Solutions 2.28 – 35

2.28

Supposons que $x(t)$ est un signal à énergie

$$\Psi_y(\tau) = \int_{-\infty}^{\infty} A x(t) e^{+j2\pi f_0 t} \cdot A x^*(t-\tau) e^{-j2\pi f_0 (t-\tau)} dt$$

$$= \int_{-\infty}^{\infty} A^2 x(t) x^*(t-\tau) e^{j2\pi f_0 \tau} dt$$

$$= A^2 e^{j2\pi f_0 \tau} \int_{-\infty}^{\infty} x(t) x^*(t-\tau) dt$$

$$\Psi_y(\tau) = A^2 e^{j2\pi f_0 \tau} \Psi_x(\tau)$$

$$G_y(f) = \mathcal{F}\{A^2 \Psi_x(\tau) e^{j2\pi f_0 \tau}\}$$
$$= \underline{\underline{A^2 G_x(f - f_0)}}$$

$$E_y = \Psi_y(0) = A^2 \Psi_x(0) e^{j2\pi f_0 (0)}$$
$$= A^2 \Psi_x(0)$$
$$= \underline{\underline{A^2 E_x}}$$

avec la densité spectrale

$$y(t) = A x(t) e^{j2\pi f_0 t}$$
$$Y(f) = A X(f - f_0)$$
$$|Y(f)|^2 = A^2 |X(f - f_0)|^2$$
$$G_y(f) = A^2 G_x(f - f_0)$$
$$\Psi_y(\tau) = \mathcal{F}^{-1}\{G_y(f)\} = A^2 \Psi_x(\tau) e^{j2\pi f_0 \tau}$$

$$E_y = \int_{-\infty} A^2 G_x(f - f_0) df \qquad \text{soit } f' = f - f_0$$
$$df' = df$$

$$E_y = \int_{-\infty}^{\infty} A^2 G_x(f')\,df'$$

$$= A^2 \int_{-\infty}^{\infty} G_x(f')\,df'$$

$$= \underline{\underline{A^2 E_x}}$$

Exercice 2.29

$$y(t) = A\,x(t)\cos(2\pi f_0 t + \phi) \qquad f_0 > B_x$$

$$R_y(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} y(t)\,y^*(t-\tau)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{T/2} A^2 x(t)\,x^*(t-\tau)\cos(2\pi f_0 t+\phi)\cos(2\pi f_0(t-\tau)+\phi)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{T/2} \frac{A^2}{2} x(t)\,x^*(t-\tau)\cos 2\pi f_0 \tau + \frac{A^2}{2} x(t)\,x^*(t-\tau)\cos(4\pi f_0 t - 2\pi f_0 \tau + 2\phi)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{T/2} \frac{A^2}{2} x(t)\,x^*(t-\tau)\cos 2\pi f_0 \tau\,dt$$

$$+ \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{T/2} \frac{A^2}{2} x(t)\,x^*(t-\tau)\cos(4\pi f_0 t - 2\pi f_0 \tau + 2\phi)\,dt$$

$$= \frac{A^2}{2} R_x(\tau)\cos 2\pi f_0 \tau$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{T/2} x(t)\,x^*(t-\tau)\,dt \cdot \frac{A^2}{2}\cos 2\pi f_0 \tau$$

0 → Parseval

$$+ \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{T/2} \frac{A^2}{2} x(t)\,x^*(t-\tau)\cos(4\pi f_0 t - 2\pi f_0 \tau + 2\phi)\,dt$$

