MGCR 341: Introduction to Finance
Winter 2017
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Midterm: Solutions

Student Last Name:___________________________________

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Student Number:_____________________________________

Friday March 10, 2017, 3:00pm to 5:00pm

Time: Do not turn past this page until the exam has begun. The exam is **120 minutes** in length.
Show your work: In order to receive credit for your answers, you must show your work (unless noted otherwise). Correct answers with no work shown will not receive any credit. Incorrect answers with partial correct work may receive partial credit.
Answer questions DIRECTLY ON THE EXAM SHEET.
Formula sheet: The exam is **closed book**, and you may not bring any notes into class. A formula sheet is provided on the last page of this exam. You may detach the formula sheet if you like. You do not need to hand in the formula sheet at the end of the exam.
Calculator: You are allowed a non-programmable non-text-storing calculator. You are allowed a financial calculator, although a simple calculator will suffice.
This exam has a total of **11 questions** on **9 pages**, including the cover page and formula sheet.
The exam is worth a total of 100 points.

GOOD LUCK!

Question	Grade	Out of
Q1		8
Q2		7
Q3		10
Q4		15
Q5		7
Q6		8
Q7		5
Q8		10
Q9		10
Q10		10
Q11		10
TOTAL		100

1) (8 points, 5 mins)

a) At t = 0 you deposit $100 in the bank, where the APR is Z. Two years later at t = 2, you have $121. What is the EAR?

Every year money gets multiplied by (1+EAR).

$100(1+EAR)^2 = 121$

→ EAR = 10%.

b) The APR is X with semi-annual compounding. The EAR is 12%. What is X?

$(1 + X/2)^2 = 1 + 0.12$

→ X = 11.66%

2) (7 points, 10 mins) Today is t = 0. The risk free rate is 5%.

Consider the following sets of cash flows

Set 1: Growing perpetuity with first cash flow equal to $100 at **t = 11** and g = 4%.

Set 2: Growing perpetuity with first cash flow equal to $100(1.02) at **t = 12** and g = 4%.

Set 3: Growing perpetuity with first cash flow equal to $100($1.02^2$) at **t = 13** and g = 4%.

Set 4: Growing perpetuity with first cash flow equal to $100($1.02^3$) at **t = 14** and g = 4%.

…(The pattern continues to Set infinity)…

a) What is the Present Value of **Set 1**? (Note: as usual, the present refers to t = 0.)

$[100/(0.05 – 0.04)](1/1.05^{10}) = 6,139.17$

b) Consider all the cash flows from all the infinite sets. What is the present value of all those cash flows? (Note: as usual, the present refers to t = 0.)

Set 1 is equivalent to 100/(0.05 – 0.04) = 10,000 at t = 10

Set 2 is equivalent to 100(1.02)/(0.05 – 0.04) = 10,000(1.02) at t = 11

Set 3 is equivalent to $100(1.02^2)/(0.05 – 0.04) = 10,000(1.02^2)$ at t = 12

etc…

This looks like a growing perpetuity where the first cash flow is 10,000 at t = 10, and the growth rate is 2%. So the PV of all cash flows from all sets is:

$[10,000/(0.05 – 0.02)](1/1.05^9) = 214,869.64$

3) (10 points, 15 mins) The risk free rate is 10%. Machine A needs to be replaced every 10 years. It costs $400 to buy Machine A at t = 0 and $400 every time it needs to be replaced. Machine A has annual maintenance costs of $100 per year. As usual, assume the maintenance costs are paid at the end of each year.

a) Write down an equation where the only unknown is the EAC of Machine A? You do not need to solve the equation.

$$400 + \frac{100}{0.1}\left(1 - \frac{1}{1.1^{10}}\right) = \frac{EAC_A}{0.1}\left(1 - \frac{1}{1.1^{10}}\right)$$

Where the left hand side is the NPV per cycle.

The risk free rate is still 10% and the real risk free rate is 8%.
Machine B has an EAC of $290 (nominal) and generates the same revenue as Machine C.
For Machine C:
- Initial purchase cost at t = 0 is 1,000; The cost of the machine increases with inflation
- The machine needs to be replaced every 10 years
- Maintenance is performed annually, for t = 1 to infinity (including the years when you replace the machine); Maintenance cost at t = 1 is 100(1+i), where i is the inflation rate
- Cost of maintenance increases with the inflation rate

b) Between Machines B and C, which should you use?

