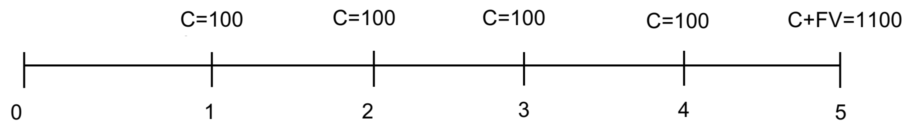
345 ANSWERS: PRESENT VALUE AND BONDS

1. **Consider a 5-year $1000 face value bond which has a 10% coupon rate. Currently, it is thought that interest rates will remain constant at 3% for the full five periods. Suddenly, there is 'news' that interest rates will almost certainly rise to 5% for the last two periods. What would happen to the bond's price, given this news?**



$$P_{bond} = \frac{100}{(1+i)} + \frac{100}{(1+i)^2} + \frac{100}{(1+i)^3} + \frac{100}{(1+i)^4} + \frac{100}{(1+i)^5} + \frac{1000}{(1+i)^5}$$

Before the news, interest rates were supposed to remain constant at 3%. Then:

$$P_{before} = \frac{100}{(1.03)} + \frac{100}{(1.03)^2} + \frac{100}{(1.03)^3} + \frac{100}{(1.03)^4} + \frac{100}{(1.03)^5} + \frac{1000}{(1.03)^5}$$

$$P_{before} = \$1320.58$$

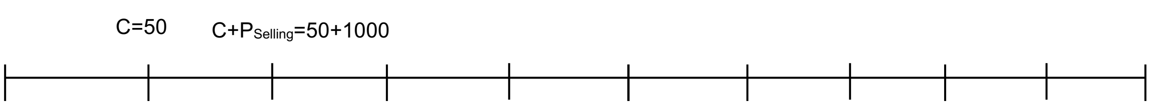
After the 'news', you would have likely substituted 5% into the last 3 terms, and derived the following lower price:

$$P_{after} = \frac{100}{(1.03)} + \frac{100}{(1.03)^2} + \frac{100}{(1.03)^3} + \frac{100}{(1.05)^4} + \frac{100}{(1.05)^5} + \frac{1000}{(1.05)^5}$$

$$P_{after} = \$1227.01$$

A complication: Since discounting still takes place at 3% for three periods, the discount rate for the 4th period is really $(1.03)^3$ x 1.05 while, for the 5th period, it is $(1.03)^3$ x $(1.05)^2$. Taking this into account, the exact answer is higher: $1283.11.

2. **Consider a 10-year $1000 face value bond with a 5% coupon rate. The investor buys the bond for $1100 but a liquidity shock forces him to sell it after two periods for exactly the face value. Coupons are not reinvested. What is the annualized yield?**



$$\text{RET(i)} = \frac{\frac{C_1+C_2+\cdots+C_i+(P_{t+i}-P_t)}{P_t}}{i}$$, i = number of periods

$$\text{Annualized yield} = \frac{\frac{50+50+(1000-1100)}{1100}}{2} = 0\%$$

A. The annualized yield is 0%.

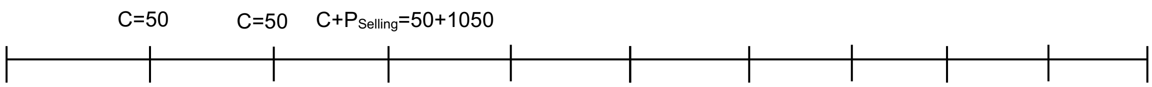
3. **In the above, what difference in annualized yield would there be if the first coupon was in fact reinvested at 5%?**

$$\mathrm{RET(i)} = \frac{\frac{C_1+C_2+\cdots+C_i+(P_{t+i}-P_t)}{P_t}}{i}$$, i = number of periods

$$\text{Annualized yield} = \frac{\frac{50(1.05)+50+(1000-1100)}{1100}}{2} = \frac{0.23\%}{2} = 0.115\%$$

   A. The annualized yield would be greater than 0. It would be 0.115%.

4. **Likewise, what difference would there be in annualized yield if the coupons were not reinvested but the investor sold instead after *three* periods for $1050?**

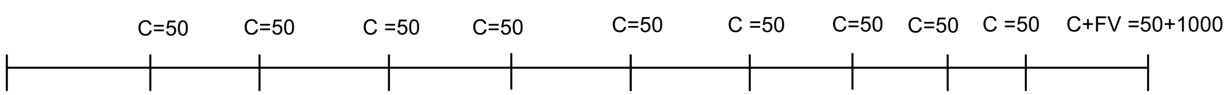


$$\mathrm{RET(i)} = \frac{\frac{C_1+C_2+\cdots+C_i+(P_{t+i}-P_t)}{P_t}}{i}$$, i = number of periods

$$\text{Annualized yield} = \frac{\frac{50+50+50+(1050-1100)}{1100}}{3} = \frac{9.09\%}{3} = 3.03\%$$

