MAT 2322A-Fall 2019-FINAL EXAM
Professor: Fabrizio Donzelli

Family Name ________________________________

Given Name ________________________________

Student Number ________________________________

Seat Number ________________________________

- This is a closed book exam.
- Only basic scientific calculators are allowed. Graphing or programmable calculators are not permitted.
- The exam has 12 questions worth a total of 60 points (5 points per question).
- The exam has 14 pages.
- Please write your answers in a complete and clear way. You may use the back of the pages or the additional pages if you need more space for your work.
- You must answer all the questions.

Please read carefully:

Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: academic fraud allegations will be filed which may result in your obtaining a 0 (zero) for the exam.

By signing below, you acknowledge that you have read and ensured that you are complying with the above statement.

Signature:

Problem 1 [5 points]:	
Problem 2 [5 points]:	
Problem 3 [5 points]:	
Problem 4 [5 points]:	
Problem 5 [5 points]:	
Problem 6 [5 points]:	
Problem 7 [5 points]:	
Problem 8 [5 points]:	
Problem 9 [5 points]:	
Problem 10 [5 points]:	
Problem 11 [5 points]:	
Problem 12 [5 points]:	
Total:	**/60**

→ NO CASES LIKE THAT IN THIS EXAM

CP LOCAL

Question 1 [5pts]
Find and classify the critical points of the function $f(x, y) = x^3 + y^3 - 3xy$.

Solve $\nabla f = \langle 3x^2 - 3y, 3y^2 - 3x \rangle$

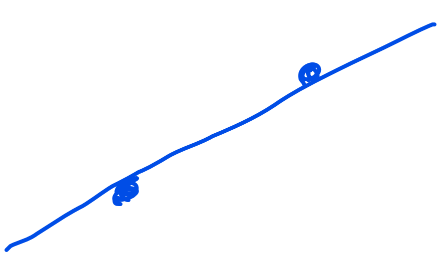
$= \langle 0, 0 \rangle$

C.P

Then Hessian test

NON-LINEAR

POLYNOMIAL EQN
GRÖBNER BASIS



Question 2 [5 pts]
Use the method of Lagrange multipliers to find the global maximum and minimum of $f(x, y) = x + y$ along the constraint $x^2 + y^2 = 1$.

Solve $\begin{cases} \nabla f = \boxed{\lambda \nabla g} \\ x^2 + y^2 = 1 \end{cases}$ NON-LINEAR

