CEG3185 Winter 2021

ASSIGNMENT 1

Due date: 5pm on Monday, February 8, 2021

Problem 1

Assume the use of a channel characterized by a bit error probability, P_B. Consider a frame of 4 bits size. The event {"bit i" is in error} is independent of the event {"bit j" is in error}, where "bit i" and "bit j" belong to the same frame, with the following relation holding: $(1 \le i, j \le 4, i \ne j)$. What is the probability of receiving a correct frame?

Answer

$$(1-P_B)^4$$

Problem 2

Deduce the maximum theoretical information rates associated with the following transmission channels:

a) Telex network with a bandwidth of 500 Hz and a signal-to-noise ratio of 5dB.
b) Switched telephone with a bandwidth of 3100 Hz and a signal-to-noise ratio of 20 dB.

Answer

$$C = W\, log_2\,(1 + S/N)$$

where

$$S/N = 10^{\frac{SNR(dB)}{10}}$$

5dB: $\frac{S}{N} = 10^{\frac{5}{10}} = 10^{0.5} = \sqrt{10}$

20dB: $\frac{S}{N} = 10^{\frac{20}{10}} = 10^2 = 100$

(a) $C = 500 * log_2\left(1 + \sqrt{10}\right) \approx 500 * 4.162 \approx 1000bps$
(b) $C = 3100 * log_2(1 + 100) \approx 3100 * 6.67 \approx 20{,}677bps$

Problem 3

Consider the angle-modulated signal

$$s(t) = 10[cos(\,10^8\pi t + 5\,sin(\,2\pi(10^3)t))]$$

Find the maximum phase and the maximum frequency deviation.

Answer

Maximum phase deviation: $\pm 5rads$

For maximum frequency deviation, you have to get the derivative of the phase, in respect to the time t. This gives:

$$(\frac{1}{2\pi})d\{10^8\pi t + 5\,sin(\,2\pi(10^3)t)\}/dt = \frac{1}{2}10^8 + 5 * 10^3\,cos(\,2\pi 10^3 t)$$

Maximum Frequency deviation: $\pm 50^3 Hz = 5KHz$

Problem 4

A communications system operates on the principles of Asynchronous transmission. Each character is transmitted in the form of a frame consisting of 15 bits (including the start bit). Assume that the synchronization at the start of the first bit could be early or late by at most 10% of the actual bit period, T_B. Calculate the value of the normalized error

Abs {(T_S - T_B) / T_B }

that allows correct detection of the frame. T_S represents the symbol period estimate at the receiver. Abs{ $\boldsymbol{x}$ } represents the absolute value of $\boldsymbol{x}$. Assume that errors could occur only due to synchronization offsets.

Include all steps in your solution.

Please be brief. Use the space below for your answer.

Answer
The following 2 conditions should be satisfied:

1) $(14 + 0.5)T_S + 0.1T_B < 15T_B$
2) $(14 + 0.5)T_S - 0.1T_B > 14T_B$

$Eq(1) \Leftrightarrow (14 + 0.5)T_S + 0.1T_B < 15T_B \Leftrightarrow 14.5T_S < 14.9T_B \Leftrightarrow \frac{T_S}{T_B} < 1.0275862 \Leftrightarrow \frac{T_S}{T_B} - 1 < 0.0275862 \Leftrightarrow \frac{T_S - T_B}{T_B} < 0.0275862$ $\boldsymbol{Eq.(3)}$

$Eq(2) \Leftrightarrow (14 + 0.5)T_S - 0.1T_B > 14T_B \Leftrightarrow 14.5T_S > 14.1T_B \Leftrightarrow \frac{T_S}{T_B} > 0.97241379 \Leftrightarrow \frac{T_S}{T_B} > 1 - 0.0275862 \Leftrightarrow \frac{T_S - T_B}{T_B} > -0.0275862$ $\boldsymbol{Eq.(4)}$

From equation 3 and 4:

$$\left|\frac{T_{S-T_B}}{T_B}\right| < 0.0275862 \Leftrightarrow \left|\frac{T_{S-T_B}}{T_B}\right| < 2.758\%$$

Problem 5

Assume a system transmits characters. Each character consists of ***n*** information bits (***n*** is even number) and 1 parity bit. We are assuming that bit errors are occurring randomly and are uncorrelated with each other. Determine the probability a character is damaged (one or more bits are in error). The parity check used per character is even.

Answer

We represent the bit error probability with p_b.

The probability the character is damaged equals:

$P_{CE} = 1 - (1 - p_b)^{n+1}$