   A. The annualized yield would be 3.03%.

5. **Suppose the investor managed to hold this bond to maturity. What would the yield to maturity be?**



$$\mathrm{RET(i)} = \frac{\frac{C_1+C_2+\cdots+C_i+(P_{t+i}-P_t)}{P_t}}{i}$$, i = number of periods

$$\text{Annualized yield} = \frac{\frac{50*10+(1000-1100)}{1100}}{10} = \frac{36.36\%}{10} = 3.64\%$$

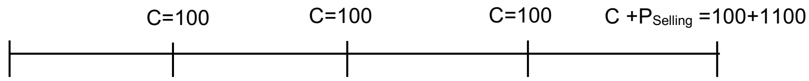
   A. The yield to maturity would be 3.64%.

   Another approach:

$$P_{bond} = \frac{50}{(1+i)} + \frac{50}{(1+i)^2} + \frac{50}{(1+i)^3} + \frac{50}{(1+i)^4} + \cdots + \frac{50}{(1+i)^9} + \frac{50}{(1+i)^{10}} + \frac{1000}{(1+i)^{10}}$$

$$P_{bond} = 50 * \frac{1-(1+i)^{-10}}{(1+i)-1} + \frac{1000}{(1+i)^{10}}, \text{ annualized yield} = 3.8\%$$

6. **Consider a 10-year $1000 face value bond with a 10% coupon rate. The investor buys the bond for $900 and sells it for $1100 after four years. Coupons are not reinvested. What is the annualized return?**



$$\text{RET(i)} = \frac{\frac{C_1+C_2+\cdots+C_i+(P_{t+i}-P_t)}{P_t}}{i}$$, i = number of periods

$$\text{Annualized yield} = \frac{\frac{100+100+100+100+(1100-900)}{900}}{4} = \frac{66.67\%}{4} = 16.67\%$$

A. The annualized return would be 16.67%.

7. Suppose in the above that the coupons were reinvested at 10%. What would his effective annualized yield be now?

$$\text{RET(i)} = \frac{\frac{C_1+C_2+\cdots+C_i+(P_{t+i}-P_t)}{P_t}}{i}$$, i = number of periods

$$\text{Annualized yield} = \frac{\frac{100(1.1)^3+100(1.1)^2+100(1.1)+100+(1100-900)}{900}}{4} = \frac{73.79\%}{4} = 18.45\%$$

A. The annualized yield would be 18.45%.

8. Consider a 5-year, \$1000 face value bond, bought for \$900 at time t, which has a price of \$1000 at t+1, \$1100 at t+2, \$1150 at t+3, \$1050 at t+4, and \$1000 at maturity. If the investor wanted to maximize his annualized return, when should they sell this bond?

Consider, for example, that the **coupon rate is 10%.**

$$\text{RET(i)} = \frac{\frac{C_1+C_2+\cdots+C_i+(P_{t+i}-P_t)}{P_t}}{i}$$, i = number of periods

$$\text{RET}(1) = \frac{\frac{100+(1000-900)}{900}}{1} = \frac{22.22\%}{1} = 22.22\%$$

$$\text{RET}(2) = \frac{\frac{100+100+(1100-900)}{900}}{2} = \frac{44.44\%}{2} = 22.22\%$$

$$\text{RET}(3) = \frac{\frac{100+100+100+(1150-900)}{900}}{3} = \frac{61.11\%}{3} = 20.37\%$$

$$\text{RET}(4) = \frac{\frac{100+100+100+100+(1050-900)}{900}}{4} = \frac{61.11\%}{4} = 15.28\%$$

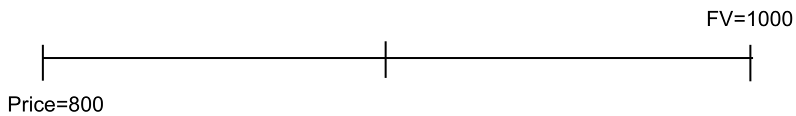
$$\text{RET}(5) = \frac{\frac{100+100+100+100+100+(1000-900)}{900}}{5} = \frac{66.67\%}{5} = 13.33\%$$

A. In principle, to maximize annualized return, the investor should sell the bond at t+1 or t+2. However, if the chances of getting 22.22% for an extra period are unlikely, then the prospect of selling at t+2 -- and getting 44% for 2 periods -- is the better decision.

9. **At the 5 year maturity date, what would you expect this annualized return to converge to?**

A. The annualized return will converge to RET(5) above, since the bond was originally bought at $900. That is its **yield to maturity**. It would converge to 10% only if it was bought at $1000, its face value.