where $f = x + y$

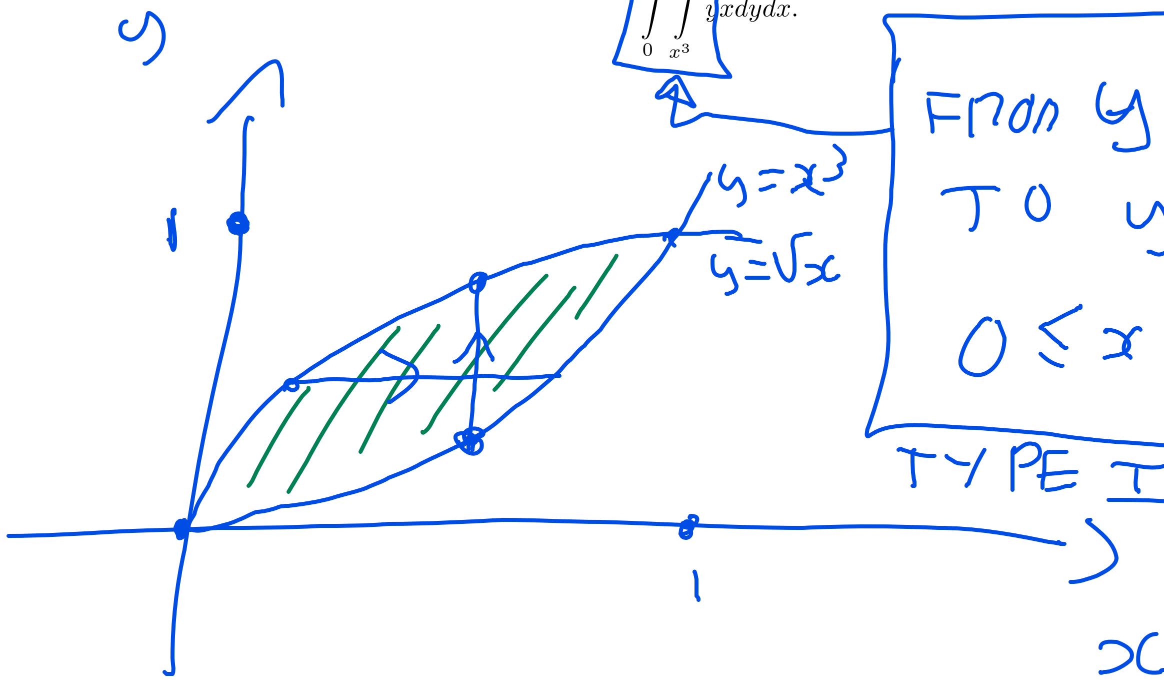
$g = x^2 + y^2 - 1$

Pick the **absolute** max/min

DRAW A GRAPH

Question 3 [5 pts]
Compute the following double integral by changing the order of integration:

$$\int_0^1 \int_{x^3}^{\sqrt{x}} yx\,dy\,dx.$$



From $y = x^3$
To $y = \sqrt{x}$
$0 \le x \le 1$

TYPE I

→ SWITCH TO TYPE II

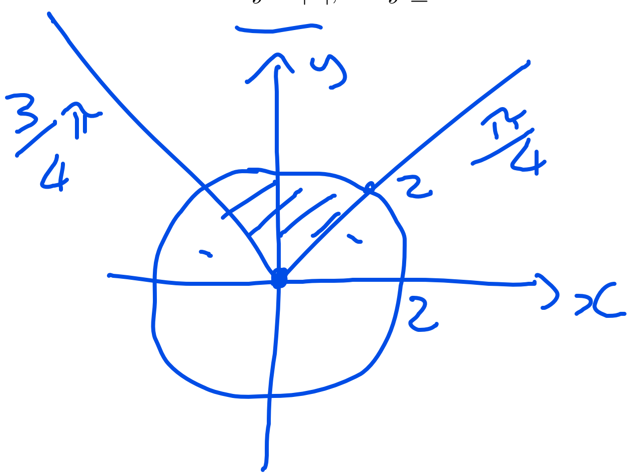
$y = \sqrt{x} \to x = y^2$
$y = x^3 \to x = \sqrt[3]{y}$

NOW y IS THE "FREE" VARIABLE $0 \le y \le 1$

TYPE II $\quad 0 \le y \le 1 \quad \to \ldots$

$y^2 \le x \le y^{1/3}$

Question 4 [5 pts]
Use polar coordinates to compute the double integral of $f(x, y) = e^{-x^2-y^2}$ on the region of the xy-plane bounded ~~by~~ below the circle $x^2 + y^2 = 4$ and ~~the~~ above the curve $y = |x|$, for $y \geq 0$.



D in polar $\frac{\pi}{4} \le \theta \le \frac{3}{4}\pi$

$0 \le \rho \le 2$

$$dA = dx\,dy = \rho\, d\rho\, d\theta$$

USING z $\qquad x + iy = e^{i\theta}\rho$

$\mathbb{C}$

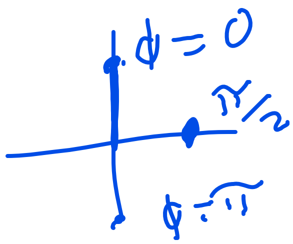
$$\sqrt{x_1^2 + x_2^2 + \cdots + x_{6,000,000}^2}$$

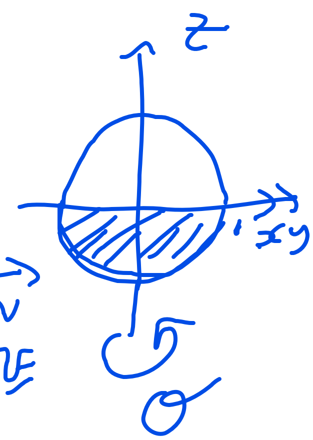
Question 5 [5 pts]
Use spherical coordinates to compute the triple integral of the function $f(x, y, z) = (x^2 + y^2 + z^2)^2$ on the solid region $\{(x, y, z) \in \mathbb{R}^3 \quad | \quad x^2 + y^2 + z^2 \leq 1, \quad z \leq 0\}$.