Since the costs of Machine C are all growing with inflation, Machine C's costs are in the form of a repeating set of real cash flows, where the machine costs 1,000 real dollars each time, and the annual maintenance costs are 100 real dollars. Note: you can't use the nominal EAC approach for Machine C because when viewed in nominal terms, the cycles are not identical, so the nominal EACs would be different for different cycles, and you cannot link them together into a perpetuity.

The NPV per cycle (left hand side) can be set equal to the PV of a real constant annuity.

$$NPV_{Cycle,A,real} = 1{,}000 + \frac{100}{0.08}\left(1 - \frac{1}{1.08^{10}}\right) = \frac{EAC_{C,r}}{0.08}\left(1 - \frac{1}{1.08^{10}}\right)$$

$$\rightarrow EAC_{C,r} = 249$$

Total NPV to infinity of the costs of C is equal to 249/0.08 = 3,113.

Total NPV to infinity of the costs of B is equal to 290/0.10 = 2,900.

So, you prefer to use Machine B.

4) (15 points, 15 mins) Today is t = 0. The **real** risk free rate is 1% per month, and the inflation rate is 0.3% per month. At t = 0 a hamburger costs $1.

Your pension will pay you 1,000 **real dollars** at t = 80. Each month thereafter your pension will pay you 0.6% more real dollars than the previous month, up to and including t = 200.

You have 2 goals for your retirement:

i) consume a certain number of hamburgers at t = 170, and then consume 4% fewer hamburgers each month until t = 190, and then consume 2% more hamburgers each month until t = 200
ii) have enough money in the bank at t = 200, such that if you donate this amount of money to the save-a-basset foundation at t = 200, they will be able to generate a perpetual stream of **100 real dollar** cash flows from t = 201 to infinity, using only this money and access to a bank account

Based only on the pension income stream, and having access to the 1% monthly real interest rate for borrowing or lending, write down one equation where the only unknown is C, where **C represents the amount of money you are spending on hamburgers at t = 170** such that you would be able to satisfy your 2 retirement goals.

Real approach:

$$\frac{1,000}{0.01-0.006}\left(1-\frac{(1.006)^{121}}{(1.01)^{121}}\right)\frac{1}{(1.01)^{79}}$$
$$=\frac{C/(1.003)^{170}}{0.01-(-0.04)}\left(1-\frac{(1-0.04)^{21}}{(1+0.01)^{21}}\right)\frac{1}{(1.01)^{169}}$$
$$+\frac{[C/(1.003)^{170}](1-0.04)^{20}(1.02)}{0.01-0.02}\left(1-\frac{(1+0.02)^{10}}{(1+0.01)^{10}}\right)\frac{1}{(1.01)^{190}}+\frac{\left[\frac{100}{0.01}\right]}{1.01^{200}}$$

One could also use the nominal approach for all, or some, terms.

5) (7 points, 10 mins) You are given the following information: at t = 0: the price of a 10-year zero coupon bond with FV = $5,000 is $3,500; $f_{3,12}$ = 6%. A bank is offering the following product: for every $1 that you give the bank at t = 10, the bank will give you back $1.5 at t = 15, or for every $1 that you borrow from the bank at t = 10, you will have to pay back $1.5 at t = 15. Write down one equation where the only unknown is $r_{0,3}$. You do not need to solve the equation.

$$(1+r_{0,3})^3(1.06^{12})=\frac{5,000}{3,500}(1.5)$$

6) (8 points, 10 mins) Today is t = 0 and the risk free rate is 3%. Risky Project A has a 5% cost of capital, requires an initial cost of 100 at t = 0 and will provide cash inflows in the form of a constant perpetuity, where the first cash flow is at t = 2. Project A has an IRR of 10%. What is the NPV of Project A?

Let Z equal the level of the first cash flow at t = 2. The IRR equation satisfies:

0 = -100 + [Z/IRR]/(1+IRR) = -100 + [Z/0.10]/(1.10)

- **➔ Z = 11**

NPV = -100 + [11/0.05]/1.05 = 109.52

7) (5 points, 5 mins) Given at t = 0:

- Bond A: 2-year, 3% coupon bond with face value $1,000 is selling for $1,000
- Bond B: 1-year, 4% coupon bond with face value $1,000 is selling for $950

Write down one equation where the only unknown is $r_{0,2}$. You do not need to solve the equation.

