

WEEK 1

7.1 VECTORS IN 2-SPACE

In Problems 1–8, find (a) $3\mathbf{a}$, (b) $\mathbf{a} + \mathbf{b}$, (c) $\mathbf{a} - \mathbf{b}$, (d) $\|\mathbf{a} + \mathbf{b}\|$, and (e) $\|\mathbf{a} - \mathbf{b}\|$.

1. $\mathbf{a} = 2\mathbf{i} + 4\mathbf{j}$, $\mathbf{b} = -\mathbf{i} + 4\mathbf{j}$

21. Determine which of the following vectors are parallel to $\mathbf{a} = 4\mathbf{i} + 6\mathbf{j}$.

(a) $-4\mathbf{i} - 6\mathbf{j}$

(b) $-\mathbf{i} - \frac{3}{2}\mathbf{j}$

(c) $10\mathbf{i} + 15\mathbf{j}$

(d) $2(\mathbf{i} - \mathbf{j}) - 3(\frac{1}{2}\mathbf{i} - \frac{5}{12}\mathbf{j})$

(e) $8\mathbf{i} + 12\mathbf{j}$

(f) $(5\mathbf{i} + \mathbf{j}) - (7\mathbf{i} + 4\mathbf{j})$

In Problems 29 and 30, $\mathbf{a} = \langle 2, 8 \rangle$ and $\mathbf{b} = \langle 3, 4 \rangle$. Find a unit vector in the same direction as the given vector.

30. $2\mathbf{a} - 3\mathbf{b}$

In Problems 41 and 42, express the vector $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ as a linear combination of the given vectors \mathbf{b} and \mathbf{c} .

41. $\mathbf{b} = \mathbf{i} + \mathbf{j}$, $\mathbf{c} = \mathbf{i} - \mathbf{j}$

50. An airplane starts from an airport located at the origin O and flies 150 miles in the direction 20° north of east to city A . From A , the airplane then flies 200 miles in the direction 23° west of north to city B . From B , the airplane flies 240 miles in the direction 10° south of west to city C . Express the location of city C as a vector \mathbf{r} as shown in **FIGURE 7.1.21**. Find the distance from O to C .

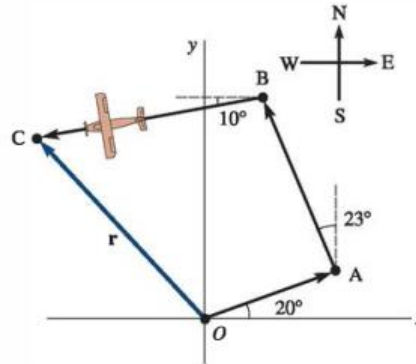


FIGURE 7.1.21 Airplane in Problem 50

7.2 VECTORS IN 3-SPACE

11. Give the coordinates of the vertices of the rectangular parallelepiped whose sides are the coordinate planes and the planes $x = 2$, $y = 5$, $z = 8$.

24. Find the distance from the point $(-6, 2, -3)$ to (a) the xz -plane and (b) the origin.

In Problems 31 and 32, solve for the unknown.

32. $P_1(x, x, 1)$, $P_2(0, 3, 5)$; $d(P_1, P_2) = 5$

In Problems 33 and 34, find the coordinates of the midpoint of the line segment between the given points.

34. $(0, 5, -8)$, $(4, 1, -6)$

52. Find a vector \mathbf{b} for which $\|\mathbf{b}\| = \frac{1}{2}$ that is parallel to $\mathbf{a} = \langle -6, 3, -2 \rangle$ but has the opposite direction.

7.3 DOT PRODUCT

In Problems 1–12, $\mathbf{a} = \langle 2, -3, 4 \rangle$, $\mathbf{b} = \langle -1, 2, 5 \rangle$, and $\mathbf{c} = \langle 3, 6, -1 \rangle$. Find the indicated scalar or vector.

12. $(\mathbf{c} \cdot \mathbf{b})\mathbf{a}$

15. Determine which pairs of the following vectors are orthogonal:

(a) $\langle 2, 0, 1 \rangle$

(b) $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

(c) $2\mathbf{i} - \mathbf{j} - \mathbf{k}$

(d) $\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$

(e) $\langle 1, -1, 1 \rangle$

(f) $\langle -4, 3, 8 \rangle$

In Problems 21–24, find the angle θ between the given vectors.

23. $\mathbf{a} = \langle 2, 4, 0 \rangle$, $\mathbf{b} = \langle -1, -1, 4 \rangle$

29. Find the angle between the diagonal \overrightarrow{AD} of the cube shown in **FIGURE 7.3.9** and the edge AB . Find the angle between the diagonal \overrightarrow{AD} of the cube and the diagonal \overrightarrow{AC} .

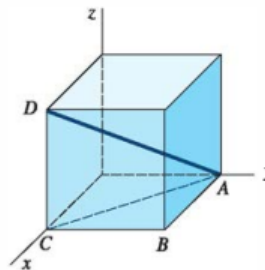


FIGURE 7.3.9 Diagonal in Problem 29

31. An airplane is 4 km high, 5 km south, and 7 km east of an airport. See **FIGURE 7.3.10**. Find the direction angles of the plane.

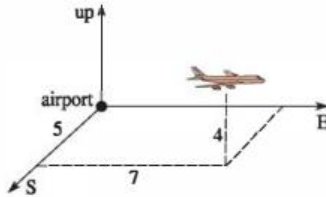


FIGURE 7.3.10 Airplane in Problem 31

In Problems 39–42, find $\text{proj}_{\mathbf{b}}\mathbf{a}$.

41. $\mathbf{a} = -\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$, $\mathbf{b} = 6\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$

48. A constant force \mathbf{F} of magnitude 3 lb is applied to the block shown in **FIGURE 7.3.12**. \mathbf{F} has the same direction as the vector $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$. Find the work done in the direction of motion if the block moves from $P_1(3, 1)$ to $P_2(9, 3)$. Assume distance is measured in feet.

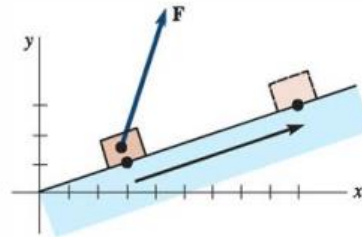
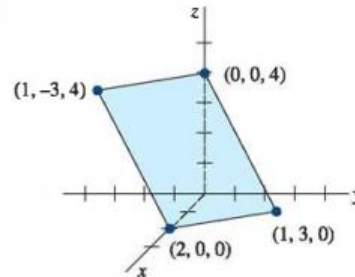


FIGURE 7.3.12 Block in Problem 48

7.4 CROSS PRODUCT

In Problems 45 and 46, (a) verify that the given quadrilateral is a parallelogram, and (b) find the area of the parallelogram.

45.



In Problems 47–50, find the area of the triangle determined by the given points.

49. $P_1(1, 2, 4)$, $P_2(1, -1, 3)$, $P_3(-1, -1, 2)$

In Problems 51 and 52, find the volume of the parallelepiped for which the given vectors are three edges.

52. $\mathbf{a} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 4\mathbf{j} + \mathbf{k}$, $\mathbf{c} = \mathbf{i} + \mathbf{j} + 5\mathbf{k}$

WEEK 2

7.5 LINES AND PLANES IN SPACE

In Problems 1–6, find a vector equation for the line through the given points.

5. $(1, 1, -1)$, $(-4, 1, -1)$

In Problems 7–12, find parametric equations for the line through the given points.

12. $(-3, 7, 9)$, $(4, -8, -1)$

In Problems 13–18, find symmetric equations for the line through the given points.

17. $(5, 10, -2)$, $(5, 1, -14)$

24. Find symmetric equations for the line through $(4, -11, -7)$ that is parallel to the line $x = 2 + 5t$, $y = -1 + \frac{1}{3}t$, $z = 9 - 2t$.

In Problems 31–34, determine whether the given lines intersect. If so, find the point of intersection.

33. $x = 2 - t$, $y = 3 + t$, $z = 1 + t$
 $x = 4 + s$, $y = 1 + s$, $z = 1 - s$

The angle between two lines \mathcal{L}_a and \mathcal{L}_b is the angle between their direction vectors \mathbf{a} and \mathbf{b} . In Problems 35 and 36, find the angle between the given lines.

36. $\frac{x-1}{2} = \frac{y+5}{7} = \frac{z-1}{-1}$; $\frac{x+3}{-2} = y-9 = \frac{z}{4}$

In Problems 39–44, find an equation of the plane that contains the given point and is perpendicular to the indicated vector.

39. $(5, 1, 3)$; $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$

In Problems 45–50, find, if possible, an equation of a plane that contains the given points.

49. $(1, 2, -1)$, $(4, 3, 1)$, $(7, 4, 3)$

In Problems 51–60, find an equation of the plane that satisfies the given conditions.

57. Contains the parallel lines $x = 1 + t$, $y = 1 + 2t$, $z = 3 + t$;
 $x = 3 + s$, $y = 2s$, $z = -2 + s$

61. Let \mathcal{P}_1 and \mathcal{P}_2 be planes with normal vectors \mathbf{n}_1 and \mathbf{n}_2 , respectively. \mathcal{P}_1 and \mathcal{P}_2 are orthogonal if \mathbf{n}_1 and \mathbf{n}_2 are orthogonal and parallel if \mathbf{n}_1 and \mathbf{n}_2 are parallel. Determine which of the following planes are orthogonal and which are parallel.

- (a) $2x - y + 3z = 1$ (b) $x + 2y + 2z = 9$
(c) $x + y - \frac{3}{2}z = 2$ (d) $-5x + 2y + 4z = 0$
(e) $-8x - 8y + 12z = 1$ (f) $-2x + y - 3z = 5$

In Problems 65–68, find parametric equations for the line of intersection of the given planes.

66. $x + 2y - z = 2$
 $3x - y + 2z = 1$

In Problems 75 and 76, find an equation of the plane that contains the given line and is orthogonal to the indicated plane.

75. $x = 4 + 3t$, $y = -t$, $z = 1 + 5t$; $x + y + z = 7$

9.1 VECTOR FUNCTIONS

In Problems 1–10, graph the curve traced by the given vector function.

1. $\mathbf{r}(t) = 2 \sin t \mathbf{i} + 4 \cos t \mathbf{j} + t \mathbf{k}$; $t \geq 0$
4. $\mathbf{r}(t) = 4\mathbf{i} + 2 \cos t \mathbf{j} + 3 \sin t \mathbf{k}$
10. $\mathbf{r}(t) = \langle t \cos t, t \sin t, t^2 \rangle$

In Problems 17–20, find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$ for the given vector function.

18. $\mathbf{r}(t) = \langle t \cos t - \sin t, t + \cos t \rangle$

In Problems 25 and 26, find parametric equations of the tangent line to the given curve at the indicated value of t .

25. $x = t$, $y = \frac{1}{2}t^2$, $z = \frac{1}{3}t^3$; $t = 2$

In Problems 33–36, evaluate the given integral.

34. $\int_0^4 (\sqrt{2t+1} \mathbf{i} - \sqrt{t} \mathbf{j} + \sin \pi t \mathbf{k}) dt$

36. $\int \frac{1}{1+t^2} (\mathbf{i} + t \mathbf{j} + t^2 \mathbf{k}) dt$

In Problems 37–40, find a vector function \mathbf{r} that satisfies the indicated conditions.

39. $\mathbf{r}''(t) = 12t \mathbf{i} - 3t^{-1/2} \mathbf{j} + 2 \mathbf{k}$; $\mathbf{r}'(1) = \mathbf{j}$, $\mathbf{r}(1) = 2\mathbf{i} - \mathbf{k}$

In Problems 41–44, find the length of the curve traced by the given vector function on the indicated interval.

42. $\mathbf{r}(t) = t \mathbf{i} + t \cos t \mathbf{j} + t \sin t \mathbf{k}$; $0 \leq t \leq \pi$

45. Express the vector equation of a circle $\mathbf{r}(t) = a \cos t \mathbf{i} + a \sin t \mathbf{j}$ as a function of arc length s . Verify that $\mathbf{r}'(s)$ is a unit vector.