donc $R_y(\tau) = \frac{A^2}{2} R_x(\tau) \cos 2\pi f_0 \tau$

$P_y = R_y(0) = \frac{A^2}{2} R_x(0)$

$= \frac{A^2}{2} P_x$

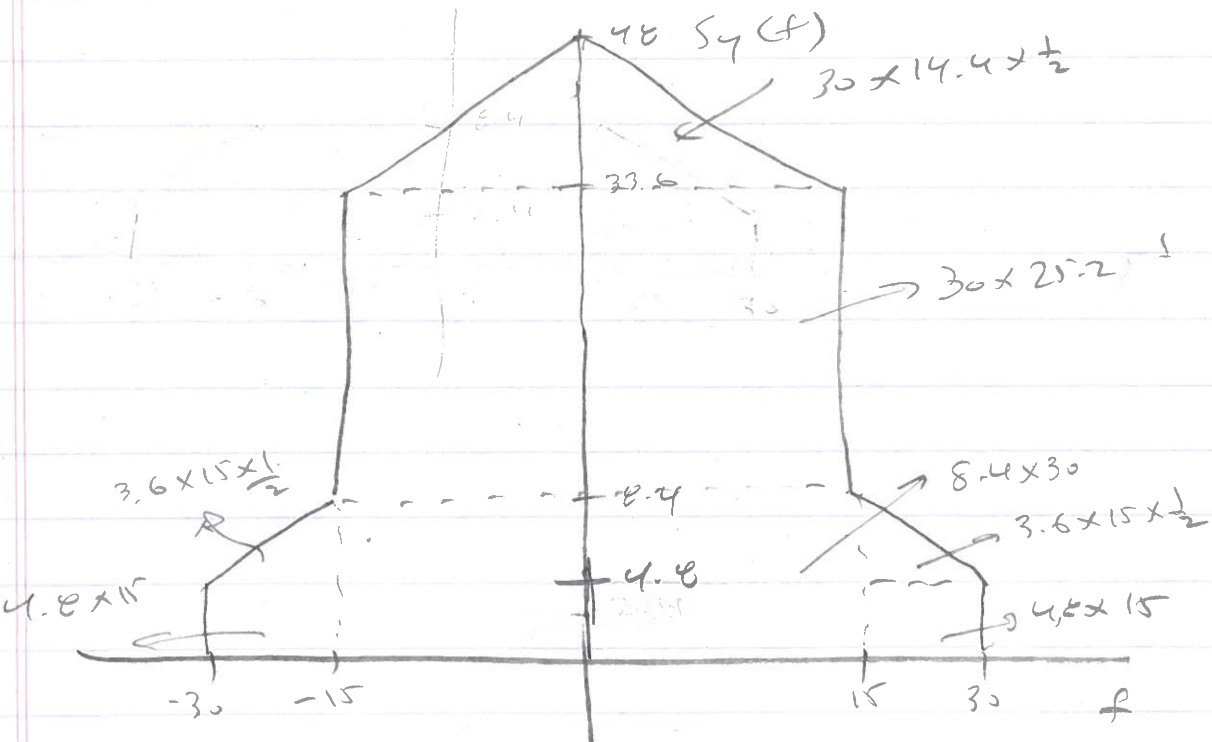
$S_y(f) = \mathcal{F}\{R_y(\tau)\} = \frac{A^2}{4} S_x(f-f_0) + \frac{A^2}{4} S_x(f+f_0)$

Exercice 2.30

(a) $P_x = \int_{-\infty}^{\infty} S_x(f)\, df = 100 \times 12 \times \frac{1}{2} = 600\,W$

(b) $S_y(f) = S_x(f)\,|H(f)|^2$

$$= \begin{cases} 4 S_x(f) & -15 \le f \le 15 \\ S_x(f) & -30 \le f \le -15 \text{ et } 15 \le f \le 30 \\ 0 & \text{autrement} \end{cases}$$



(c) P_y = la somme des surfaces = $\underline{1422\,W}$

Exercice 2.31

$H_{HT}(f) = -j\,\mathrm{sgn}(f)$

$\mathrm{sgn}(t) \xrightarrow{\mathcal{F}} \dfrac{1}{j\pi f}$

donc $\mathcal{F}\left\{\dfrac{1}{j\pi t}\right\} = \mathrm{sgn}(-f) = -\mathrm{sgn}(f)$

et $\dfrac{j}{j\pi t} = \dfrac{1}{\pi t} \xrightarrow{\mathcal{F}} j \cdot -\mathrm{sgn}(f) = -j\,\mathrm{sgn}(f)$
$= H_{HT}(t)$

$\boxed{\text{donc } h_{HT}(t) = \frac{1}{\pi t}}$

Exercice 2.32

$s(t) = 3\sin(2\pi 200t) + 2\cos(2\pi 20t)\cos(2\pi 210t)$
$= 3\sin(2\pi 200t) + \cos(2\pi 230t) + \cos(2\pi 190t)$

(a) $S(f) = \frac{3}{2j}\delta(f-200) - \frac{3}{2j}\delta(f+200)$
$+ \frac{1}{2}\delta(f-230) + \frac{1}{2}\delta(f+230)$
$+ \frac{1}{2}\delta(f-190) + \frac{1}{2}\delta(f+190)$