First back out $r_{0,1}$ and $r_{0,2}$

$$950 = \frac{1{,}040}{(1+r_{0,1})}$$

→ $r_{0,1}$ = 9.47%

$$1{,}000 = \frac{30}{(1+r_{0,1})} + \frac{1030}{(1+r_{0,2})^2} = \frac{30}{(1+0.0947)} + \frac{1030}{(1+r_{0,2})^2}$$

8) (10 points, 15 mins) You are given the following information:
- Bond A is a 1-year, 6% coupon bond with face value $8,000 and YTM = 4%
- Bond B is a 2-year, zero coupon bond with face value $30,000 and YTM = 6%
- Bond C is a 3-year, 7% coupon bond with face value $20,000 and YTM = 7%
- Bond D is a 4-year, zero coupon bond with face value $8,000 and YTM = 9%
- Bond E is a 5-year, zero coupon bond with face value $2,000 and YTM = 11%

- A financial institution is offering the following product:
- the client pays the financial institution $200,000 at t = 0 and another $100,000 at t = 1
- the financial institution pays the client $40,000 at t = 2, X at t = 3, and $60,000 at t = 4

You may take it as given that if X were equal to 268,053.61, the product would be fairly priced.

Suppose, however, that X = (268,053.61 + 200). You are a Hedge Fund manager. You want to create an arbitrage by buying 1 unit of the product from the financial institution and trading the various bonds listed above at t = 0, such that your arbitrage profit is realized at t = 0.

a) Set up a system of equations and unknowns such that if you were to solve it, you would know how many units of each bond to trade. **For each variable in your equation, make sure to specify if you are assuming that the value of that variable corresponds to the number of units you are SHORTING or if it represents the number of units BUYING.** You do not need to solve the equations.

In the solutions below, each variable represents the number of units you are shorting of each bond. If you defined one or more variables as the number of units long, then they would appear on the other side of the equation.

outflows = inflows

t = 1 condition:
100,000 + C(1,400) + A(8,480) =0

t = 2 condition:
B(30,000) + C(1,400) = 40,000

t = 3 condition:
C(21,400) = (268,053.61 + 200)

t = 4 condition:
D(8,000) = 60,000

b) Repeat part a, but this time construct the arbitrage in such a way that the profit is realized at t = 3.

t = 0 condition:

$$200,000 = A\frac{8,480}{1.04} + B\frac{30,000}{1.06^2} + C(20,000) + (D)\frac{8,000}{1.09^4}$$

t = 1 condition:
100,000 + C(1,400) + A(8,480) =0

t = 2 condition:
B(30,000) + C(1,400) = 40,000

t = 4 condition:
D(8,000) = 60,000

9) (10 points, 10 mins) At t = 0 you buy a 4-year, 4% coupon bond with FV = $10,000 and 4% YTM.
At t = 1 the risk free rate turned out to be 10% for all maturities.
At t = 2 $r_{2,1}$ = 5%, $r_{2,2}$ = 8%, $r_{2,3}$ = 7%, $r_{2,4}$ = 8.5%, $r_{2,5}$ = 9%, $r_{2,6}$ = 10%, $r_{2,7}$ = 11%
At t = 3 : $r_{3,1}$ = 3%, $r_{3,2}$ = 2%, $r_{3,3}$ = 1%, $r_{3,4}$ = 1%, $r_{3,5}$ = 1%, $r_{3,6}$ = 1%, $r_{3,7}$ = 1%
At t = 4 : $r_{4,1}$ = 5%, $r_{4,2}$ = 6%, $r_{4,3}$ = 7%, $r_{4,4}$ = 8%, $r_{4,5}$ = 9%, $r_{4,6}$ = 10%, $r_{4,7}$ = 11%
At t = 5 the risk free rate turned out to be 9% for all maturities.

a) What is the price of the bond at t = 0?