WEEK 3

9.2 MOTION ON A CURVE

In Problems 1–8, $\mathbf{r}(t)$ is the position vector of a moving particle. Graph the curve and the velocity and acceleration vectors at the indicated time. Find the speed at that time.

4. $\mathbf{r}(t) = 2 \cos t \mathbf{i} + (1 + \sin t) \mathbf{j}$; $t = \pi/3$
9. Suppose $\mathbf{r}(t) = t^2 \mathbf{i} + (t^3 - 2t) \mathbf{j} + (t^2 - 5t) \mathbf{k}$ is the position vector of a moving particle. At what points does the particle
11. A shell is fired from ground level with an initial speed of 480 ft/s at an angle of elevation of 30° . Find:
(a) a vector function and parametric equations of the shell's trajectory,
(b) the maximum altitude attained,
(c) the range of the shell, and
(d) the speed at impact.
13. A used car is pushed off an 81-ft-high sheer seaside cliff with a speed of 4 ft/s. Find the speed at which the car hits the water.
14. A small projectile is launched from ground level with an initial speed of 98 m/s. Find the possible angles of elevation so that its range is 490 m.

19. If a projectile is launched from level ground, the initial conditions are $\mathbf{r}(0) = \mathbf{0}$, $\mathbf{r}'(0) = \mathbf{v}_0 = (v_0 \cos \theta) \mathbf{i} + (v_0 \sin \theta) \mathbf{j}$, and if linear air resistance is taken into consideration, the vector function analogue of (1) is

$$\mathbf{r}(t) = \frac{mv_0 \cos \theta}{\beta} (1 - e^{-\beta t/m}) \mathbf{i} + \left[\left(\frac{mv_0 \sin \theta}{\beta} + \frac{m^2 g}{\beta^2} \right) (1 - e^{-\beta t/m}) - \frac{mg}{\beta} t \right] \mathbf{j},$$

This function is equivalent to the solution $x(t)$, $y(t)$ of the system of linear differential equations (11) in Problem 22 of Exercises 4.6 subject to the same initial conditions. Here $\beta > 0$ is a constant of proportionality related to the air resistance or drag. If $m = \frac{1}{4}$ slug, $g = 32$ ft/s², $\beta = 0.02$, $v_0 = 300$ ft/s, and $\theta = 38^\circ$, use a calculator or CAS to find the impact speed of the projectile. See part (b) of Problem 22 in Exercises 4.6.

22. In army field maneuvers sturdy equipment and supply packs are simply dropped from planes that fly horizontally at a slow speed and a low altitude. A supply plane flies horizontally over a target at an altitude of 1024 ft at a constant speed of 180 mi/h. Use (1) to determine the horizontal distance a supply pack travels relative to the point from which it was dropped. At what line-of-sight angle α should the supply pack be released in order to hit the target indicated in FIGURE 9.2.7?

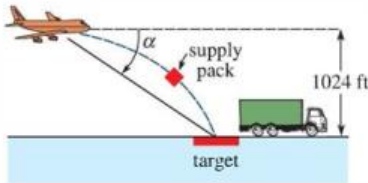


FIGURE 9.2.7 Supply plane in Problem 22

27. The velocity of a particle moving in a fluid is described by means of a **velocity field** $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$, where the components v_1 , v_2 , and v_3 are functions of x , y , z , and time t . If the velocity of the particle is $\mathbf{v}(t) = 6t^2x\mathbf{i} - 4ty^2\mathbf{j} + 2t(z+1)\mathbf{k}$, find $\mathbf{r}(t)$. [Hint: Use separation of variables.]
28. Suppose m is the mass of a moving particle. Newton's second law of motion can be written in vector form as

$$\mathbf{F} = m\mathbf{a} = \frac{d}{dt}(m\mathbf{v}) = \frac{d\mathbf{p}}{dt},$$

where $\mathbf{p} = m\mathbf{v}$ is called **linear momentum**. The angular momentum of the particle with respect to the origin is defined to be $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, where \mathbf{r} is its position vector. If the torque of the particle about the origin is $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times d\mathbf{p}/dt$, show that $\boldsymbol{\tau}$ is the time rate of change of angular momentum.

9.3 CURVATURE. COMPONENTS OF ACCELERATION

In Problems 1 and 2, for the given position function, find the unit tangent.

1. $\mathbf{r}(t) = (t \cos t - \sin t)\mathbf{i} + (t \sin t + \cos t)\mathbf{j} + t^2\mathbf{k}$, $t > 0$

In Problems 5 and 6, find an equation of the osculating plane to the given space curve at the point that corresponds to the indicated value of t .

6. The twisted cubic of Example 4; $t = 1$

In Problems 7–16, $\mathbf{r}(t)$ is the position vector of a moving particle. Find the tangential and normal components of the acceleration at any t .

9. $\mathbf{r}(t) = t^2\mathbf{i} + (t^2 - 1)\mathbf{j} + 2t^2\mathbf{k}$

16. $\mathbf{r}(t) = t\mathbf{i} + (2t - 1)\mathbf{j} + (4t + 2)\mathbf{k}$

17. Find the curvature of an elliptical helix that is described by $\mathbf{r}(t) = a \cos t\mathbf{i} + b \sin t\mathbf{j} + ct\mathbf{k}$, $a > 0$, $b > 0$, $c > 0$.

29. Suppose the Sun is located at the origin. The gravitational force \mathbf{F} exerted on a planet of mass m by the Sun of mass M is

$$\mathbf{F} = -k \frac{Mm}{r^2} \mathbf{u}.$$

\mathbf{F} is a **central force**—that is, a force directed along the position vector \mathbf{r} of the planet. Here k is the gravitational constant, $r = \|\mathbf{r}\|$, $\mathbf{u} = \mathbf{r}/r$ is a unit vector in the direction of \mathbf{r} , and the minus sign indicates that \mathbf{F} is an attractive force—that is, a force directed toward the Sun. See FIGURE 9.2.9.

- (a) Use Problem 28 to show that the torque acting on the planet due to this central force is $\mathbf{0}$.
- (b) Explain why the angular momentum \mathbf{L} of a planet is constant.

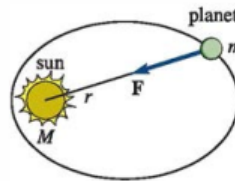


FIGURE 9.2.9 Force \mathbf{F} in Problem 29

20. Find the curvature at $t = \pi$ of the cycloid that is described by

$$\mathbf{r}(t) = a(t - \sin t)\mathbf{i} + a(1 - \cos t)\mathbf{j}, \quad a > 0.$$

In Problems 23 and 24, use the result of Problem 22 to find the curvature and radius of curvature of the curve at the indicated points. Decide at which point the curve is “sharper.”

23. $y = x^2$; $(0, 0)$, $(1, 1)$

WEEK 4

9.4 PARTIAL DERIVATIVES

In Problems 1–6, sketch some of the level curves associated with the given function.

2. $f(x, y) = y^2 - x$

3. $f(x, y) = \sqrt{x^2 - y^2 - 1}$

In Problems 7–10, describe the level surfaces but do not graph.

9. $F(x, y, z) = x^2 + 3y^2 + 6z^2$

In Problems 13–32, find the first partial derivatives of the given function.

15. $z = 5x^4y^3 - x^2y^6 + 6x^5 - 4y$

21. $z = \cos^2 5x + \sin^2 5y$

24. $f(\theta, \phi) = \phi^2 \sin \frac{\theta}{\phi}$

26. $f(x, y) = \frac{xy}{(x^2 - y^2)^2}$

27. $g(u, v) = \ln(4u^2 + 5v^3)$

In Problems 35 and 36 verify that the given function satisfies the wave equation:

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$

36. $u = \cos(x + at) + \sin(x - at)$

In Problems 39–48, use the Chain Rule to find the indicated partial derivatives.

39. $z = e^{uv^2}; u = x^3, v = x - y^2; \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$

42. $z = \frac{x - y}{x + y}; x = \frac{u}{v}, y = \frac{v^2}{u}; \frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$

48. $s = p^2 + q^2 - r^2 + 4t; p = \phi e^{3\theta}, q = \cos(\phi + \theta), r = \phi\theta^2, t = 2\phi + 8\theta; \frac{\partial s}{\partial \phi}, \frac{\partial s}{\partial \theta}$

In Problems 49–52, use (8) to find the indicated derivative.

49. $z = \ln(u^2 + v^2); u = t^2, v = t^{-2}; \frac{dz}{dt}$

51. $w = \cos(3u + 4v); u = 2t + \frac{\pi}{2}, v = -t - \frac{\pi}{4}; \frac{dw}{dt} \Big|_{t=\pi}$

If dT/dt and dV/dt are rates at which the temperature and volume change, respectively, use the Chain Rule to find dP/dt .

55. The equation of state for a thermodynamic system is $F(P, V, T) = 0$, where P, V , and T are pressure, volume, and temperature, respectively. If the equation defines V as a function of P and T , and also defines T as a function of V and P , show that

$$\frac{\partial V}{\partial T} = -\frac{\frac{\partial F}{\partial T}}{\frac{\partial F}{\partial V}} = -\frac{1}{\frac{\partial T}{\partial V}}$$

56. The voltage across a conductor is increasing at a rate of 2 volts/min and the resistance is decreasing at a rate of 1 ohm/min. Use $I = E/R$ and the Chain Rule to find the rate at which the current passing through the conductor is changing when $R = 50$ ohms and $E = 60$ volts.

57. The length of the side labeled x of the triangle in **FIGURE 9.4.8** increases at a rate of 0.3 cm/s, the side labeled y increases at a rate of 0.5 cm/s, and the included angle θ increases at a rate of 0.1 rad/s. Use the Chain Rule to find the rate at which the area of the triangle is changing at the instant $x = 10$ cm, $y = 8$ cm, and $\theta = \pi/6$.

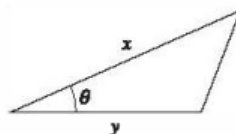


FIGURE 9.4.8 Triangle in Problem 57

9.5 DIRECTIONAL DERIVATIVE

In Problems 1–4, compute the gradient for the given function.

3. $F(x, y, z) = \frac{xy^2}{z^3}$

In Problems 5–8, find the gradient of the given function at the indicated point.

6. $f(x, y) = \sqrt{x^3y - y^4}; (3, 2)$

In Problems 11–20, find the directional derivative of the given function at the given point in the indicated direction.

12. $f(x, y) = 4x + xy^2 - 5y; (3, -1), \theta = \pi/4$

14. $f(x, y) = \frac{xy}{x + y}; (2, -1), 6\mathbf{i} + 8\mathbf{j}$

15. $f(x, y) = (xy + 1)^2; (3, 2)$, in the direction of $(5, 3)$

18. $F(x, y, z) = \frac{x^2 - y^2}{z^2}; (2, 4, -1), \mathbf{i} - 2\mathbf{j} + \mathbf{k}$

In Problems 23–26, find a vector that gives the direction in which the given function increases most rapidly at the indicated point. Find the maximum rate.

24. $f(x, y) = xy e^{x-y}; (5, 5)$

In Problems 27–30, find a vector that gives the direction in which the given function decreases most rapidly at the indicated point. Find the minimum rate.

27. $f(x, y) = \tan(x^2 + y^2); (\sqrt{\pi/6}, \sqrt{\pi/6})$

28. $f(x, y) = x^3 - y^3; (2, -2)$

33. Suppose $\nabla f(a, b) = 4\mathbf{i} + 3\mathbf{j}$. Find a unit vector \mathbf{u} so that
 (a) $D_{\mathbf{u}}f(a, b) = 0$,
 (b) $D_{\mathbf{u}}f(a, b)$ is a maximum, and
 (c) $D_{\mathbf{u}}f(a, b)$ is a minimum.

41. The temperature at a point (x, y) on a rectangular metal plate is given by $T(x, y) = 100 - 2x^2 - y^2$. Find the path a heat-seeking particle will take, starting at $(3, 4)$, as it moves in the direction in which the temperature increases most rapidly.