SOUTH

$$\rho = \sqrt{x^2+y^2+z^2}$$

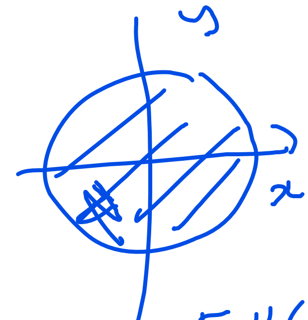
$$f = (\rho^2)^2 = \rho^4$$





SOUTHERN HEMISPHERE

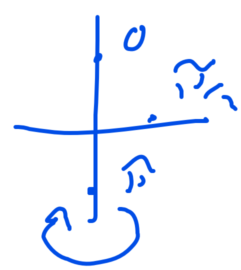
$$V: \quad \pi/2 \leq \phi \leq \pi$$
$$0 \leq \theta \leq 2\pi$$
$$0 \leq \rho \leq 1$$



FULL θ

$$dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$\rho^{\dim(\text{DOMAIN})-1}$$



$r'(t)\,dt$

Question 6 [5 pts]
Find the arc length of the curve C parametrized by

$$r(t) = \left(\frac{2}{3}t^{3/2}, 2\sin t, 2\cos t\right), \text{ for } \overset{a}{0} \le t \le \overset{b}{1}.$$

$$\boxed{\ell(C) = \int_a^b \underbrace{\|r'(t)\|\,dt}_{\text{FIRST ORDER}}}$$

$$r'(t) = \langle t^{1/2},\ 2\cos t,\ -2\sin t \rangle$$

...

COMPARE WITH SIMILAR IDEA IN MAT 2384 (Euler method)

Question 7 [5 pts]
Consider the vector field $\vec{F}(x,y,z) = < 2x + e^{yz}, xze^{yz}, xye^{yz} >$.

(a) Show that $\vec{F}$ is conservative, and compute a potential function of $\vec{F}$.

$$\text{curl } \vec{F} = \vec{0}$$

SOLVE

$\boxed{\text{MONDAY DEC 7}}$

$$\begin{cases} f_x = 2x + e^{yz} \\ f_y = xze^{yz} \\ f_z = xye^{yz} \end{cases}$$

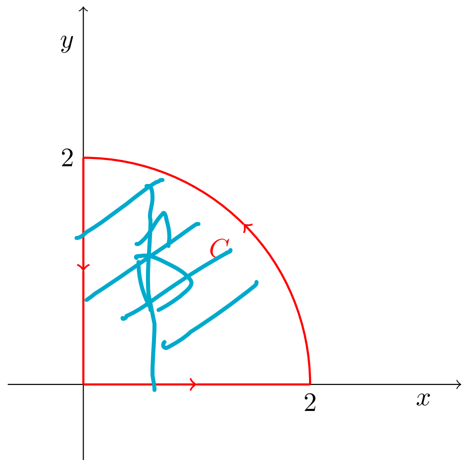
(b) Using the result from part a), compute the line integral of $\vec{F}$ along the curve C parametrized by $r(t) = (\cos t, t + e^{-t^2}\sin t, t^2)$, for $0 \le t \le \pi$.

$$f(r(\pi)) - f(r(0))$$

$x = \rho \cos\theta$

Question 8 [5 pts] Use Green's Theorem to compute the integral of the vector field $\vec{F}(x,y) =< y, x^3 >$ along the **closed** curve C, in the picture below, consisting of the arc of circle and two segments, oriented counterclockwise.

$F_{2x} - F_{1y} = \boxed{3x^2 - 1}$



CONVERT $\oint$ INTO $\iint_D$

where D is the domain whose boundary is C

$\partial D = C$

$$\oint_C \vec{F} \cdot dr = \iint_D (F_{2x} - F_{1y})\, dy\, dx =$$

$$\int_0^{\pi/2} \int_0^2 (3\rho^2 \cos^2\theta - 1)\, \rho\, d\rho\, d\theta$$

$\rightarrow$ FUBINI

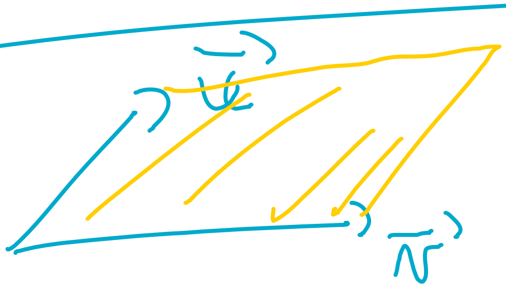
December 3

Question 9 [5 pts] Compute the surface integral of $f(x, y, z) = -3xyz$ along the cone parametrized by $(x(u, v), y(u, v), z(u, v)) = (u\cos v, u\sin v, u)$, for $0 \leq u \leq 1$, $0 \leq v \leq \pi$.