10. **A fussy investor always wants at least 15% from every investment made. A 2 year, $1000 face value *discount* (zero-coupon) bond is selling for $800. Would they buy it?**



$$\mathrm{RET}(2) = \frac{\frac{(1000-800)}{800}}{2} = \frac{25.00\%}{2} = 12.50\% < 15.00\%$$

A. They should not buy it, because the annualized return is lower than the 15% required.

Another approach:

$800 = \frac{1000}{(1+i)^2}$, then i=11.8%< 15.00%

11. **An investor buys at par a $1000 face value bond with a 10% coupon rate. A week after buying it, the corporation announces severe liquidity problems and states that there is only a 50/50 chance that the coupons and final payment will be paid. What would be the yield on this bond, given the announcement?**

$$\mathrm{RET(i)} = \frac{\frac{C_1+C_2+\cdots+C_i+(P_{t+i}-P_t)}{P_t}}{i}$$, i = number of periods

Consider a **10 year bond:**

$$\text{Annualized yield} = \frac{\frac{50*10+(500-1000)}{1000}}{10} = 0\%$$

Another approach:

$$P_{bond} = \frac{50}{(1+i)} + \frac{50}{(1+i)^2} + \frac{50}{(1+i)^3} + \cdots + \frac{50}{(1+i)^8} + \frac{50}{(1+i)^9} + \frac{50}{(1+i)^{10}} + \frac{500}{(1+i)^{10}}$$

$$1000 = 50 * \frac{1-(1+i)^{-10}}{(1+i)-1} + \frac{500}{(1+i)^{10}}$$

$\text{Annualized yield} = 0\%$

A. The yield, given the announcement, would be 0%.

12. Explain how this announcement could make this bond trade with a high risk premium.

A. As there is now a probability of default, the calculated yield on this bond is now lower, and some or many agents would want to sell it, given the existence of a safer alternative. With *excess supply*, the bond's price will be lower, implying a positive *risk premium* for any potential purchaser.

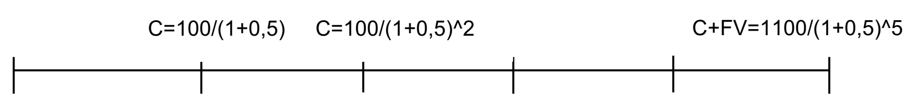
13. Suppose that a number of investors just learned that interest rates would move increasingly downward for a number of years. Would that bias investors to holding more long-term bonds in his portfolio?

A. All bond prices would move up since rates are now expected to fall, producing the prospect of a capital gain. However, since long term bond prices are more volatile upward than shorter term bonds, a portfolio switch towards long term makes sense. The investors better operate fast though: once the news of falling interest rates becomes fully public, bond prices will have already gone up.

14. Suppose that current global uncertainty and high household debt levels now make the probability of liquidity shocks much greater than before. Would that actually make long-term bonds 'safer' relative to short-term bonds?

A. It is now more likely that a short-term bond sustains a liquidity shock. If so, this would remove the option to hold to maturity, and makes it more likely an investor will have to sell a short term bond when prices are down. In this sense, short-term bonds become more risky than they were before, so the risk difference between long term and short term is narrowed. Long term bonds are still absolutely more risky but here they become *relatively* less risky.

15. Consider the same \$1000, 5 year, 10% coupon bond as above. Just after you have bought it, it recognized that there will be 5% inflation per period, thus depreciating the real value of future coupons by 5% in each subsequent period. How will this affect its yield to maturity, i? What will happen if people start selling this bond?



$$\text{RET(i)} = \frac{\frac{C_1+C_2+\cdots+C_i+(P_{t+i}-P_t)}{P_t}}{i}$$, i = number of periods

$$\text{Annualized yield} = \frac{\frac{(95.23+90.70+86.38+82.27+78.35+783.53-1000)}{1000}}{5} = \frac{21.65\%}{5} = 4.33\%$$

Another approach:

$$1000 = \frac{95.23}{(1+i)} + \frac{90.70}{(1+i)^2} + \frac{86.38}{(1+i)^3} + \frac{82.27}{(1+i)^4} + \frac{78.35 + 783.53}{(1+i)^5}$$

$$\text{Annualized yield} = 4.76\%$$

A. Writing the coupons as C/Pt, the yield is now approximately 5%. As the expected cash flows are declining by 5% per period, it follows that the yield to maturity will be lower, reflecting this. There is an incentive for agents to sell this bond when more inflation-proof assets exist. In turn, the excess supply will lower its price. The lower price will create an **inflation premium**, registered in a higher yield, i. This is a case of complete insurance: if the inflation premium is about 5%, that will bring the anticipated yield back to (approximately) 10%. This illustrates a version of the **Fisher Equation**: the nominal rate of interest = the real rate of interest + expected inflation. Thus:

Nominal yield (10%) = real yield (5%) + inflation premium (5%).

If there were no inflation, we would get:

Nominal yield (10%) = Real yield (10%)