43. Find a function f such that

$$\nabla f = (3x^2 + y^3 + ye^{xy})\mathbf{i} + (-2y^2 + 3xy^2 + xe^{xy})\mathbf{j}$$

44. Let f_x, f_y, f_{xy}, f_{yx} be continuous and \mathbf{u} and \mathbf{v} be unit vectors. Show that $D_{\mathbf{u}}D_{\mathbf{v}}f = D_{\mathbf{v}}D_{\mathbf{u}}f$.

WEEK 5

9.6 TANGENT PLANES AND NORMAL LINES

In Problems 1–12, sketch the level curve or surface passing through the indicated point. Sketch the gradient at the point.

3. $f(x, y) = y - x^2$; (2, 5) 4. $f(x, y) = x^2 + y^2$; (-1, 3)

In Problems 13 and 14, find the points on the given surface at which the gradient is parallel to the indicated vector.

14. $x^3 + y^2 + z = 15$; $27\mathbf{i} + 8\mathbf{j} + \mathbf{k}$

In Problems 15–24, find an equation of the tangent plane to the graph of the given equation at the indicated point.

15. $x^2 + y^2 + z^2 = 9$; (-2, 2, 1)

In Problems 25 and 26, find the points on the given surface at which the tangent plane is parallel to the indicated plane.

25. $x^2 + y^2 + z^2 = 7$; $2x + 4y + 6z = 1$

In Problems 33 and 34, find parametric equations for the normal line at the indicated point. In Problems 35 and 36, find symmetric equations for the normal line.

34. $z = 2x^2 - 4y^2$; (3, -2, 2)

38. Two surfaces are said to be **orthogonal** at a point P of intersection if their normal lines at P are orthogonal. Prove that the surfaces given by $F(x, y, z) = 0$ and $G(x, y, z) = 0$ are orthogonal at P if and only if $F_x G_x + F_y G_y + F_z G_z = 0$.

In Problems 39 and 40, use the result of Problem 38 to show that the given surfaces are orthogonal at a point of intersection.

39. $x^2 + y^2 + z^2 = 25$; $-x^2 + y^2 + z^2 = 0$

9.7 CURL AND DIVERGENCE: 7,11,15,21,24,27,30,39,40,43,44

In Problems 7–16, find the curl and the divergence of the given vector field.

7. $\mathbf{F}(x, y, z) = xz\mathbf{i} + yz\mathbf{j} + xy\mathbf{k}$

11. $\mathbf{F}(x, y, z) = 3x^2y\mathbf{i} + 2xz^3\mathbf{j} + y^4\mathbf{k}$

15. $\mathbf{F}(x, y, z) = xye^x\mathbf{i} - x^3yz^2\mathbf{j} + xy^2e^y\mathbf{k}$

In Problems 17–24, let \mathbf{a} be a constant vector and $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Verify the given identity.

21. $\nabla \cdot (\mathbf{a} \times \mathbf{r}) = 0$

24. $\nabla \cdot [(\mathbf{r} \cdot \mathbf{r})\mathbf{a}] = 2(\mathbf{r} \cdot \mathbf{a})$

In Problems 25–32, verify the given identity. Assume continuity of all partial derivatives.

27. $\nabla \cdot (f\mathbf{F}) = f(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla f$

30. $\text{div}(\text{curl } \mathbf{F}) = 0$

39. Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ be the position vector of a mass m_1 and let the mass m_2 be located at the origin. If the force of gravitational attraction is

$$\mathbf{F} = -\frac{Gm_1m_2}{\|\mathbf{r}\|^3} \mathbf{r},$$

verify that $\text{curl } \mathbf{F} = \mathbf{0}$ and $\text{div } \mathbf{F} = 0$, $\mathbf{r} \neq \mathbf{0}$.

40. Suppose a body rotates with a constant angular velocity $\boldsymbol{\omega}$ about an axis. If \mathbf{r} is the position vector of a point P on the body measured from the origin, then the linear velocity vector \mathbf{v} of rotation is $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$. See **FIGURE 9.7.8**. If $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $\boldsymbol{\omega} = \omega_1\mathbf{i} + \omega_2\mathbf{j} + \omega_3\mathbf{k}$, show that $\boldsymbol{\omega} = \frac{1}{2} \text{curl } \mathbf{v}$.

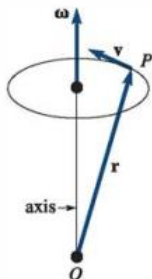


FIGURE 9.7.8 Rotating body in Problem 40

43. The velocity vector field for the two-dimensional flow of an ideal fluid around a cylinder is given by

$$\mathbf{F}(x, y) = A \left[\left(1 - \frac{x^2 - y^2}{(x^2 + y^2)^2} \right) \mathbf{i} - \frac{2xy}{(x^2 + y^2)^2} \mathbf{j} \right]$$

for some positive constant A . See **FIGURE 9.7.9**.

- (a) Show that when the point (x, y) is far from the origin, $\mathbf{F}(x, y) \approx A\mathbf{i}$.
 (b) Show that \mathbf{F} is irrotational.
 (c) Show that \mathbf{F} is incompressible.

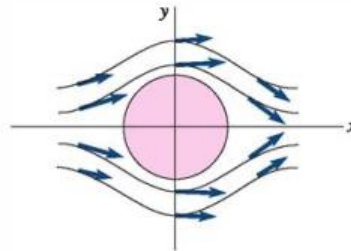


FIGURE 9.7.9 Vector field in Problem 43

44. If $\mathbf{E} = \mathbf{E}(x, y, z, t)$ and $\mathbf{H} = \mathbf{H}(x, y, z, t)$ represent electric and magnetic fields in empty space, then Maxwell's equations are

$$\text{div } \mathbf{E} = 0, \quad \text{curl } \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t},$$

$$\text{div } \mathbf{H} = 0, \quad \text{curl } \mathbf{H} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t},$$

where c is the speed of light. Use the identity in Problem 36(a) to show that \mathbf{E} and \mathbf{H} satisfy

$$\nabla^2 \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}, \quad \nabla^2 \mathbf{H} = \frac{1}{c^2} \frac{\partial^2 \mathbf{H}}{\partial t^2}.$$

ANSWERS
WEEK 1
7.1 VECTORS IN 2-SPACE

1. (a) $6\mathbf{i} + 12\mathbf{j}$

21. $a(= -\mathbf{a})$, $b(= -\frac{1}{4}\mathbf{a})$, $c(= \frac{5}{2}\mathbf{a})$, $e(= 2\mathbf{a})$, and $f(= -\frac{1}{2}\mathbf{a})$ are parallel to \mathbf{a} .

30. $\|2\mathbf{a} - 3\mathbf{b}\| = \|\langle -5, 4 \rangle\| = \sqrt{25 + 16} = \sqrt{41}$; $\mathbf{u} = \frac{1}{\sqrt{41}}\langle -5, 4 \rangle = \langle -\frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}} \rangle$

41. From $2\mathbf{i} + 3\mathbf{j} = k_1\mathbf{b} + k_2\mathbf{c} = k_1(\mathbf{i} + \mathbf{j}) + k_2(\mathbf{i} - \mathbf{j}) = (k_1 + k_2)\mathbf{i} + (k_1 - k_2)\mathbf{j}$ we obtain the system of equations $k_1 + k_2 = 2$, $k_1 - k_2 = 3$. Solving, we find $k_1 = \frac{5}{2}$ and $k_2 = -\frac{1}{2}$. Then $\mathbf{a} = \frac{5}{2}\mathbf{b} - \frac{1}{2}\mathbf{c}$.

50. We have $\vec{OA} = 150 \cos 20^\circ \mathbf{i} + 150 \sin 20^\circ \mathbf{j}$, $\vec{AB} = 200 \cos 113^\circ \mathbf{i} + 200 \sin 113^\circ \mathbf{j}$, $\vec{BC} = 240 \cos 190^\circ \mathbf{i} + 240 \sin 190^\circ \mathbf{j}$.
Then

$$\begin{aligned} \mathbf{r} &= (150 \cos 20^\circ + 200 \cos 113^\circ + 240 \cos 190^\circ)\mathbf{i} + (150 \sin 20^\circ + 200 \sin 113^\circ + 240 \sin 190^\circ)\mathbf{j} \\ &\approx -173.55\mathbf{i} + 193.73\mathbf{j} \end{aligned}$$

and $\|\mathbf{r}\| \approx 260.09$ miles.

7.2 VECTORS IN 3-SPACE

11. $(2, 0, 0)$, $(2, 5, 0)$, $(2, 0, 8)$, $(2, 5, 8)$, $(0, 5, 0)$, $(0, 5, 8)$, $(0, 0, 8)$, $(0, 0, 0)$

24. (a) 2; (b) $d = \sqrt{(-6)^2 + 2^2 + (-3)^2} = 7$

32. $\sqrt{(0-x)^2 + (3-x)^2 + (5-1)^2} = 5 \implies 2x^2 - 6x + 25 = 25 \implies x^2 - 3x = 0 \implies x = 0, 3$

34. $\left(\frac{0+4}{2}, \frac{5+1}{2}, \frac{-8+(-6)}{2}\right) = (2, 3, -7)$

52. $\|\mathbf{a}\| = \sqrt{36 + 9 + 4} = 7$; $\mathbf{b} = -\frac{1}{2}\left(\frac{1}{7}\right)\langle -6, 3, -2 \rangle = \langle \frac{3}{7}, -\frac{3}{14}, \frac{1}{7} \rangle$

7.3 DOT PRODUCT

12. $(2\mathbf{a}) \cdot (\mathbf{a} - 2\mathbf{b}) = 4(4) + (-6)(-7) + 8(-6) = 10$

15. \mathbf{a} and \mathbf{f} , \mathbf{b} and \mathbf{e} , \mathbf{c} and \mathbf{d}

23. $\mathbf{a} \cdot \mathbf{b} = 2(-1) + 4(-1) + 0(4) = -6$; $\|\mathbf{a}\| = 2\sqrt{5}$, $\|\mathbf{b}\| = 3\sqrt{2}$

$$\cos \theta = \frac{-6}{(2\sqrt{5})(3\sqrt{2})} = -\frac{1}{\sqrt{10}} \implies \theta = \cos^{-1}(-1/\sqrt{10}) \approx 1.89 \text{ rad} \approx 108.43^\circ$$

29. Let θ be the angle between \vec{AD} and \vec{AB} and a be the length of an edge of the cube. Then $\vec{AD} = a\mathbf{i} + a\mathbf{j} + a\mathbf{k}$, $\vec{AB} = a\mathbf{i}$ and

$$\cos \theta = \frac{\vec{AD} \cdot \vec{AB}}{\|\vec{AD}\| \|\vec{AB}\|} = \frac{a^2}{\sqrt{3a^2} \sqrt{a^2}} = \frac{1}{\sqrt{3}}$$

so $\theta \approx 0.955317$ radian or 54.7356° . Letting ϕ be the angle between \vec{AD} and \vec{AC} and noting that $\vec{AC} = a\mathbf{i} + a\mathbf{j}$ we have

$$\cos \phi = \frac{\vec{AD} \cdot \vec{AC}}{\|\vec{AD}\| \|\vec{AC}\|} = \frac{a^2 + a^2}{\sqrt{3a^2} \sqrt{2a^2}} = \sqrt{\frac{2}{3}}$$

so $\phi \approx 0.61548$ radian or 35.2644° .

31. $\mathbf{a} = \langle 5, 7, 4 \rangle$; $\|\mathbf{a}\| = 3\sqrt{10}$; $\cos \alpha = 5/3\sqrt{10}$, $\alpha \approx 58.19^\circ$; $\cos \beta = 7/3\sqrt{10}$, $\beta \approx 42.45^\circ$; $\cos \gamma = 4/3\sqrt{10}$, $\gamma \approx 65.06^\circ$

41. $\text{comp}_{\mathbf{b}} \mathbf{a} = \mathbf{a} \cdot \mathbf{b} / \|\mathbf{b}\| = (-\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}) \cdot (6\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}) / 7 = -2$
 $\text{proj}_{\mathbf{b}} \mathbf{a} = (\text{comp}_{\mathbf{b}} \mathbf{a})\mathbf{b} / \|\mathbf{b}\| = -2(6\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}) / 7 = -\frac{12}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{4}{7}\mathbf{k}$

48. Using $\mathbf{d} = 6\mathbf{i} + 2\mathbf{j}$ and $\mathbf{F} = 3(\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j})$, $W = \mathbf{F} \cdot \mathbf{d} = \langle \frac{9}{5}, \frac{12}{5} \rangle \cdot \langle 6, 2 \rangle = \frac{78}{5}$ ft-lb.