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ \cos v & \sin v & 0 \\ -u\sin v & u\cos v & 1 \end{vmatrix}$$

$$\|r_u \times r_v\| = \sqrt{2}\,u$$

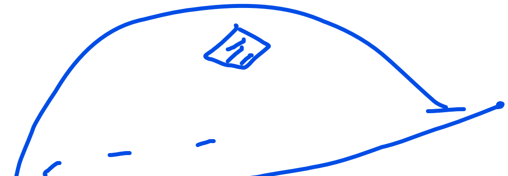
$$-3\sqrt{2}\int_0^{\pi}\int_0^{1} u^{3+1}\cos v \sin v \, du\, dv = \dots$$



$\|\vec{v} \times \vec{w}\| =$

AREA OF PAR.

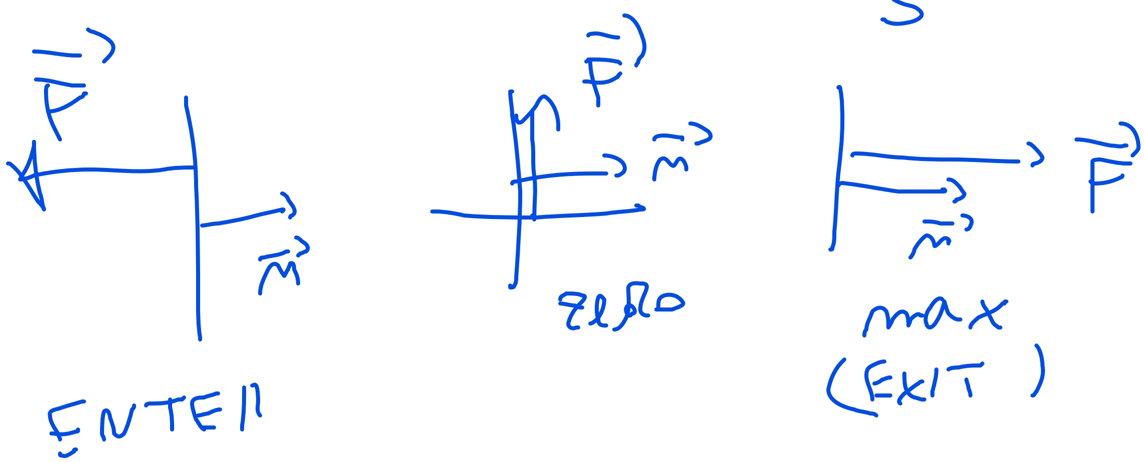
$\|r_u \times r_v\|\, du\, dv =$ area of infinitesimal region



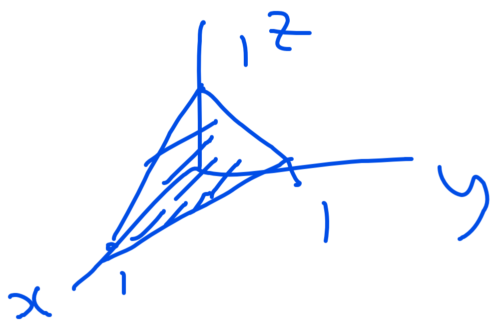
Monday 07 Dec

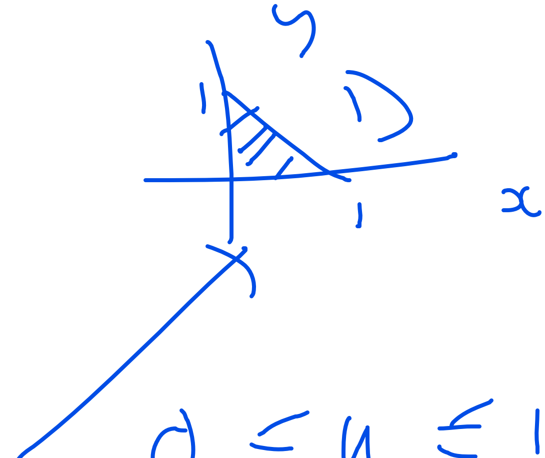
Question 10 [5 pts] Compute the flux of the vector fields $\vec{F}(x,y,z) = < x, y^2, 1 >$ across the portion of the plane $x+y+z=1$ on the first octant, with orientation pointing toward the positive x direction. (Do not use Stokes' theorem)

