precedence of logical connectives.* Assume that Jack and Nancy are inhabitants of the Island of Knights & Knave.

- (a): A sufficient condition for swimming around the island to be safe is that the shark is not hungry.
- (b): (If the shark attacks Nancy, then Jack rescues Nancy) unless Nancy is a knave.
- (c): The shark attacks Nancy only if the shark is hungry and Nancy is a Knave.
- (d): Either Jack is a knight or the shark does not attack Nancy.
- (e): A necessary condition for Jack to rescue Nancy is that (Nancy goes swimming around the island whenever swimming around the island is unsafe)
- (f): Whenever the shark is hungry, Nancy goes swimming around the island, but not conversely.

**Bonus** Suppose that all six of the propositions (a)-(f) are true. What ( if any thing ) can you conclude ? Circle the best answer for each questions. You do not need to show any justification for this, but you will only earn the bonus points if all of your answers are correct.[2 points]

Does the shark attack Nancy?	Circle:	Yes	No	unable to determine.
Is the shark hungry?	Circle:	Yes	No	unable to determine.
Is Jack a knight?	Circle:	Yes	No	unable to determine.
Is Nancy a knight?	Circle:	Yes	No	unable to determine.
Does Nancy go swimming around the island ?	Circle:	Yes	No	unable to determine.

**Q2.** For this question, you will prove in two ways that the propositions  $X$  and  $Y$ , given below, are **logically equivalent**.

$$X : (a \wedge \neg b) \vee (\neg c \wedge \neg d) \qquad Y : (a \rightarrow b) \rightarrow \neg(c \vee d)$$

- (a) [3 points] Use an appropriate **truth table** to verify that  $X \equiv Y$ . Make sure you briefly explain how your truth table demonstrates that  $X \equiv Y$ .

**(Q2 continued)**

(b) [3 points] Use the Laws from the Table of Logical Equivalences to prove that  $X \equiv Y$ .

You may use **one (and only one!)** law at each step, and you must write the name of the law used at each step.

(c) [1 point] Write a disjunctive normal form (DNF) for  $X$ . You do not need to justify your answer for this part.

**Q3.** [7 points] Let  $P$  be the following compound proposition consisting of atoms  $A$ ,  $B$ , and  $C$ :

$$P : ((A \vee B) \wedge C) \rightarrow (A \wedge C)$$

For this question, you will grow two truth trees. For each tree, you must use the *official* branching rules and apply the branching rules to the propositions as they are written (i.e. do not use logical equivalences to change the propositions in your tree – stick to the official branching rules, seen in class). Clearly label each path as active(alive) or inactive (dead).

**Tree 1. Complete truth tree with root  $P$**

**Tree 2. Complete truth tree with root  $\neg P$**

**Q3 continued.**

**3a.** Is  $P$  a tautology?

*Circle:*    **YES**            **NO**

Briefly explain. Your explanation must make reference to Tree 1 or Tree 2 explicitly, its root, and any relevant paths in the tree.

**3b.** Is  $P$  a contradiction?

*Circle:*    **YES**            **NO**

Briefly explain. Your explanation must make reference to Tree 1 or Tree 2 explicitly, its root, and any relevant paths in the tree.

**3c.** Based on one of your two trees, give a disjunctive normal form for  $P$ .

**DNF for  $P$ :**

Which tree did you use to find your DNF for  $P$ ?

*Circle:*    **Tree 1**            **Tree 2**