P = FV = 10,000 since c = YTM.

b) **<u>At t = 2</u>** you sell the bond right after the t = 2 coupon has been paid. Assume that the coupon you received at t = 1 was invested in a **2-year 10%-coupon bond**. Write down an equation for the realized compound yield of your two-year investment. You do not need to solve the equation.

$$y_{realized} = \left[\frac{40 + \frac{440}{1.05} + 400 + \left(\frac{400}{1.05} + \frac{10,400}{1.08^2}\right)}{10,000}\right]^{\frac{1}{2}} - 1$$

With the $400 you got at t = 1, you bought a 2-yr, 10% coupon bond, which matures at t = 3, and whose FV you can confirm equals $400 since $440/1.10^2 + 40/1.10 = 400$ (Recall, at t = 1 the yield curve is flat at 5%.) At t = 2, this bond pays out a $40 coupon, which is the first term in the numerator. At t = 2, you sell that bond (which has one cash flow left at t = 3) at a price of 440/1.05, since the 1-year rate at t = 2 is 5%. This is the second term.

At t = 2 you also receive a $400 coupon from the original bond. This is the third term.

Finally, at t = 2, you sell the original bond, whose price is given in brackets, and is obtained by discounting the remaining two cash flows at the 1-year and 2-year rates prevailing at t = 2.

10) (10 points, 10 mins) For each of the below, CIRCLE a, b, or c. NO EXPLANATION REQUIRED.

A. Interest rate is positive and the inflation rate is positive. Do you prefer receiving:
   a. $1 at t = 0
   b. 1 real dollar at t = 10
   c. Not enough information to determine whether a or b is preferred

   PV of b is $1/(1+r_r)^{10}$. Since we don't know whether the real interest rate, r_r, is positive or negative, we don't know whether the PV of b is greater or less than $1. (Recall also that at t = 0, 1 real dollar is equivalent to $1.)

B. Interest rate is 5% and the inflation rate is 5%. Do you prefer receiving:
   a. 10 real dollars at t = 10
   b. 11 real dollars at t = 12
   c. Not enough information to determine whether a or b is preferred

   Since the nominal rate equals the inflation rate, the real interest rate is 0%. Discounting 11 real dollars from t = 12 back to t = 10 thus gives 11 real dollars at t = 10, which is better than 10 real dollars at t = 10.

C. Inflation rate is positive and the real risk free rate is positive. Do you prefer receiving:
   a.$10 at t = 10
   b. $11 at t = 11
   c. Not enough information to determine whether a or b is preferred

   Discounting the $11 cash flow from t = 11 to t = 10 gives 11/(1+r) at t = 10. But since we don't know whether the interest rate is greater or less than 10%, we don't know whether 11/(1+r) is greater or less than $10.

D. Real interest rate is negative and inflation is positive. Do you prefer:
   a. 1 real dollar at t = 10
   b. $1 at t = 0
   c. Not enough information to determine whether a or b is preferred

   PV of option a is $1/(1+r_r)^{10}$. Since the real interest rate, r_r, is negative, the PV of option a is greater than $1. (Recall also that at t = 0, 1 real dollar is equivalent to $1.)

11) (10 points, 10 mins) For each of the below, indicate True or False to the LEFT of the statement. No explanation is required.

a) Given: At t = 0 you bought a 3-year, 9% coupon bond with a 7% YTM. You held the position until t = 3. Each coupon that was received prior to t = 3 was reinvested and rolled over at a 7% interest rate. Statement: The realized compound yield on the investment was 7%.
True. For a coupon paying bond held to maturity, and where coupons are reinvested and rolled over at a rate equal to the bond's initial YTM, the realized compound yield will equal the bond's initial YTM. (Initial YTM refers to the YTM when you bought the bond).

b) Given: At t = 0 you bought a 4-year zero coupon bond with a 9% YTM. Two years later you sold the bond when it was trading at a 12% YTM. Statement: The realized compound yield on your two-year investment was somewhere between 9% and 12%.
False, it will be below 9%. Recall that for a zero coupon bond bought and sold at the same YTM< the realized compound yield will equal that YTM. In this case, since the final YTM is higher, then final bond price is lower than if the YTM were unchanged, so the realized compound yield is less than the initial YTM.