7.4 CROSS PRODUCT

3. $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 1 \\ 2 & 0 & 4 \end{vmatrix} = \begin{vmatrix} -3 & 1 \\ 0 & 4 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & -3 \\ 2 & 0 \end{vmatrix} \mathbf{k} = \langle -12, -2, 6 \rangle$

13. $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 7 & -4 \\ 1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 7 & -4 \\ 1 & -1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & -4 \\ 1 & -1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & 7 \\ 1 & 1 \end{vmatrix} \mathbf{k} = -3\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$

is perpendicular to both \mathbf{a} and \mathbf{b} .

22. $\mathbf{i} \times (\mathbf{j} \times \mathbf{k}) = \mathbf{i} \times \mathbf{i} = \mathbf{0}$

28. $\mathbf{i} \cdot [\mathbf{j} \times (-\mathbf{k})] = \mathbf{i} \cdot [-(\mathbf{j} \times \mathbf{k})] = \mathbf{i} \cdot (-\mathbf{i}) = -(\mathbf{i} \cdot \mathbf{i}) = -1$

41. $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -3 & 6 \\ 2 & 4 & -1 \end{vmatrix} = \begin{vmatrix} -3 & 6 \\ 4 & -1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 4 & 6 \\ 2 & -1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 4 & -3 \\ 2 & -4 \end{vmatrix} \mathbf{k} = -21\mathbf{i} + 16\mathbf{j} + 22\mathbf{k}$

42. $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = 4(2) + (-3)4 + 6(-1) = -10$

45. (a) Let $A = (1, 3, 0)$, $B = (2, 0, 0)$, $C = (0, 0, 4)$, and $D = (1, -3, 4)$. Then $\overrightarrow{AB} = \mathbf{i} - 3\mathbf{j}$, $\overrightarrow{AC} = -\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$, $\overrightarrow{CD} = \mathbf{i} - 3\mathbf{j}$, and $\overrightarrow{BD} = -\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$. Since $\overrightarrow{AB} = \overrightarrow{CD}$ and $\overrightarrow{AC} = \overrightarrow{BD}$, the quadrilateral is a parallelogram.

(b) Computing

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 0 \\ -1 & -3 & 4 \end{vmatrix} = -12\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$$

we find that the area is $\| -12\mathbf{i} - 4\mathbf{j} - 6\mathbf{k} \| = \sqrt{144 + 16 + 36} = 14$.

49. $\overrightarrow{P_1P_2} = -3\mathbf{j} - \mathbf{k}$; $\overrightarrow{P_2P_3} = -2\mathbf{i} - \mathbf{k}$

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_2P_3} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -3 & -1 \\ -2 & 0 & -1 \end{vmatrix} = \begin{vmatrix} -3 & -1 \\ 0 & -1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 0 & -1 \\ -2 & -1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 0 & -3 \\ -2 & 0 \end{vmatrix} \mathbf{k} = 3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$$

$A = \frac{1}{2} \| 3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k} \| = \frac{7}{2}$ sq. units

52. $\mathbf{b} \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & 1 \\ 1 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 4 & 1 \\ 1 & 5 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} \mathbf{k} = 19\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$

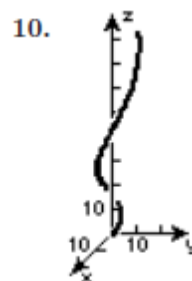
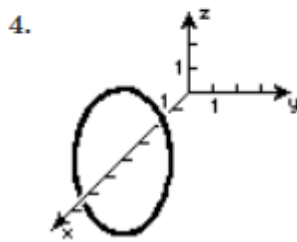
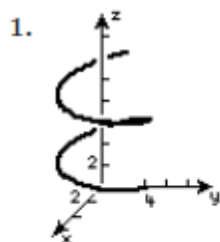
$\mathbf{v} = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})| = |(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \cdot (19\mathbf{i} - 4\mathbf{j} - 3\mathbf{k})| = |57 - 4 - 3| = 50$ cu. units

WEEK 2

7.5 LINES AND PLANES IN SPACE

5. $\mathbf{a} = \langle 1 - (-4), 1 - 1, -1 - (-1) \rangle = \langle 5, 0, 0 \rangle$; $\langle x, y, z \rangle = \langle 1, 1, -1 \rangle + t\langle 5, 0, 0 \rangle$
12. $\mathbf{a} = \langle -3 - 4, 7 - (-8), 9 - (-1) \rangle = \langle -7, 15, 10 \rangle$; $x = -3 - 7t$, $y = 7 + 15t$, $z = 9 + 10t$
17. $a_1 = 5 - 5 = 0$, $a_2 = 10 - 1 = 9$, $a_3 = -2 - (-14) = 12$; $x = 5$, $\frac{y - 10}{9} = \frac{z + 2}{12}$
24. A direction vector is $\langle 5, 1/3, -2 \rangle$. Symmetric equations for the line are $(x - 4)/5 = (y + 11)/(1/3) = (z + 7)/(-2)$.
33. The system of equations $2 - t = 4 + s$, $3 + t = 1 + s$, $1 + t = 1 - s$, or $t + s = -2$, $t - s = -2$, $t + s = 0$ has no solution since $-2 \neq 0$. Thus, the lines do not intersect.
36. $\mathbf{a} = \langle 2, 7, -1 \rangle$, $\mathbf{b} = \langle -2, 1, 4 \rangle$, $\mathbf{a} \cdot \mathbf{b} = -1$, $\|\mathbf{a}\| = 3\sqrt{6}$, $\|\mathbf{b}\| = \sqrt{21}$;
 $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} = \frac{-1}{(3\sqrt{6})(\sqrt{21})} = -\frac{1}{9\sqrt{14}}$; $\theta = \cos^{-1}\left(-\frac{1}{9\sqrt{14}}\right) \approx 91.70^\circ$
39. $2(x - 5) - 3(y - 1) + 4(z - 3) = 0$; $2x - 3y + 4z = 19$
49. From the points $(1, 2, -1)$ and $(4, 3, 1)$ we obtain the vector $\mathbf{u} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$. From the points $(4, 3, 1)$ and $(7, 4, 3)$ we obtain the vector $\mathbf{v} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$. From the points $(7, 4, 3)$ and (x, y, z) we obtain the vector $\mathbf{w} = (x - 7)\mathbf{i} + (y - 4)\mathbf{j} + (z - 3)\mathbf{k}$. Since $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, the points are colinear.
57. A direction vector for the two lines is $\langle 1, 2, 1 \rangle$. Points on the lines are $(1, 1, 3)$ and $(3, 0, -2)$. Thus, another vector parallel to the plane is $\langle 1 - 3, 1 - 0, 3 + 2 \rangle = \langle -2, 1, 5 \rangle$. A normal vector to the plane is $\langle 1, 2, 1 \rangle \times \langle -2, 1, 5 \rangle = \langle 9, -7, 5 \rangle$. Using the point $(3, 0, -2)$ in the plane, the equation of the plane is $9(x - 3) - 7(y - 0) + 5(z + 2) = 0$ or $9x - 7y + 5z = 17$.
61. Normal vectors to the planes are (a) $\langle 2, -1, 3 \rangle$, (b) $\langle 1, 2, 2 \rangle$, (c) $\langle 1, 1, -3/2 \rangle$, (d) $\langle -5, 2, 4 \rangle$, (e) $\langle -8, -8, 12 \rangle$, (f) $\langle -2, 1, -3 \rangle$. Parallel planes are (c) and (e), and (a) and (f). Perpendicular planes are (a) and (d), (b) and (c), (b) and (e), and (d) and (f).
66. Letting $y = t$ in both equations and solving $x - z = 2 - 2t$, $3x + 2z = 1 + t$, we obtain $x = 1 - \frac{3}{5}t$, $y = t$, $z = -1 + \frac{7}{5}t$ or, letting $t = 5s$, $x = 1 - 3s$, $y = 5s$, $z = -1 + 7s$.
- In Problems 75 and 76, the cross product of the direction vector of the line with the normal vector of the given plane will be a normal vector to the desired plane.*
75. A direction vector of the line is $\langle 3, -1, 5 \rangle$ and a normal vector to the given plane is $\langle 1, 1, 1 \rangle$. A normal vector to the desired plane is $\langle 3, -1, 5 \rangle \times \langle 1, 1, 1 \rangle = \langle -6, 2, 4 \rangle$. A point on the line, and hence in the plane, is $(4, 0, 1)$. The equation of the plane is $-6(x - 4) + 2(y - 0) + 4(z - 1) = 0$ or $3x - y - 2z = 10$.

9.1 VECTOR FUNCTIONS



18. $\mathbf{r}'(t) = \langle -t \sin t, 1 - \sin t \rangle$; $\mathbf{r}''(t) = \langle -t \cos t - \sin t, -\cos t \rangle$

25. $\mathbf{r}(t) = t\mathbf{i} + \frac{1}{2}t^2\mathbf{j} + \frac{1}{3}t^3\mathbf{k}$; $\mathbf{r}(2) = 2\mathbf{i} + 2\mathbf{j} + \frac{8}{3}\mathbf{k}$; $\mathbf{r}'(t) = \mathbf{i} + t\mathbf{j} + t^2\mathbf{k}$; $\mathbf{r}'(2) = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$

Using the point $(2, 2, 8/3)$ and the direction vector $\mathbf{r}'(2)$, we have $x = 2 + t$, $y = 2 + 2t$, $z = 8/3 + 4t$.

34.
$$\int_0^4 \mathbf{r}(t) dt = \left[\int_0^4 \sqrt{2t+1} dt \right] \mathbf{i} + \left[\int_0^4 -\sqrt{t} dt \right] \mathbf{j} + \left[\int_0^4 \sin \pi t dt \right] \mathbf{k}$$

$$= \frac{1}{3}(2t+1)^{3/2} \Big|_0^4 \mathbf{i} - \frac{2}{3}t^{3/2} \Big|_0^4 \mathbf{j} - \frac{1}{\pi} \cos \pi t \Big|_0^4 \mathbf{k} = \frac{26}{3}\mathbf{i} - \frac{16}{3}\mathbf{j}$$

36.
$$\int \mathbf{r}(t) dt = \left[\int \frac{1}{1+t^2} dt \right] \mathbf{i} + \left[\int \frac{t}{1+t^2} dt \right] \mathbf{j} + \left[\int \frac{t^2}{1+t^2} dt \right] \mathbf{k}$$

$$= [\tan^{-1} t + c_1] \mathbf{i} + \left[\frac{1}{2} \ln(1+t^2) + c_2 \right] \mathbf{j} + \left[\int \left(1 - \frac{1}{1+t^2} \right) dt \right] \mathbf{k}$$

$$= [\tan^{-1} t + c_1] \mathbf{i} + \left[\frac{1}{2} \ln(1+t^2) + c_2 \right] \mathbf{j} + [t - \tan^{-1} t + c_3] \mathbf{k}$$

$$= \tan^{-1} t \mathbf{i} + \frac{1}{2} \ln(1+t^2) \mathbf{j} + (t - \tan^{-1} t) \mathbf{k} + \mathbf{c},$$

where $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$.