(b) $S_+(f) = \frac{3}{j}\delta(f-200) + \delta(f-230) + \delta(f-190)$

$\tilde{S}(f) = \frac{3}{j}\delta(f+10) + \delta(f-20) + \delta(f+20)$

$\tilde{s}(t) = \frac{3}{j}e^{-j2\pi 10t} + e^{j2\pi 20t} + e^{-j2\pi 20t}$

$$\tilde{s}(t) = \frac{3}{j}\left(\cos 2\pi 10t - j\sin 2\pi 10t\right) + 2\cos 2\pi 20t$$

$$= -3j\cos 2\pi 10t - 3\sin 2\pi 10t + 2\cos 2\pi 20t$$

$$\mathrm{Re}\{\tilde{s}(t)\} = 2\cos 2\pi 20t - 3\sin 2\pi 10t = s_I(t)$$

$$\mathrm{Im}\{\tilde{s}(t)\} = -3\cos 2\pi 10t = s_Q(t)$$

$$s(t) = s_I(t)\cos 2\pi 210t - s_Q(t)\sin 2\pi 210t$$

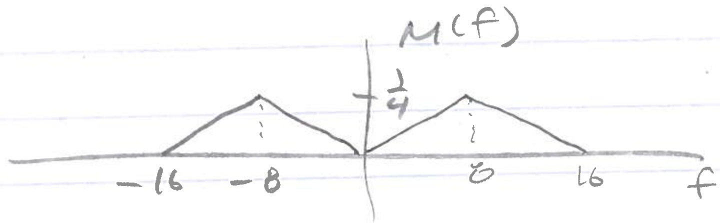
$$s(t) = \left(2\cos 2\pi 20t - 3\sin 2\pi 10t\right)\cos 2\pi 210t + 3\cos 2\pi 10t \sin 2\pi 210t$$

Exercice 3.1

$m(t) = 4\,\mathrm{sinc}^2(8t)\cos 2\pi 8t$

(a) $M(f) = 2X(f-8) + 2X(f+8)$ où $X(t) = \mathcal{F}\{\mathrm{sinc}^2(8t)\} = \frac{1}{8}\Delta\left(\frac{f}{8}\right)$

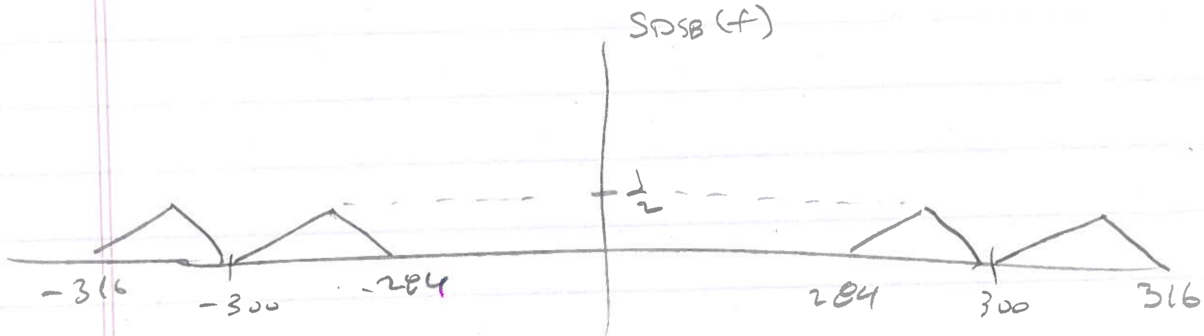
$$M(f) = \frac{1}{4}\Delta\left(\frac{f-8}{8}\right) + \frac{1}{4}\Delta\left(\frac{f+8}{8}\right)$$



(b) $s_{DSB}(t) = 4\, m(t) \cos 2\pi 300 t$

$= 16 \,\text{sinc}^2(8t) \cos 2\pi 8t \cos 2\pi 300t$

$S_{DSB}(f) = 2M(f-300) + 2M(f+300)$

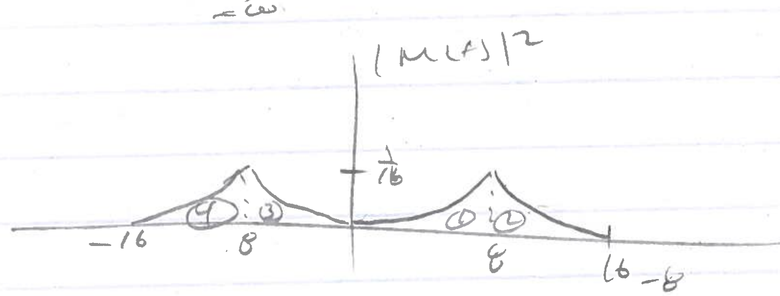


(c) $B_m = 16 \text{ Hz}$

$B_{DSB} = 316 - 284 = \underline{\underline{32 \text{ Hz}}}$

(d) $E_{DSB} = \dfrac{A^2}{2} E_m = 8 E_m$

$E_m = \displaystyle\int_{-\infty}^{\infty} |M(f)|^2 \, df$



On peut remarquer que $\displaystyle\int_{-16}^{-8} |M(f)|^2 df = \int_{-8}^{0} |M(f)|^2 df$