$$\iint_D \vec{F}(r(u,v))\cdot(r_u \times r_v)\,du\,dv$$



You need to parametrize S





$x = u$
$y = v$
$z = 1 - u - v$

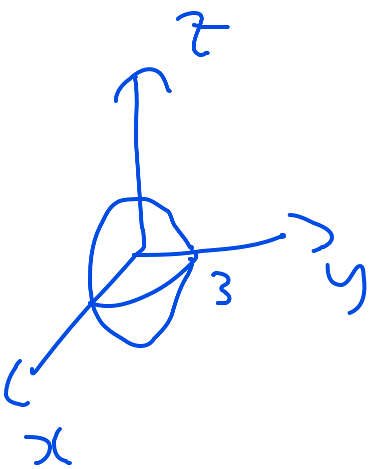
$0 \le u \le 1$
$0 \le v \le 1 - u$

$\operatorname{div}\vec{F} = F_{1x} + F_{2y} + F_{3z} = 12$

Question 11 [5 pts] Let S the sphere of equation $x^2+y^2+z^2=9$, with outward orientation. Use the divergence theorem to compute the flux of the vector field $\vec{F} =< 3x, 4y, 5z >$ across S.

CONVERT $\displaystyle\oiint_S \vec{F}\cdot\vec{n}\,dS$

TO $\displaystyle\iiint_V \operatorname{div}\vec{F}\,dV$

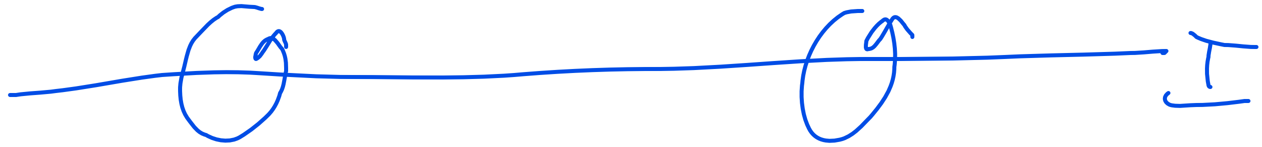


$$\oiint \vec{F}\cdot\vec{n}\,dS = \iiint_V \operatorname{div}\vec{F}\,dV$$

where $V = \boxed{\text{BALL OF RADIUS } 3}$

$$= \iiint_V 12\,dV = 12\iiint_V dV$$

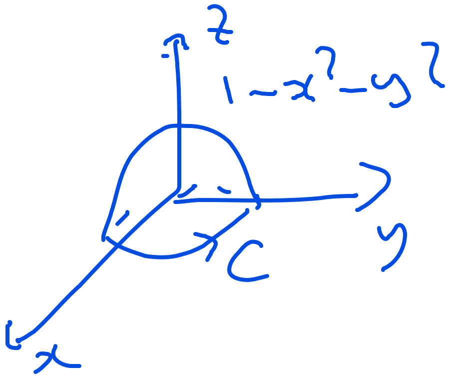
$$12\,\mathrm{Vol}(V) = 12\,\tfrac{4}{3}\pi 3^3$$



$\text{curl}\,\vec{B} = \vec{J}$

Question 12 [5 pts] $\vec{F}(x,y,z) = < -y, x+z, z >$. Let S be the part of the paraboloid $z = 1 - x^2 - y^2$ above the xy-plane with upward orientation vector field $\vec{n}$. Use Stokes Theorem to compute $\iint\limits_S curl\vec{F} \cdot \vec{n} dS$

Key: FIND $\partial S = C$



C: set $z=0$

in. $z = 1 - x^2 - y^2$

$x^2 + y^2 = 1$

CIRCLE

$r(\theta) = (\cos\theta, \sin\theta, 0)$

USING STOKES

$$\iint\limits_S \text{curl}\,\vec{F}\cdot\vec{n}\,dS = \oint\limits_C \vec{F}\cdot dr$$

$$= \int_0^{2\pi} \vec{F}(\cos\theta, \sin\theta, 0)\cdot r'(\theta)\,d\theta$$

$$\int_0^{2\pi} \langle -\sin\theta, \cos\theta, 0\rangle \cdot \overbrace{\langle -\sin\theta, \cos\theta, 0\rangle}^{r'(\theta)}\,d\theta$$