39. $\mathbf{r}'(t) = \int \mathbf{r}''(t) dt = \left[\int 12t dt \right] \mathbf{i} + \left[\int -3t^{-1/2} dt \right] \mathbf{j} + \left[\int 2 dt \right] \mathbf{k} = [6t^2 + c_1] \mathbf{i} + [-6t^{1/2} + c_2] \mathbf{j} + [2t + c_3] \mathbf{k}$

42. $\mathbf{r}'(t) = \mathbf{i} + (\cos t - t \sin t) \mathbf{j} + (\sin t + t \cos t) \mathbf{k}$

$$\|\mathbf{r}'(t)\| = \sqrt{1^2 + (\cos t - t \sin t)^2 + (\sin t + t \cos t)^2} = \sqrt{2 + t^2}$$

$$s = \int_0^\pi \sqrt{2+t^2} dt = \left(\frac{t}{2} \sqrt{2+t^2} + \ln \left| t + \sqrt{2+t^2} \right| \right) \Big|_0^\pi = \frac{\pi}{2} \sqrt{2+\pi^2} + \ln(\pi + \sqrt{2+\pi^2}) - \ln \sqrt{2}$$

45. $\mathbf{r}'(t) = -a \sin t \mathbf{i} + a \cos t \mathbf{j}$; $\|\mathbf{r}'(t)\| = \sqrt{a^2 \sin^2 t + a^2 \cos^2 t} = a$, $a > 0$; $s = \int_0^t a du = at$

$$\mathbf{r}(s) = a \cos(s/a) \mathbf{i} + a \sin(s/a) \mathbf{j}; \quad \mathbf{r}'(s) = -\sin(s/a) \mathbf{i} + \cos(s/a) \mathbf{j}$$

$$\|\mathbf{r}'(s)\| = \sqrt{\sin^2(s/a) + \cos^2(s/a)} = 1$$

WEEK 3

9.2 MOTION ON A CURVE

4. $\mathbf{v}(t) = -2 \sin t \mathbf{i} + \cos t \mathbf{j}$; $\mathbf{v}(\pi/3) = -\sqrt{3} \mathbf{i} + \frac{1}{2} \mathbf{j}$; $\|\mathbf{v}(\pi/3)\| = \sqrt{3 + 1/4} = \sqrt{13}/2$;

$$\mathbf{a}(t) = -2 \cos t \mathbf{i} - \sin t \mathbf{j}; \quad \mathbf{a}(\pi/3) = -\mathbf{i} - \frac{\sqrt{3}}{2} \mathbf{j}$$

9. The particle passes through the xy -plane when $z(t) = t^2 - 5t = 0$ or $t = 0, 5$ which gives us the points $(0, 0, 0)$ and $(25, 115, 0)$. $\mathbf{v}(t) = 2t \mathbf{i} + (3t^2 - 2) \mathbf{j} + (2t - 5) \mathbf{k}$; $\mathbf{v}(0) = -2 \mathbf{j} - 5 \mathbf{k}$, $\mathbf{v}(5) = 10 \mathbf{i} + 73 \mathbf{j} + 5 \mathbf{k}$; $\mathbf{a}(t) = 2 \mathbf{i} + 6t \mathbf{j} + 2 \mathbf{k}$;
 $\mathbf{a}(0) = 2 \mathbf{i} + 2 \mathbf{k}$, $\mathbf{a}(5) = 2 \mathbf{i} + 30 \mathbf{j} + 2 \mathbf{k}$

11. Initially we are given $\mathbf{s}_0 = \mathbf{0}$ and $\mathbf{v}_0 = (480 \cos 30^\circ) \mathbf{i} + (480 \sin 30^\circ) \mathbf{j} = 240\sqrt{3} \mathbf{i} + 240 \mathbf{j}$. Using $\mathbf{a}(t) = -32 \mathbf{j}$ we find

$$\begin{aligned} \mathbf{v}(t) &= \int \mathbf{a}(t) dt = -32t \mathbf{j} + \mathbf{c} \\ 240\sqrt{3} \mathbf{i} + 240 \mathbf{j} &= \mathbf{v}(0) = \mathbf{c} \\ \mathbf{v}(t) &= -32t \mathbf{j} + 240\sqrt{3} \mathbf{i} + 240 \mathbf{j} = 240\sqrt{3} \mathbf{i} + (240 - 32t) \mathbf{j} \\ \mathbf{r}(t) &= \int \mathbf{v}(t) dt = 240\sqrt{3} t \mathbf{i} + (240t - 16t^2) \mathbf{j} + \mathbf{b} \\ \mathbf{0} &= \mathbf{r}(0) = \mathbf{b}. \end{aligned}$$

- (a) The shell's trajectory is given by $\mathbf{r}(t) = 240\sqrt{3} t \mathbf{i} + (240t - 16t^2) \mathbf{j}$ or $x = 240\sqrt{3} t$, $y = 240t - 16t^2$.
 (b) Solving $dy/dt = 240 - 32t = 0$, we see that y is maximum when $t = 15/2$. The maximum altitude is $y(15/2) = 900$ ft.
 (c) Solving $y(t) = 240t - 16t^2 = 16t(15 - t) = 0$, we see that the shell is at ground level when $t = 0$ and $t = 15$. The range of the shell is $x(15) = 3600\sqrt{3} \approx 6235$ ft.
 (d) From (c), impact is when $t = 15$. The speed at impact is

$$\|\mathbf{v}(15)\| = |240\sqrt{3} \mathbf{i} + (240 - 32 \cdot 15) \mathbf{j}| = \sqrt{240^2 \cdot 3 + (-240)^2} = 480 \text{ ft/s}.$$

13. We are given $\mathbf{s}_0 = 81 \mathbf{j}$ and $\mathbf{v}_0 = 4 \mathbf{i}$. Using $\mathbf{a}(t) = -32 \mathbf{j}$, we have

$$\begin{aligned} \mathbf{v}(t) &= \int \mathbf{a}(t) dt = -32t \mathbf{j} + \mathbf{c} \\ 4 \mathbf{i} &= \mathbf{v}(0) = \mathbf{c} \\ \mathbf{v}(t) &= 4 \mathbf{i} - 32t \mathbf{j} \\ \mathbf{r}(t) &= \int \mathbf{v}(t) dt = 4t \mathbf{i} - 16t^2 \mathbf{j} + \mathbf{b} \\ 81 \mathbf{j} &= \mathbf{r}(0) = \mathbf{b} \\ \mathbf{r}(t) &= 4t \mathbf{i} + (81 - 16t^2) \mathbf{j}. \end{aligned}$$

Solving $y(t) = 81 - 16t^2 = 0$, we see that the car hits the water when $t = 9/4$. Then

$$\|\mathbf{v}(9/4)\| = |4 \mathbf{i} - 32(9/4) \mathbf{j}| = \sqrt{4^2 + 72^2} = 20\sqrt{13} \approx 72.11 \text{ ft/s}.$$

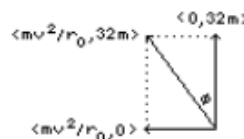
14. Let θ be the angle of elevation. Then $\mathbf{v}(0) = 98 \cos \theta \mathbf{i} + 98 \sin \theta \mathbf{j}$. Using $\mathbf{a}(t) = -9.8 \mathbf{j}$, we have

$$\begin{aligned} \mathbf{v}(t) &= \int \mathbf{a}(t) dt = -9.8t \mathbf{j} + \mathbf{c} \\ 98 \cos \theta \mathbf{i} + 98 \sin \theta \mathbf{j} &= \mathbf{v}(0) = \mathbf{c} \\ \mathbf{v}(t) &= 98 \cos \theta \mathbf{i} + (98 \sin \theta - 9.8t) \mathbf{j} \\ \mathbf{r}(t) &= 98t \cos \theta \mathbf{i} + (98t \sin \theta - 4.9t^2) \mathbf{j} + \mathbf{b}. \end{aligned}$$

Since $\mathbf{r}(0) = \mathbf{0}$, $\mathbf{b} = \mathbf{0}$ and $\mathbf{r}(t) = 98t \cos \theta \mathbf{i} + (98t \sin \theta - 4.9t^2) \mathbf{j}$. Setting $y(t) = 98t \sin \theta - 4.9t^2 = t(98 \sin \theta - 4.9t) = 0$, we see that the projectile hits the ground when $t = 20 \sin \theta$. Thus, using $x(t) = 98t \cos \theta$, $490 = x(t) = 98(20 \sin \theta) \cos \theta$ or $\sin 2\theta = 0.5$. Then $2\theta = 30^\circ$ or 150° . The angles of elevation are 15° and 75° .

19. $\mathbf{r}'(t) = \mathbf{v}(t) = -r_0\omega \sin \omega t \mathbf{i} + r_0\omega \cos \omega t \mathbf{j}$; $v = \|\mathbf{v}(t)\| = \sqrt{r_0^2\omega^2 \sin^2 \omega t + r_0^2\omega^2 \cos^2 \omega t} = r_0\omega$
 $\omega = v/r_0$; $\mathbf{a}(t) = \mathbf{r}''(t) = -r_0\omega^2 \cos \omega t \mathbf{i} - r_0\omega^2 \sin \omega t \mathbf{j}$
 $a = \|\mathbf{a}(t)\| = \sqrt{r_0^2\omega^4 \cos^2 \omega t + r_0^2\omega^4 \sin^2 \omega t} = r_0\omega^2 = r_0(v/r_0)^2 = v^2/r_0.$

22. By Problem 19, the centripetal acceleration is v^2/r_0 . Then the horizontal force is mv^2/r_0 . The vertical force is $32m$. The resultant force is $\mathbf{U} = (mv^2/r_0)\mathbf{i} + 32m\mathbf{j}$. From the figure, we see that $\tan \phi = (mv^2/r_0)/32m = v^2/32r_0$. Using $r_0 = 60$ and $v = 44$ we obtain $\tan \phi = 44^2/32(60) \approx 1.0083$ and $\phi \approx 45.24^\circ$.



27. (a) Since \mathbf{F} is directed along \mathbf{r} we have $\mathbf{F} = c\mathbf{r}$ for some constant c . Then

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (c\mathbf{r}) = c(\mathbf{r} \times \mathbf{r}) = \mathbf{0}.$$

(b) If $\boldsymbol{\tau} = \mathbf{0}$ then $d\mathbf{L}/dt = \mathbf{0}$ and \mathbf{L} is constant.

28. (a) Using Problem 27, $\mathbf{F} = -k(Mm/r^2)\mathbf{u} = m\mathbf{a}$. Then $\mathbf{a} = d^2\mathbf{r}/dt^2 = -k(M/r^2)\mathbf{u}$.

(b) Using $\mathbf{u} = \mathbf{r}/r$ we have

$$\mathbf{r} \times \mathbf{r}'' = \mathbf{r} \times \left(-k \frac{M}{r^2} \mathbf{u}\right) = -\frac{kM}{r^2} \left[\mathbf{r} \times \left(\frac{1}{r} \mathbf{r}\right)\right] = -\frac{kM}{r^3}(\mathbf{r} \times \mathbf{r}) = \mathbf{0}.$$

(c) From Theorem 9.4 (iv) we have

$$\frac{d}{dt}(\mathbf{r} \times \mathbf{v}) = \mathbf{r} \times \frac{d\mathbf{v}}{dt} + \frac{d\mathbf{r}}{dt} \times \mathbf{v} = \mathbf{r} \times \mathbf{r}'' + \mathbf{v} \times \mathbf{v} = \mathbf{0} + \mathbf{0} = \mathbf{0}.$$

(d) Since $\mathbf{r} = r\mathbf{u}$ we have $\mathbf{c} = \mathbf{r} \times \mathbf{v} = r\mathbf{u} \times r\mathbf{u}' = r^2(\mathbf{u} \times \mathbf{u}')$.

(e) Since $\mathbf{u} = (1/r)\mathbf{r}$ is a unit vector, $\mathbf{u} \cdot \mathbf{u} = 1$ and

$$\frac{d}{dt}(\mathbf{u} \cdot \mathbf{u}) = \mathbf{u} \cdot \frac{d\mathbf{u}}{dt} + \frac{d\mathbf{u}}{dt} \cdot \mathbf{u} = 2\mathbf{u} \cdot \frac{d\mathbf{u}}{dt} = \frac{d}{dt}(1) = 0.$$

Thus, $\mathbf{u} \cdot \mathbf{u}' = 0$.

