University of Ottawa

Department of Mathematics and Statistics

MAT 1302C : Mathematical Methods II

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First Midterm Exam – Version A

October 2, 2018

Surname ______________________ First Name ______________________

Student # ______________________ DGD ______________________

Instructions:

(a) You have 80 minutes to complete this exam.
(b) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this **clearly**. Otherwise, the work written on the reverse side of pages will not be considered for marks.
(c) Write your student number at the top of each page in the space provided.
(d) No notes, books, scrap paper, calculators or other electronic devices are allowed.
(e) You are strongly recommended to write in **pen**, not pencil.
(f) You may use the last page of the exam as scrap paper.
(g) Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. **Phones and devices must be turned off and put away in your bag**. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

Signature ______________________

Please do not write in the table below.

Question	1	2	3	4	5	6	7	Total
Maximum	5	3	4	4	2	4	2	24
Grade								

1. Consider the following linear system:

$$\begin{cases} y_1 - 2y_5 &= 3y_2 + 3y_3 + 4 \\ y_1 - y_3 &= 3y_2 + 4y_5 + 2 \\ 2y_1 + y_4 - 9 &= 6y_2 + 6y_3 + y_5. \end{cases}$$

(a) [**4 points**] Write down the general solution of this linear system.

(b) [**1 point**] Give an explicit solution of this linear system such that $y_2 = -1$ and $y_5 = 3$.

2. Set

$$A = \begin{bmatrix} 1 & 0 & -2 & 0 & -3 \\ 0 & 1 & 3 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \text{ and } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}.$$

(a) [**2 points**] Suppose that $\mathbf{b} = \begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix}$. Write down the general solution of the linear system $A\mathbf{x} = \mathbf{b}$ in vector parametric form.

(b) [**1 point**] Does there exist a vector $\mathbf{b} \in \mathbb{R}^3$ for which the linear system $A\mathbf{x} = \mathbf{b}$ is inconsistent? You should justify your answer.

3. [**4 points**] Determine all the values of the parameter a for which the linear system

$$\begin{bmatrix} 1 & 2 & a \\ 0 & 1 & -2 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

is consistent. For each of these values of a, how many solutions does the linear system have? You should justify your answers.

4. [**4 points**] For each of the following statements, indicate if it is true or false. You will receive 1 point for every correct answer and lose .5 points for every incorrect answer (thus, if you leave the answer space blank, you will not receive negative points). You will not receive a negative total score on this question.

_____ If the rightmost column of the coefficient matrix of a linear system contains a pivot position, then the linear system must be inconsistent.

_____ A homogeneous linear system with 4 equations and 5 variables always has infinitely many solutions.

_____ For any two given vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3$, and any two given scalars $c_1, c_2 \in \mathbb{R}$, we always have $c_1\mathbf{a} + c_2\mathbf{b} \in \text{Span}\{\mathbf{a}, \mathbf{b}\}$.

_____ Two row-equivalent matrices which are both in echelon form must be equal.

5. Compute the following:

(a) [**1 point**] $c_1 \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} - c_3 \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}$ for $c_1 = -2$, $c_2 = 1$, and $c_3 = 3$.

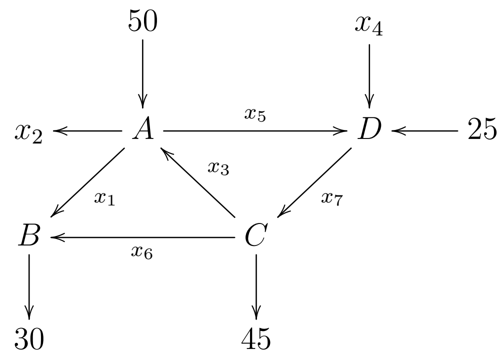
(b) [**1 point**] $\begin{bmatrix} -2 & 1 & -\frac{1}{2} \\ 3 & 1 & 4 \\ -2 & -5 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$.

6. [**4 points**] Set

$$\mathbf{a}_1 = \begin{bmatrix} 3 \\ 0 \\ 0 \\ -3 \end{bmatrix}, \ \mathbf{a}_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 3 \end{bmatrix}, \text{ and } \mathbf{a}_3 = \begin{bmatrix} 4 \\ 1 \\ -3 \\ -2 \end{bmatrix}.$$

Does the vector $\mathbf{b} = \begin{bmatrix} -1 \\ 2 \\ -7 \\ 2 \end{bmatrix}$ belong to $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$? You should justify your answer.

7. [**2 points**] Consider the traffic flow described by the following diagram. The letters A through D label intersections. The arrows indicate the direction of flow (all roads are one-way) and their labels indicate flow in cars per minute.



Write down a linear system describing the traffic flow, i.e., all constraints on the variables $x_i, i = 1, \ldots, 7$. (You do not need to solve the linear system.)

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