$= \displaystyle\int_{0}^{8} |M(f)|^2 df = \int_{8}^{16} |M(f)|^2 df$

$|M(f)|^2 = \left\{ \dfrac{1}{16}\left(\dfrac{f}{8}\right)^2 \quad 0 \le f \le 8 \right.$

$= \dfrac{f^2}{1024} \quad 0 \le f \le 8$

$E_m = 4 \displaystyle\int_{0}^{8} \frac{f^2}{1024} df$

$$E_m = \frac{4}{3} \cdot \frac{f^3}{1024} \Big|_0^8$$

$$= \frac{4 \cdot 8^3}{3072} = \underline{\underline{\frac{2}{3} J}}$$

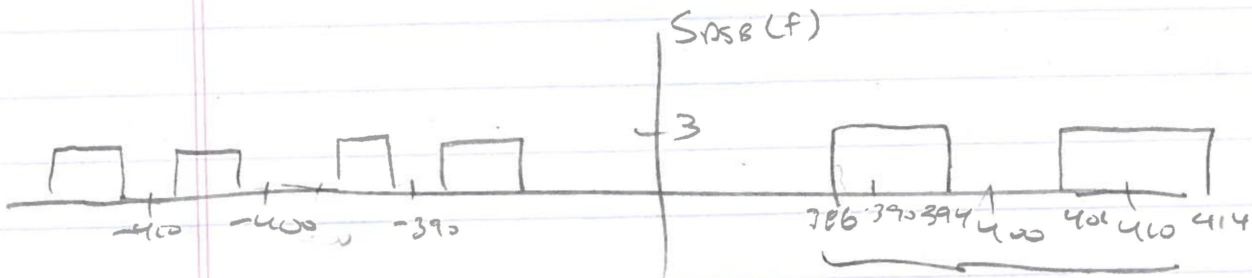
$$E_s = 8 \times \frac{2}{3} = \underline{\underline{5.333 J}}$$

Exercice 3,2

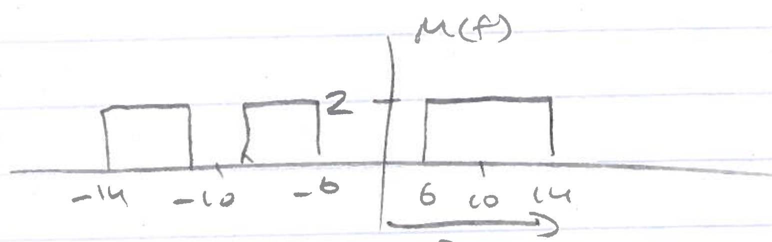
$$S_{DSB}(f) = \frac{3}{2} M(f-400) + \frac{3}{2} M(f+400)$$