(f)
$$\begin{aligned} \frac{d}{dt}(\mathbf{v} \times \mathbf{c}) &= \mathbf{v} \times \frac{d\mathbf{c}}{dt} + \frac{d\mathbf{v}}{dt} \times \mathbf{c} = \mathbf{v} \times \mathbf{0} + \mathbf{a} \times \mathbf{c} = -\frac{kM}{r^2} \mathbf{u} \times \mathbf{c} = -\frac{kM}{r^2} \mathbf{u} \times [r^2(\mathbf{u} \times \mathbf{u}')] \\ &= -kM[\mathbf{u} \times (\mathbf{u} \times \mathbf{u}')] = -kM = -kM[(\mathbf{u} \cdot \mathbf{u}')\mathbf{u} - (\mathbf{u} \cdot \mathbf{u})\mathbf{u}'] \quad \text{by (10) of 7.4} \\ &= -kM[\mathbf{0} - \mathbf{u}'] = kM\mathbf{u}' = kM \frac{d\mathbf{u}}{dt} \end{aligned}$$

(g) Since

$$\begin{aligned} \mathbf{r} \cdot (\mathbf{v} \times \mathbf{c}) &= (\mathbf{r} \times \mathbf{v}) \cdot \mathbf{c} \quad \text{by Problem 61 in 7.4} \\ &= \mathbf{c} \cdot \mathbf{c} = c^2 \quad \text{where } c = \|\mathbf{c}\| \end{aligned}$$

and

$$\begin{aligned} (kM\mathbf{u} + \mathbf{d}) \cdot \mathbf{r} &= (kM\mathbf{u} + \mathbf{d}) \cdot r\mathbf{u} = kMr\mathbf{u} \cdot \mathbf{u} + r\mathbf{d} \cdot \mathbf{u} \\ &= kMr + rd \cos \theta \quad \text{where } d = \|\mathbf{d}\| \end{aligned}$$

we have $c^2 = kMr + rd \cos \theta$ or $r = \frac{c^2}{kM + d \cos \theta} = \frac{c^2/kM}{1 + (d/kM) \cos \theta}$.

(h) First note that $c > 0$ (otherwise there is no orbit) and $d > 0$ (since the orbit is not a circle). We recognize the equation in (g) to be that of a conic section with eccentricity $e = d/kM$. Since the orbit of the planet is closed it must be an ellipse.

(i) At perihelion $c = \|\mathbf{c}\| = \|\mathbf{r} \times \mathbf{v}\| = r_0v_0 \sin(\pi/r) = r_0v_0$. Since r is minimum at this point, we want the denominator in the equation $r_0 = [c^2/kM]/[1 + (d/kM) \cos \theta]$ to be maximum. This occurs when $\theta = 0$. In this case

$$r_0 = \frac{r_0^2v_0^2/kM}{1 + d/kM} \quad \text{and} \quad d = r_0v_0^2 - kM.$$

29. a)

$$\frac{dl}{dt} = r \times \frac{dp}{dt} = \tau$$

$$F = \frac{-k M_m}{r^2}(u) = \frac{dp}{dt}$$

$$\tau = r \times \left(\frac{-k m_m}{r^2} \right) u$$

$$\cdot = (r \times u) \left(\frac{-k m_m}{r^2} \right) = 0 \quad \left[As = u = \frac{r}{|r|} \right]$$

b)

$$\tau = 0$$

$$\frac{dl}{dt} = \tau = 0$$

$$dl = 0 \times dt$$

$$l = \text{constant}$$

9.3 CURVATURE. COMPONENTS OF ACCELERATION

1. $\mathbf{r}'(t) = -t \sin t \mathbf{i} + t \cos t \mathbf{j} + 2t \mathbf{k}$; $|\mathbf{r}'(t)| = \sqrt{t^2 \sin^2 t + t^2 \cos^2 t + 4t^2} = \sqrt{5}t$;

$$\mathbf{T}(t) = -\frac{\sin t}{\sqrt{5}} \mathbf{i} + \frac{\cos t}{\sqrt{5}} \mathbf{j} + \frac{2}{\sqrt{5}} \mathbf{k}$$

6. From Problem 4, a normal to the osculating plane is $\mathbf{B}(1) = \frac{1}{\sqrt{6}}(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. The point on the curve when $t = 1$ is $(1, 1/2, 1/3)$. An equation of the plane is $(x - 1) - 2(y - 1/2) + (z - 1/3) = 0$ or $x - 2y + z = 1/3$.

9. $\mathbf{v}(t) = 2t\mathbf{i} + 2t\mathbf{j} + 4t\mathbf{k}$, $|\mathbf{v}(t)| = 2\sqrt{6}t$, $t > 0$; $\mathbf{a}(t) = 2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$; $\mathbf{v} \cdot \mathbf{a} = 24t$, $\mathbf{v} \times \mathbf{a} = \mathbf{0}$;

$$a_T = \frac{24t}{2\sqrt{6}t} = 2\sqrt{6}, \quad a_N = 0, \quad t > 0$$

16. $\mathbf{v}(t) = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$, $|\mathbf{v}(t)| = \sqrt{21}$; $\mathbf{a}(t) = \mathbf{0}$; $\mathbf{v} \cdot \mathbf{a} = 0$, $\mathbf{v} \times \mathbf{a} = \mathbf{0}$, $|\mathbf{v} \times \mathbf{a}| = 0$; $a_T = 0$, $a_N = 0$

17. $\mathbf{v}(t) = -a \sin t \mathbf{i} + b \cos t \mathbf{j} + c \mathbf{k}$, $|\mathbf{v}(t)| = \sqrt{a^2 \sin^2 t + b^2 \cos^2 t + c^2}$; $\mathbf{a}(t) = -a \cos t \mathbf{i} - b \sin t \mathbf{j}$;

$$\mathbf{v} \times \mathbf{a} = bc \sin t \mathbf{i} - ac \cos t \mathbf{j} + ab \mathbf{k}, \quad |\mathbf{v} \times \mathbf{a}| = \sqrt{b^2 c^2 \sin^2 t + a^2 c^2 \cos^2 t + a^2 b^2}$$

$$\kappa = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3} = \frac{\sqrt{b^2 c^2 \sin^2 t + a^2 c^2 \cos^2 t + a^2 b^2}}{(a^2 \sin^2 t + b^2 \cos^2 t + c^2)^{3/2}};$$

20. $\mathbf{v}(t) = a(1 - \cos t)\mathbf{i} + a \sin t \mathbf{j}$; $\mathbf{v}(\pi) = 2a\mathbf{i}$, $|\mathbf{v}(\pi)| = 2a$; $\mathbf{a}(t) = a \sin t \mathbf{i} + a \cos t \mathbf{j}$, $\mathbf{a}(\pi) = -a\mathbf{j}$;

$$|\mathbf{v} \times \mathbf{a}| = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2a & 0 & 0 \\ 0 & -a & 0 \end{vmatrix} = -2a^2 \mathbf{k}; \quad |\mathbf{v} \times \mathbf{a}| = 2a^2; \quad \kappa = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3} = \frac{2a^2}{8a^3} = \frac{1}{4a}$$

23. $F(x) = x^2$, $F(0) = 0$, $F(1) = 1$; $F'(x) = 2x$, $F'(0) = 0$, $F'(1) = 2$; $F''(x) = 2$, $F''(0) = 2$, $F''(1) = 2$;

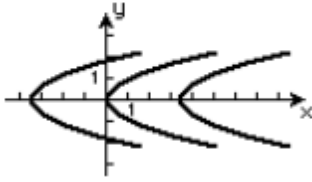
$$\kappa(0) = \frac{2}{(1+0^2)^{3/2}} = 2; \quad \rho(0) = \frac{1}{2}; \quad \kappa(1) = \frac{2}{(1+2^2)^{3/2}} = \frac{2}{5\sqrt{5}} \approx 0.18;$$

$$\rho(1) = \frac{5\sqrt{5}}{2} \approx 5.59; \quad \text{Since } 2 > 2/5\sqrt{5}, \text{ the curve is "sharper" at } (0, 0).$$

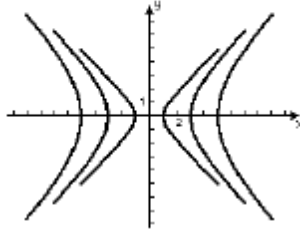
WEEK 4

9.4 PARTIAL DERIVATIVES

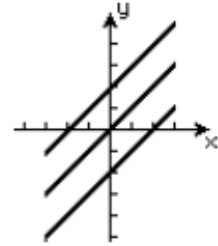
2. $x = y^2 - c$



3. $x^2 - y^2 = 1 + c^2$



6. $y = x + \tan c, -\pi/x < c < \pi/2$



9. $x^2 + 3y^2 + 6z^2 = c$; ellipsoid

15. $z_x = 20x^3y^3 - 2xy^6 + 30x^4$; $z_y = 15x^4y^2 - 6x^2y^5 - 4$

21. $z_x = 2(\cos 5x)(-\sin 5x)(5) = -10 \sin 5x \cos 5x$; $z_y = 2(\sin 5y)(\cos 5y)(5) = 10 \sin 5y \cos 5y$

24. $f_\theta = \phi^2 \left(\cos \frac{\theta}{\phi} \right) \left(\frac{1}{\phi} \right) = \phi \cos \frac{\theta}{\phi}$; $f_\phi = \phi^2 \left(\cos \frac{\theta}{\phi} \right) \left(-\frac{\theta}{\phi^2} \right) + 2\phi \sin \frac{\theta}{\phi} = -\theta \cos \frac{\theta}{\phi} + 2\phi \sin \frac{\theta}{\phi}$

26. $f_x = \frac{(x^2 - y^2)^2 y - xy[2(x^2 - y^2)2x]}{(x^2 - y^2)^4} = \frac{-3x^2y - y^3}{(x^2 - y^2)^3}$;
 $f_y = \frac{(x^2 - y^2)^2 x - xy[2(x^2 - y^2)(-2y)]}{(x^2 - y^2)^4} = \frac{3xy^2 + x^3}{(x^2 - y^2)^3}$

27. $g_u = \frac{8u}{4u^2 + 5v^3}$; $g_v = \frac{15v^2}{4u^2 + 5v^3}$

36. $\frac{\partial u}{\partial x} = -\sin(x + at) + \cos(x - at)$, $\frac{\partial^2 u}{\partial x^2} = -\cos(x + at) - \sin(x - at)$;
 $\frac{\partial u}{\partial t} = -a \sin(x + at) - a \cos(x - at)$, $\frac{\partial^2 u}{\partial t^2} = -a^2 \cos(x + at) - a^2 \sin(x - at)$;
 $a^2 \frac{\partial^2 u}{\partial x^2} = -a^2 \cos(x + at) - a^2 \sin(x - at) = \frac{\partial^2 u}{\partial t^2}$

39. $z_x = v^2 e^{uv^2} (3x^2) + 2uve^{uv^2} (1) = 3x^2 v^2 e^{uv^2} + 2uve^{uv^2}$; $z_y = v^2 e^{uv^2} (0) + 2uve^{uv^2} (-2y) = -4yuve^{uv^2}$

42. $z_u = \frac{2y}{(x+y)^2} \left(\frac{1}{v} \right) + \frac{-2x}{(x+y)^2} \left(-\frac{v^2}{u^2} \right) = \frac{2y}{v(x+y)^2} + \frac{2xv^2}{u^2(x+y)^2}$;
 $z_v = \frac{2y}{(x+y)^2} \left(-\frac{u}{v^2} \right) + \frac{-2x}{(x+y)^2} \left(\frac{2v}{u} \right) = -\frac{2yu}{v^2(x+y)^2} - \frac{4xv}{u(x+y)^2}$

48. $s_\phi = 2pe^{3\theta} + 2q[-\sin(\phi + \theta)] - 2r\theta^2 + 4(2) = 2pe^{3\theta} - 2q \sin(\phi + \theta) - 2r\theta^2 + 8$;
 $s_\theta = 2p(3\phi e^{3\theta}) + 2q[-\sin(\phi + \theta)] - 2r(2\phi\theta) + 4(8) = 6p\phi e^{3\theta} - 2q \sin(\phi + \theta) - 4r\phi\theta + 32$

49. $\frac{dz}{dt} = \frac{2u}{u^2 + v^2} (2t) + \frac{2v}{u^2 + v^2} (-2t^{-3}) = \frac{4ut - 4vt^{-3}}{u^2 + v^2}$