c) Given: At t = 0 a 10-year zero coupon bond had an 8% YTM. You bought the bond at t = 2 when it had 8 years left to maturity and was trading at a 7% YTM. You sold the bond 3 years later when its YTM was greater than 7%. Statement: Your realized compound yield from t = 2 to t = 5 was less than 7%. **True. Same logic as part b. Note, what matters is the YTM when you bought and sold the bond. The YTM at t = 0 is irrelevant here.**

d) Given: From t = 3 to t = 4 the price of a risk free bond **decreased**, and its YTM also **decreased**. Statement: This must be a premium bond. **True. If YTM remains unchanged, then after 1 year, a discount bond's price (where c < YTM) will increase (see pull to par graph). If the YTM decreased, the price would increase even further. So If price and YTM decrease it must be for a premium bond (where c > YTM). Note: while if there is no passage of time, a decrease in YTM would imply an increase in price. But if there is a passage of time, it is possible for YTM and price to both decrease.**

MGCR 341: Midterm Formula Sheet

$$HPR = \frac{\text{ending price}}{\text{beginning price}} - 1$$

$$\text{annualized } HPR = (1+HPR)^{1/t} - 1 = \left(\frac{\text{ending price}}{\text{beginning price}}\right)^{1/t} - 1$$

$$E[r] = p_1 r_1 + ... + p_S r_S = \sum_{i=1}^{S} p_i r_i$$

$$P_{Bond}\left(1+y_{realized}\right)^T = FW$$

$$\rho_{A,B} = \frac{\text{cov}[r_A, r_B]}{\sigma_A \sigma_B}$$

$$y_{\text{realized}} = \left(\frac{\text{FW}}{\text{beginning price}}\right)^{1/t} - 1$$

$$E(a+X) = a + E(X)$$
$$E(aX) = aE(X)$$
$$E(X+Y) = E(X) + E(Y)$$
$$E(aX+bY) = aE(X) + bE(Y)$$

$$Var(a+X) = Var(X)$$
$$Var(aX) = a^2 Var(X)$$
$$Cov(aX, bY) = abCov(X,Y)$$
$$Var(aX+bY) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X,Y)$$
$$Var(X) = 0 \text{ if X is a constant}$$
$$Cov(X,X) = Var(X)$$
$$Cov(X,Y) = 0 \text{ if either X or Y is a constant}$$

$$S = \frac{E(r_p) - r_f}{\sigma_p}$$

$$\text{E}(\text{r}_\text{p}) = \sum_{i=1}^{n} \text{w}_\text{i} \cdot \text{E}(\text{r}_\text{i})$$

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 \cdot \sigma_i^2 + 2\sum_{i \neq j} w_i w_j \text{cov}(r_i, r_j)$$

$$E[r_i] = r_f + \beta_i \left(E[r_m] - r_f\right)$$

$$\sigma_p^2 = w_A^2 \cdot \sigma_A^2 + w_B^2 \cdot \sigma_B^2 + 2 w_A w_B \text{cov}(r_A, r_B)$$

$$\beta_i = \frac{\text{cov}(r_m, r_i)}{\text{var}(r_m)} = \frac{\sigma_{i,m}}{\sigma_m^2} = \frac{\sigma_i}{\sigma_m}\rho_{i,m}$$

$$\beta_P = w_1 \times \beta_1 + ... + w_N \times \beta_N$$

$$D = 1 \times \frac{C_1/(1+y)}{\text{P}} + 2 \times \frac{C_2/(1+y)^2}{\text{P}} + ... + n \times \frac{(F+C_n)/(1+y)^n}{\text{P}}$$

$$\text{MD} = \text{D}/(1+\text{YTM})$$

$$\frac{\Delta \text{P}}{\text{P}} \approx -\text{MD} \times \Delta \text{y}$$

$$\text{PV (perpetuity)} = \text{C/r}$$

$$PV(annuity) = \frac{\text{C}}{\text{r}}\left(1 - \frac{1}{(1+r)^n}\right)$$

$$\text{PV (growing perpetuity)} = \text{C/(r - g)}$$

$$PV(growing\_annuity) = \frac{\text{C}}{\text{r - g}}\left(1 - \frac{(1+g)^n}{(1+r)^n}\right)$$

$$1 + EAR = \left(1 + \frac{APR}{k}\right)^k$$

$$1 + r_r = \frac{1+r}{1+i}$$

$$CF_{real,t} = \frac{CF_{nominal,t}}{(1+i)^t}$$