$$= 3\Pi\left(\frac{f-410}{8}\right) + 3\Pi\left(\frac{f-390}{8}\right)$$

$$+ 3\Pi\left(\frac{f+390}{8}\right) + 3\Pi\left(\frac{f-410}{8}\right)$$



$$BW = 414 - 386 = \underline{\underline{28 Hz}}$$



$$B_m = \underline{\underline{14 Hz}}$$

Exercice 3.3

$k_a = 0.3 \quad m_p = 9V$

$\mu_a = 0.3 \times 9 = 2.7 > 1$

donc $S_{AM}(t)$ est surmodulé

Exercice 3.4

$m(t) = 3\cos 2\pi 20t + \cos 2\pi 35t$

$S_{AM}(t) = 8[1 + k_a m(t)] \cos 2\pi 500t$

(a) $\mu_a = 0.8 = k_a m_p$

$m_p = 4V$

donc $k_a = \frac{0.8}{4} = 0.2$

$S_{AM}(t) = 8[1 + 0.2m(t)] \cos 2\pi 500t$

(b) $P_{AM} = \frac{8^2}{2} + \frac{8^2(0.2)^2}{2} P_m$

$P_m = \frac{3^2+1}{2} = 5W$

$P_{AM} = \frac{64}{2} + \frac{64(0.2)^2 \cdot 5}{2}$

$= 32 + 6.4$

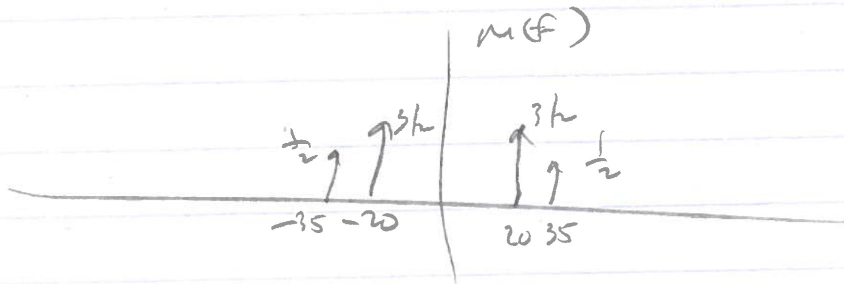
$= 38.4W$

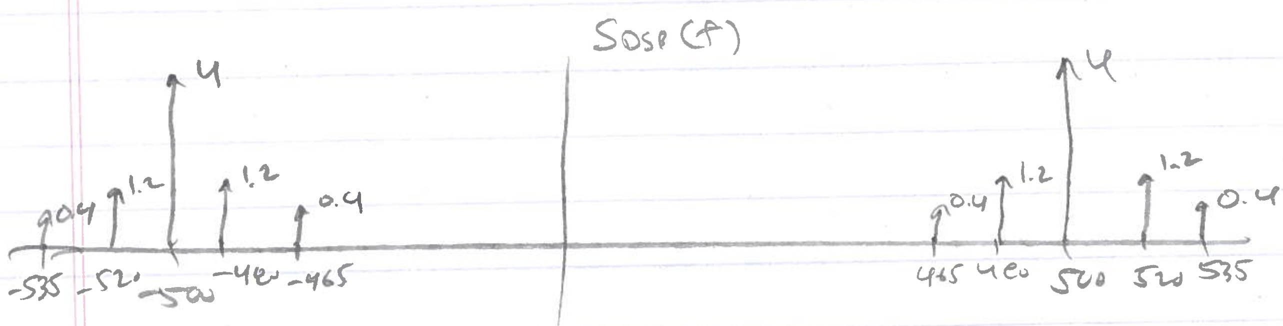
(c) $\eta = \dfrac{6.4\,W}{38.4\,W} = \underline{\underline{0.1667}}$

(d)

$$S_{AM}(f) = \mathcal{F}\{8[1 + k_a m(t)] \cos 2\pi 500 t\}$$
$$= \mathcal{F}\{8 \cos 2\pi 500 t + 1.6\, m(t) \cos 2\pi 500 t\}$$
$$= 4\delta(f-500) + 4\delta(f+500) + 0.8 M(f-500)$$
$$+ 0.8 M(f-500)$$

$$M(f) = \frac{3}{2}\delta(f-20) + \frac{3}{2}\delta(f+20)$$
$$+ \frac{1}{2}\delta(f-35) + \frac{1}{2}\delta(f+35)$$





Exercice 3.5

$$s_{QAM} = A_c m_1(t) \cos 2\pi f_c t + A_c m_2(t) \sin 2\pi f_c t$$

$$s_{QAM}^2(t) = A_c^2 m_1^2(t) \cos^2 2\pi f_c t + A_c^2 m_2^2(t) \sin^2 2\pi f_c t + 2A_c m_1(t) m_2(t) \cos 2\pi f_c t \sin 2\pi f_c t$$

$$P_{QAM} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} s_{QAM}^2(t)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T/2}^{T/2} A_c^2 m_1^2(t) \cos^2 2\pi f_c t\, dt$$

puissance de $A_c m_1(t) \cos 2\pi f_c t$

$$+ \lim_{T\to\infty} \frac{1}{2T} \int_{-T/2}^{T/2} A_c^2 m_2(t) \sin^2 2\pi f_c t\, dt$$

puissance de $A_c m_2(t) \sin 2\pi f_c t$

$$\underbrace{+ \lim_{T\to\infty} \frac{1}{2T} \int_{-T/2}^{T/2} 2A_c m_1(t) m_2(t) \underbrace{\cos 2\pi f_c t \sin 2\pi f_c t}_{\frac{1}{2}\sin 4\pi f_c t}\, dt}_{=0 \text{ si on fait par Parseval, } 2f_c \gg B_{m_1} + B_{m_2}}$$

Donc $P_{QAM} = \frac{A_c^2}{2} P_{m_1} + \frac{A_c^2}{2} P_{m_2}$