51. $\frac{dw}{dt} = -3 \sin(3u + 4v)(2) - 4 \sin(3u + 4v)(-1)$; $u(\pi) = 5\pi/2$, $v(\pi) = -5\pi/4$;
 $\frac{dw}{dt} \Big|_{\pi} = -6 \sin \left(\frac{15\pi}{2} - 5\pi \right) + 4 \sin \left(\frac{15\pi}{2} - 5\pi \right) = -2 \sin \frac{5\pi}{2} = -2$

55. Since $dI/dT = 1$ and $\partial P/\partial T = 0$,

$$0 = E_T = \frac{\partial F}{\partial P} \frac{\partial P}{\partial T} + \frac{\partial F}{\partial V} \frac{\partial V}{\partial T} + \frac{\partial F}{\partial T} \frac{dT}{dT} \implies \frac{\partial V}{\partial T} = -\frac{\partial F/\partial T}{\partial F/\partial V} = -\frac{1}{\partial T/\partial V}.$$

56. We are given $dE/dt = 2$ and $dR/dt = -1$. Then $\frac{dI}{dt} = \frac{\partial I}{\partial E} \frac{dE}{dt} + \frac{\partial I}{\partial R} \frac{dR}{dt} = \frac{1}{R}(2) - \frac{E}{R^2}(-1)$, and

$$\text{when } E = 60 \text{ and } R = 50, \quad \frac{dI}{dt} = \frac{2}{50} + \frac{60}{50^2} = \frac{1}{25} + \frac{3/5}{25} = \frac{8}{125} \text{ amp/min.}$$

57. Since the height of the triangle is $x \sin \theta$, the area is given by $A = \frac{1}{2}xy \sin \theta$. Then

$$\frac{dA}{dt} \frac{\partial A}{\partial x} \frac{dx}{dt} + \frac{\partial A}{\partial y} \frac{dy}{dt} + \frac{\partial A}{\partial \theta} \frac{d\theta}{dt} = \frac{1}{2}y \sin \theta \frac{dx}{dt} + \frac{1}{2}x \sin \theta \frac{dy}{dt} + \frac{1}{2}xy \cos \theta \frac{d\theta}{dt}.$$

When $x = 10$, $y = 8$, $\theta = \pi/6$, $dx/dt = 0.3$, $dy/dt = 0.5$, and $d\theta/dt = 0.1$,

$$\begin{aligned} \frac{dA}{dt} &= \frac{1}{2}(8) \left(\frac{1}{2}\right) (0.3) + \frac{1}{2}(10) \left(\frac{1}{2}\right) (0.5) + \frac{1}{2}(10)(8) \left(\frac{\sqrt{3}}{2}\right) (0.1) \\ &= 0.6 + 1.25 + 2\sqrt{3} = 1.85 + 2\sqrt{3} \approx 5.31 \text{ cm}^2/\text{s}. \end{aligned}$$

9.5 DIRECTIONAL DERIVATIVE

$$3. \nabla F = \frac{y^2}{z^3} \mathbf{i} + \frac{2xy}{z^3} \mathbf{j} - \frac{3xy^2}{z^4} \mathbf{k}$$

$$6. \nabla f = \frac{3x^2}{2\sqrt{x^3y - y^4}} \mathbf{i} + \frac{x^3 - 4y^3}{2\sqrt{x^3y - y^4}} \mathbf{j}; \quad \nabla f(3, 2) = \frac{27}{\sqrt{38}} \mathbf{i} - \frac{5}{2\sqrt{38}} \mathbf{j}$$

$$12. \mathbf{u} = \frac{\sqrt{2}}{2} \mathbf{i} + \frac{\sqrt{2}}{2} \mathbf{j}; \quad \nabla f = (4 + y^2)\mathbf{i} + (2xy - 5)\mathbf{j}; \quad \nabla f(3, -1) = 5\mathbf{i} - 11\mathbf{j};$$

$$D_{\mathbf{u}}f(3, -1) = \frac{5\sqrt{2}}{2} - \frac{11\sqrt{2}}{2} = -3\sqrt{2}$$

$$14. \mathbf{u} = \frac{6}{10} \mathbf{i} + \frac{8}{10} \mathbf{j} = \frac{3}{5} \mathbf{i} + \frac{4}{5} \mathbf{j}; \quad \nabla f = \frac{y^2}{(x+y)^2} \mathbf{i} + \frac{x^2}{(x+y)^2} \mathbf{j}; \quad \nabla f(2, -1) = \mathbf{i} + 4\mathbf{j}$$

$$D_{\mathbf{u}}f(2, -1) = \frac{3}{5} + \frac{16}{5} = \frac{19}{5}$$

$$15. \mathbf{u} = (2\mathbf{i} + \mathbf{j})/\sqrt{5}; \quad \nabla f = 2y(xy + 1)\mathbf{i} + 2x(xy + 1)\mathbf{j}; \quad \nabla f(3, 2) = 28\mathbf{i} + 42\mathbf{j}$$

$$D_{\mathbf{u}}f(3, 2) = \frac{2(28)}{\sqrt{5}} + \frac{42}{\sqrt{5}} = \frac{98}{\sqrt{5}}$$

$$18. \mathbf{u} = \frac{1}{\sqrt{6}} \mathbf{i} - \frac{2}{\sqrt{6}} \mathbf{j} + \frac{1}{\sqrt{6}} \mathbf{k}; \quad \nabla F = \frac{2x}{z^2} \mathbf{i} - \frac{2y}{z^2} \mathbf{j} + \frac{2y^2 - 2x^2}{z^3} \mathbf{k}; \quad \nabla F(2, 4, -1) = 4\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}$$

$$D_{\mathbf{u}}F(2, 4, -1) = \frac{4}{\sqrt{6}} - \frac{16}{\sqrt{6}} - \frac{24}{\sqrt{6}} = -6\sqrt{6}$$

$$24. \nabla f = (xye^{x-y} + ye^{x-y})\mathbf{i} + (-xye^{x-y} + xe^{x-y})\mathbf{j}; \quad \nabla f(5, 5) = 30\mathbf{i} - 20\mathbf{j}$$

The maximum $D_{\mathbf{u}}f$ is $[30^2 + (-20)^2]^{1/2} = 10\sqrt{13}$ in the direction $30\mathbf{i} - 20\mathbf{j}$.

27. $\nabla f = 2x \sec^2(x^2 + y^2)\mathbf{i} + 2y \sec^2(x^2 + y^2)\mathbf{j}$;
 $\nabla f(\sqrt{\pi/6}, \sqrt{\pi/6}) = 2\sqrt{\pi/6} \sec^2(\pi/3)(\mathbf{i} + \mathbf{j}) = 8\sqrt{\pi/6}(\mathbf{i} + \mathbf{j})$
 The minimum $D_{\mathbf{u}}$ is $-8\sqrt{\pi/6}(1^2 + 1^2)^{1/2} = -8\sqrt{\pi/3}$ in the direction $-(\mathbf{i} + \mathbf{j})$.
28. $\nabla f = 3x^2\mathbf{i} - 3y^2\mathbf{j}$; $\nabla f(2, -2) = 12\mathbf{i} - 12\mathbf{j} = 12(\mathbf{i} - \mathbf{j})$
 The minimum $D_{\mathbf{u}}$ is $-12[1^2 + (-1)^2]^{1/2} = -12\sqrt{2}$ in the direction $-(\mathbf{i} - \mathbf{j}) = -\mathbf{i} + \mathbf{j}$.
33. (a) Vectors perpendicular to $4\mathbf{i} + 3\mathbf{j}$ are $\pm(3\mathbf{i} - 4\mathbf{j})$. Take $\mathbf{u} = \pm\left(\frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}\right)$.
- (b) $\mathbf{u} = (4\mathbf{i} + 3\mathbf{j})/\sqrt{16+9} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}$
- (c) $\mathbf{u} = -\frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$
41. Let $x(t)\mathbf{i} + y(t)\mathbf{j}$ be the vector equation of the path. At (x, y) on this curve, the direction of a tangent vector is $x'(t)\mathbf{i} + y'(t)\mathbf{j}$. Since we want the direction of motion to be $\nabla T(x, y)$, we have $x'(t)\mathbf{i} + y'(t)\mathbf{j} = \nabla T(x, y) = -4x\mathbf{i} - 2y\mathbf{j}$. Separating variables in $dx/dt = -4x$ we obtain $dx/x = -4 dt$, $\ln x = -4t + c_1$ and $x = C_1 e^{-4t}$. Separating variables in $dy/dt = -2y$ we obtain $dy/y = -2 dt$, $\ln y = -2t + c_2$ and $y = C_2 e^{-2t}$. Since $x(0) = 3$ and $y(0) = 4$, we have $x = 3e^{-4t}$ and $y = 4e^{-2t}$. The equation of the path is $3e^{-4t}\mathbf{i} + 4e^{-2t}\mathbf{j}$, or eliminating the parameter, $16x = 3y^2$, $y \geq 0$.
43. Since $\nabla f = f_x(x, y)\mathbf{i} + f_y(x, y)\mathbf{j}$, we have $\partial f/\partial x = 3x^2 + y^3 + ye^{xy}$. Integrating, we obtain $f(x, y) = x^3 + xy^3 + e^{xy} + g(y)$. Then $f_y = 3xy^2 + xe^{xy} + g'(y) = -2y^2 + 3xy^2 + xe^{xy}$. Thus, $g'(y) = -2y^2$, $g(y) = -\frac{2}{3}y^3 + c$, and $f(x, y) = x^3 + xy^3 + e^{xy} - \frac{2}{3}y^3 + C$.
44. Let $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$ and $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j}$. $D_{\mathbf{v}}f = (f_x\mathbf{i} + f_y\mathbf{j}) \cdot \mathbf{v} = v_1f_x + v_2f_y$
 $D_{\mathbf{u}}D_{\mathbf{v}}f = \left[\frac{\partial}{\partial x}(v_1f_x + v_2f_y)\mathbf{i} + \frac{\partial}{\partial y}(v_1f_x + v_2f_y)\mathbf{j}\right] \cdot \mathbf{u} = [(v_1f_{xx} + v_2f_{yx})\mathbf{i} + (v_1f_{xy} + v_2f_{yy})\mathbf{j}] \cdot \mathbf{u}$
 $= u_1v_1f_{xx} + u_1v_2f_{yx} + u_2v_1f_{xy} + u_2v_2f_{yy}$
 $D_{\mathbf{u}}f = (f_x\mathbf{i} + f_y\mathbf{j}) \cdot \mathbf{u} = u_1f_x + u_2f_y$
 $D_{\mathbf{v}}D_{\mathbf{u}}f = \left[\frac{\partial}{\partial x}(u_1f_x + u_2f_y)\mathbf{i} + \frac{\partial}{\partial y}(u_1f_x + u_2f_y)\mathbf{j}\right] \cdot \mathbf{v} = [(u_1f_{xx} + u_2f_{yx})\mathbf{i} + (u_1f_{xy} + u_2f_{yy})\mathbf{j}] \cdot \mathbf{v}$
 $= u_1v_1f_{xx} + u_2v_1f_{yx} + u_1v_2f_{xy} + u_2v_2f_{yy}$
- Since the second partial derivatives are continuous, $f_{xy} = f_{yx}$ and $D_{\mathbf{u}}D_{\mathbf{v}}f = D_{\mathbf{v}}D_{\mathbf{u}}f$. [Note that this result is a generalization of $f_{xy} = f_{yx}$ since $D_1D_2f = f_{yx}$ and $D_2D_1f = f_{xy}$.]

WEEK 5

9.6 TANGENT PLANES AND NORMAL LINES

3. Since $f(2, 5) = 1$, the level curve is $y = x^2 + 1$.
 $\nabla f = -2x\mathbf{i} + \mathbf{j}$; $\nabla f(2, 5) = -10\mathbf{i} + \mathbf{j}$
4. Since $f(-1, 3) = 10$, the level curve is $x^2 + y^2 = 10$.
 $\nabla f = 2x\mathbf{i} + 2y\mathbf{j}$; $\nabla f(-1, 3) = -2\mathbf{i} + 6\mathbf{j}$
14. $F(x, y, z) = x^3 + y^2 + z$; $\nabla F = 3x^2\mathbf{i} + 2y\mathbf{j} + \mathbf{k}$. We want $\nabla F = c(27\mathbf{i} + 8\mathbf{j} + \mathbf{k})$ or $3x^2 = 27c$, $2y = 8c$, $1 = c$. From $c = 1$ we obtain $x = \pm 3$ and $y = 4$. Since $z = 15 - x^3 - y^2 = 15 - (\pm 3)^3 - 16 = -1 \mp 27$, the points on the surface are $(3, 4, -28)$ and $(-3, 4, 26)$.
15. $F(x, y, z) = x^2 + y^2 + z^2$; $\nabla F = 2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$. $\nabla F(-2, 2, 1) = -4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$. The equation of the tangent plane is $-4(x + 2) + 4(y - 2) + 2(z - 1) = 0$ or $-2x + 2y + z = 9$.

