Final Exam – Econ 302 – W2011T2

Please write your name clearly, as appears on official lists

Write your answers on the exam. Use booklets for scratch paper only.

This is a closed book exam. You may not use notes, text, or a calculator

You have 50 minutes

Name:

Question 1 (20 points)
For each of the following statements, determine whether it is true or false. (Note that you must explain or provide a counter-example; guessing without explaining will not be granted with any points).

(a) Nominal GDP is always higher than Real GDP.
False. We know that depending on the choice of the base year, and the growth calculation method, the GDP deflator can be smaller or larger than 1, which means that Nominal GDP can be either higher or lower than real GDP.

(b) If one Canadian Dollar can be exchanged with one US dollar, then we can say that purchasing power parity holds (or: neither currency is over or under valued).
False. This is just a statement about the nominal exchange rate. Over or under valuation is a statement about the real exchange rate.

(c) If we employ more labor (N), then the labor share of income must increase.
False. Example: Cobb-Douglas production function: the labor share is constant.

(d) If two firms have the same production function and the same level of TFP, then efficiency requires that they employ the same **levels** of capital and labor.

False. Example (from problem set #2): A Cobb Douglas production function implies that both firms have to employ the same **ratio** $\frac{K}{N}$, but not necessarily the same levels of K and N.

Question 2 (40 points)
Assume all economies in the world produce output using a production function that uses capital (K), labor (N), and land (L) as inputs. The production function is Cobb-Douglas: $Y = AK^{\alpha}L^{\beta}N^{1-\alpha-\beta}$, with $\alpha < 1$ and $\beta < 1$. The purpose of this example is to think about the development accounting exercise in this context.
(a) Provide an example for potential differences in TFP that can be "land specific".
(b) Show that this production function is Constant Returns to Scale in capital, land, and labor.
(c) What could be a useful data proxy (or estimate) for β.
(d) Denote the capital labor ratio by k ($k = \frac{K}{N}$) and the land-labor ratio by l ($l = \frac{L}{N}$). Show that GDP per capita ($y = \frac{Y}{N}$) can be expressed as $y = Ak^{\alpha}l^{\beta}$.
(e) Assume you have data on GDP per capita, the capital labor ratio, and the land-labor ratio for two economies. Briefly describe how would you calculate the difference in TFP between the two economies.

(a) The simplest example is thinking how fertile land is, which is an actual difference between countries. Another related example can be described as the efficiency in utilization of land. (For example: in agricultural industries, do we use our land for the most suitable crops).

(b) $AF(\lambda K, \lambda L, \lambda N) = A(\lambda K)^{\alpha}(\lambda L)^{\beta}(\lambda N)^{1-\alpha-\beta} = \lambda^{\alpha+\beta+1-\alpha-\beta}AK^{\alpha}L^{\beta}N^{1-\alpha-\beta} = \lambda AF(K, L, N)$

(c) One way is to try and directly estimate. An easier proxy to find is the income share of land. We know that with a cobb-douglas production function the income share equals the elasticity.

(d) $y = \frac{AK^{\alpha}L^{\beta}N^{1-\alpha-\beta}}{N} = AK^{\alpha}L^{\beta}N^{-\alpha-\beta} = AK^{\alpha}N^{-\alpha}L^{\beta}N^{-\beta} = Ak^{\alpha}l^{\beta}$

(e) First stage is to find estimates to α and β. A proxy can be the income shares.
Second stage is just substitution into the following:

$$\frac{A^1}{A^2} = \frac{\frac{y^1}{(k^1)^{\alpha}(l^1)^{\beta}}}{\frac{y^2}{(k^2)^{\alpha}(l^2)^{\beta}}}$$

Question 3 (40 points)
Assume a Solow Model with a constant population growth rate $n > 0$, a constant capital depreciation rate $\delta > 0$, and a constant exogenous saving rate $s > 0$.
Production in this economy involves a different structure than usual. In particular, when $k < \bar{k}$ the production function is **convex**. Once the level of k is "high enough" ($k > \bar{k}$), the production function is concave as usual.
You can think about the following example for such a production function:

$$
\begin{aligned}
y &= Ak^2 && \text{if} \quad k \leq \bar{k} \\
y &= Ak^{\alpha} \quad , \alpha < 1 && \text{if} \quad k > \bar{k}
\end{aligned}
$$

(a) Plot the production function using a diagram with k on the horizontal axis, and output per worker (y) on the vertical axis. (note: you don't have to be "exact"; just illustrate how the production function should look).
(b) Add the "required investment line" and the saving curve to the graph. Using the graph, show that there may be two potential steady state levels of k (not including $k = 0$).
(c) Are these steady states "stable"? i.e. for each potential steady state, consider levels of k that are below and above the steady state levels and explain whether the economy converges back to this steady state.
(d) Assume that the economy receives foreign aid in the form of final output y. Is it possible that the aid would be large enough such that there will be a single steady state? If so, is that steady state stable?

Graph: a function that is convex until $\bar{k}$ and then concave: "S Shaped" starting from zero.
With the required investment line, we can see that as long as the linear line is not too steep, it will cross the saving curve once in the convex area, and once in the concave area.

Let's call the steady state in the convex area k_1^{ss}.
If $k < k_1^{ss}$ then actual investment is lower than the required investment. This means that k will be lower next period, therefore not moving towards the steady state point. (It will actually converge to zero).
If $k > k_1^{ss}$ then actual investment is higher than the required investment, which means that k will be higher next period. Again, this means that k "moves away" from this steady state.
Conclusion: k_1^{ss} is not a stable steady state.

Denote the crossing of saving and required investment at the concave area by k_2^{ss}. This steady state is stable using the same logic as we described in class. At any point below k_2^{ss} we invest more than enough to compensate for depreciation and population growth rate $\rightarrow k$ converges back to steady state. At any point above k_2^{ss} we invest less than required, which means that we'll converge to k_2^{ss}.

Foreign aid implies a vertical shift of the production (and therefore saving) curves. (See problem set for a similar example). It is easy to see that when the vertical shift is large enough, we avoid the first crossing (at the convex area), and remain with one steady state. By the same argument as before, this is a stable steady state.