25. The gradient of $F(x, y, z) = x^2 + y^2 + z^2$ is $\nabla F = 2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$, so the normal vector to the surface at (x_0, y_0, z_0) is $2x_0\mathbf{i} + 2y_0\mathbf{j} + 2z_0\mathbf{k}$. A normal vector to the plane $2x + 4y + 6z = 1$ is $2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$. Since we want the tangent plane to be parallel to the given plane, we find c so that $2x_0 = 2c$, $2y_0 = 4c$, $2z_0 = 6c$ or $x_0 = c$, $y_0 = 2c$, $z_0 = 3c$. Now, (x_0, y_0, z_0) is on the surface, so $c^2 + (2c)^2 + (3c)^2 = 14c^2 = 7$ and $c = \pm 1/\sqrt{2}$. Thus, the points on the surface are $(\sqrt{2}/2, \sqrt{2}, 3\sqrt{2}/2)$ and $(-\sqrt{2}/2, -\sqrt{2}, -3\sqrt{2}/2)$.

34. $F(x, y, z) = 2x^2 - 4y^2 - z$; $\nabla F = 4x\mathbf{i} - 8y\mathbf{j} - \mathbf{k}$; $\nabla F(3, -2, 2) = 12\mathbf{i} + 16\mathbf{j} - \mathbf{k}$. Parametric equations of the line are $x = 3 + 12t$, $y = -2 + 16t$, $z = 2 - t$.

39. Let $F(x, y, z) = x^2 + y^2 + z^2 - 25$ and $G(x, y, z) = -x^2 + y^2 + z^2$. Then

$$F_x G_x + F_y G_y + F_z G_z = (2x)(-2x) + (2y)(2y) + (2z)(2z) = 4(-x^2 + y^2 + z^2).$$

For (x, y, z) on both surfaces, $F(x, y, z) = G(x, y, z) = 0$. Thus, $F_x G_x + F_y G_y + F_z G_z = 4(0) = 0$ and the surfaces are orthogonal at points of intersection.

9.7 CURL AND DIVERGENCE

7. $\text{curl } \mathbf{F} = (x - y)\mathbf{i} + (x - y)\mathbf{j}$; $\text{div } \mathbf{F} = 2z$

11. $\text{curl } \mathbf{F} = (4y^3 - 6xz^2)\mathbf{i} + (2z^3 - 3x^2)\mathbf{k}$; $\text{div } \mathbf{F} = 6xy$

15. $\text{curl } \mathbf{F} = (xy^2 e^y + 2xye^y + x^3 ye^z + x^3 yze^z)\mathbf{i} - y^2 e^y \mathbf{j} + (-3x^2 yze^z - xe^x)\mathbf{k}$; $\text{div } \mathbf{F} = xye^x + ye^x - x^3 ze^z$

21. $\nabla \cdot (\mathbf{a} \times \mathbf{r}) = \begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ a_1 & a_2 & a_3 \\ x & y & z \end{vmatrix} = \frac{\partial}{\partial x}(a_2 z - a_3 y) - \frac{\partial}{\partial y}(a_1 z - a_3 x) + \frac{\partial}{\partial z}(a_1 y - a_2 x) = 0$

24. $\mathbf{r} \cdot \mathbf{a} = a_1 x + a_2 y + a_3 z$; $\mathbf{r} \cdot \mathbf{r} = x^2 + y^2 + z^2$; $\nabla \cdot [(\mathbf{r} \cdot \mathbf{r})\mathbf{a}] = 2xa_1 + 2ya_2 + 2za_3 = 2(\mathbf{r} \cdot \mathbf{a})$

27. $\nabla \cdot (f\mathbf{F}) = \nabla \cdot (fP\mathbf{i} + fQ\mathbf{j} + fR\mathbf{k}) = fP_x + Pf_x + fQ_y + Qf_y + fR_z + Rf_z$
 $= f(P_x + Q_y + R_z) + (Pf_x + Qf_y + Rf_z) = f(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot (\nabla f)$

30. Assuming continuous second partial derivatives,

$$\begin{aligned} \text{div}(\text{curl } \mathbf{F}) &= \nabla \cdot [(R_y - Q_z)\mathbf{i} - (R_x - P_z)\mathbf{j} + (Q_x - P_y)\mathbf{k}] \\ &= (R_{yx} - Q_{zx} - (R_{xy} - P_{zy})) + (Q_{xz} - P_{yz}) = 0. \end{aligned}$$

39. $\text{curl } \mathbf{F} = -Gm_1 m_2 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x/|\mathbf{r}|^3 & y/|\mathbf{r}|^3 & z/|\mathbf{r}|^3 \end{vmatrix}$
 $= -Gm_1 m_2 [(-3yz/|\mathbf{r}|^5 + 3yz/|\mathbf{r}|^5)\mathbf{i} - (-3xz/|\mathbf{r}|^5 + 3xz/|\mathbf{r}|^5)\mathbf{j} + (-3xy/|\mathbf{r}|^5 + 3xy/|\mathbf{r}|^5)\mathbf{k}]$
 $= \mathbf{0}$

$$\text{div } \mathbf{F} = -Gm_1 m_2 \left[\frac{-2x^2 + y^2 + z^2}{|\mathbf{r}|^{5/2}} + \frac{x^2 - 2y^2 + z^2}{|\mathbf{r}|^{5/2}} + \frac{x^2 + y^2 - 2z^2}{|\mathbf{r}|^{5/2}} \right] = 0$$

40. $\frac{1}{2} \text{curl } \mathbf{v} = \frac{1}{2} \text{curl}(\boldsymbol{\omega} \times \mathbf{r}) = \frac{1}{2} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \omega_2 z - \omega_3 y & \omega_3 x - \omega_1 z & \omega_1 y - \omega_2 x \end{vmatrix}$
 $= \frac{1}{2} [(\omega_1 + \omega_1)\mathbf{i} - (-\omega_2 - \omega_2)\mathbf{j} + (\omega_3 + \omega_3)\mathbf{k}] = \omega_1 \mathbf{i} + \omega_2 \mathbf{j} + \omega_3 \mathbf{k} = \boldsymbol{\omega}$

43. (a) Expressing the vertical component of \mathbf{V} in polar coordinates, we have

$$\frac{2xy}{(x^2 + y^2)^2} = \frac{2r^2 \sin \theta \cos \theta}{r^4} = \frac{\sin 2\theta}{r^2}.$$

Similarly,

$$\frac{x^2 - y^2}{(x^2 + y^2)^2} = \frac{r^2(\cos^2 \theta - \sin^2 \theta)}{r^4} = \frac{\cos 2\theta}{r^2}.$$

Since $\lim_{r \rightarrow \infty} (\sin 2\theta)/r^2 = \lim_{r \rightarrow \infty} (\cos 2\theta)/r^2 = 0$, $\mathbf{V} \approx A\mathbf{i}$ for r large or (x, y) far from the origin.

(b) Identifying $P(x, y) = A \left[1 - \frac{x^2 - y^2}{(x^2 + y^2)^2} \right]$, $Q(x, y) = -\frac{2Axy}{(x^2 + y^2)^2}$, and $R(x, y) = 0$, we have

$$P_y = \frac{2Ay(3x^2 - y^2)}{(x^2 + y^2)^3}, \quad Q_x = \frac{2Ay(3x^2 - y^2)}{(x^2 + y^2)^3}, \quad \text{and} \quad P_z = Q_z = R_x = R_y = 0.$$

Thus, $\text{curl } \mathbf{V} = (R_y - Q_z)\mathbf{i} + (P_z - R_x)\mathbf{j} + (Q_x - P_y)\mathbf{k} = 0$ and \mathbf{V} is irrotational.

(c) Since $P_x = \frac{2Ax(x^2 - 3y^2)}{(x^2 + y^2)^3}$, $Q_y = \frac{2Ax(3y^2 - x^2)}{(x^2 + y^2)^3}$, and $R_z = 0$, $\nabla \cdot \mathbf{F} = P_x + Q_y + R_z = 0$ and \mathbf{V} is incompressible.

44. We first note that $\text{curl } (\partial \mathbf{H} / \partial t) = \partial(\text{curl } \mathbf{H}) / \partial t$ and $\text{curl } (\partial \mathbf{E} / \partial t) = \partial(\text{curl } \mathbf{E}) / \partial t$. Then, from Problem 36,

$$\begin{aligned} -\nabla^2 \mathbf{E} &= -\nabla^2 \mathbf{E} + \mathbf{0} = -\nabla^2 \mathbf{E} + \text{grad } 0 = -\nabla^2 \mathbf{E} + \text{grad } (\text{div } \mathbf{E}) = \text{curl } (\text{curl } \mathbf{E}) \\ &= \text{curl} \left(-\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} \right) = -\frac{1}{c} \frac{\partial}{\partial t} \text{curl } \mathbf{H} = -\frac{1}{c} \frac{\partial}{\partial t} \left(\frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \right) = -\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \end{aligned}$$

and $\nabla^2 \mathbf{E} = \frac{1}{c^2} \partial^2 \mathbf{E} / \partial t^2$. Similarly,

$$\begin{aligned} -\nabla^2 \mathbf{H} &= -\nabla^2 \mathbf{H} + \text{grad } (\text{div } \mathbf{H}) = \text{curl } (\text{curl } \mathbf{H}) = \text{curl} \left(\frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \right) = \frac{1}{c} \frac{\partial}{\partial t} \text{curl } \mathbf{E} \\ &= \frac{1}{c} \frac{\partial}{\partial t} \left(-\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} \right) = -\frac{1}{c^2} \frac{\partial^2 \mathbf{H}}{\partial t^2} \end{aligned}$$

and $\nabla^2 \mathbf{H} = \frac{1}{c^2} \partial^2 \mathbf{H} / \partial t^2$.

WEEK 6

9.8 LINE INTEGRALS 3,6,9,15,21,25,27,28,30,33,36,40

9.9 INDEPENDENCE OF PATH 3,6,15,18,21,24,26,27,28,30

WEEK 7

9.10 DOUBLE INTEGRALS: 3,5,9,15,18,21,24,27,33,36,39,42,45,52,62,65,68

WEEK 7: TERM TEST 1 – CHAP 7 + SECTIONS 9.1 TO 9.7

WEEK 8

9.11 DOUBLE INTEGRAL IN POLAR COORDINATES: 3,6,11,12,19,24,27,29,30,33,34

9.12 GREEN'S THEOREM: 3,4,6,8,12,18,19,23,24,25,27,33

WEEK 9

9.13 SURFACE INTEGRALS: 2,4,6,8,10,11,15,17,18,24,28, 29,32,33,36,37,39

WEEK 10

9.14 STOKES THEOREM: 3,4,6,9,10,12,13,14,18

WEEK 10: TERM TEST 2 – SECTION 9.8 TO 9.13

WEEK 11

9.15 TRIPLE INTEGRALS: 3,6,9,13,14,15,21,23,24,27,32,34,45,48,

WEEK 12

9.15 TRIPLE INTEGRALS: 51,54,57,68,69,72,75,76,78,81

9.16 DIVERGENCE THEOREM: 2,3,6,9,11,12,13,15,17,21,22

WEEK 13

9.17 CHANGE OF VARIABLES IN MULTIPLE INTEGRAL: 3,5,7,8,9,10,13,15,17,22,23,25,27

TIME PERMITTED: REVIEW: 1-20,24,26,29,30,32,36,38,43,46,50,51,53,54,56,57,58,60,63,65