CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	209	All except EC
Examination	**Date**	**Pages**
Final	April 2018	2
Instructors		**Course Examiner**
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Special Instructions

- ▷ Ruled booklets to be used.
- ▷ Only approved calculators allowed.

[MARKS]

[10] **1.** Find the derivatives for each of the following functions: (DO NOT SIMPLIFY):

(a) $g(x) = \left(e\sqrt{x} - \frac{7}{x^3}\right)\left(e^4 - x^4\right)$

(b) $h(x) = x^3 ln(3x) - e^{(-x^3+x)}$

[10] **2.** Graph $x^2 - 100 = y^2$, find y' by implicit differentiation, and find the slopes of the graph when $x = -10$.

[10] **3.** Use the price-demand equation $x = (40 - p)1000$ to find the values of p for which the demand is elastic and for which the demand is inelastic.

[10] **4.** A discount store is presently selling 200 television sets monthly. If the store invests x thousand dollars in an advertising campaign, the ad company estimates that sales will increase to $N(x) = 4x^3 - 0.25x^4 + 500$, for $0 \le x \le 12$. When is the rate of change of sales increasing and when is it decreasing? What is the point of diminishing returns?

[10] **5.** (a) Find $\displaystyle\lim_{x\to-\infty} \frac{7 - x^2}{x^4 + 13x}$.

(b) Give an example of a function f defined for all real numbers which has the property that $\displaystyle\lim_{x\to-\infty} f = 7$ and $\displaystyle\lim_{x\to\infty} f = -\infty$.

–PLEASE TURN OVER

[10] **6.** A point is moving on the graph of $x^2y = 12$. When the point is at $(-2, 3)$, its x coordinate is increasing by 7 units per second. How fast is the y coordinate changing at that moment?

[5] **7.** Find the differential dh if $h = 2x^2 - 3x, x = 2$ and the change in x is 0.1.

[7] **8.** You are told a country has Lorenz curve $y = \frac{x^2}{10}$. You want to find its Gini index. What conclusion can you draw?

[8] **9.** For $f(x) = 12x - x^3$ find the absolute maximum and minimum, if either exists, on the interval [-3, 3].

[10] **10.** Find the following:

(a) $\int (\frac{2}{\sqrt{x}} - \frac{1}{x^4})dx$

(b) $\int_3^6 \frac{-7}{x+3}dx$

[10] **11.** Find the area bounded by $f(x) = 6 - x^2$ and $g(x) = x$.

#1

(a) $g(x) = \left(e\sqrt{x} - \frac{7}{x^2}\right)\left(e^4 - x^4\right)$

$= e^5 x^{1/2} - e x^{9/2} - 7e^4 x^{-2} + 7x^2$ Foil to make more simple

$g'(x) = \frac{e^5}{2} x^{-1/2} - \frac{9ex^{7/2}}{2} + 14e^4 x^{-3} + 14x$ Power Rule

(b) $h(x) = \underbrace{x^3}\underbrace{\ln(3x)} - e^{(-x^3+x)}$

$u = x^3$ $v = \ln(3x)$

$u' = 3x^2$ $v' = \frac{1}{3x}(3)$

$h'(x) = 3x^2 \ln(3x) + x^3\left(\frac{1}{x}\right) - e^{-x^3+x}(-3x^2+1)$

prod rule = $u'v + uv'$

#2 Graph $x^2 - 100 = y^2$, find y' by implicit diff, and find the slopes of the graph when $x = -10$

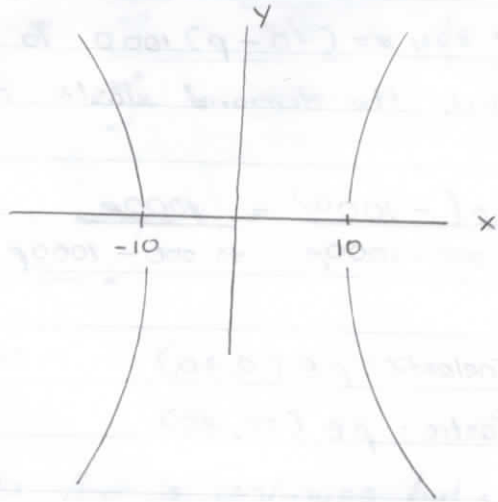
slope

$x^2 - 100 = y^2$

$2x = 2yy'$

$\frac{2x}{2y} = y'$

$y' = \frac{x}{y} = \frac{-10}{0}$ und



#3 Use price-demand eqN $x = (40 - p)1000$ to find values of p which the demand elastic and inelastic

$$\boxed{E = \frac{-px'}{x}} = \frac{-p(-1000)}{40\,000 - 1000p} = \frac{1000p}{40\,000 - 1000p} = \frac{p}{40 - p} = 1$$

$p = 40 - p$

$2p = 40$

$p = 20$

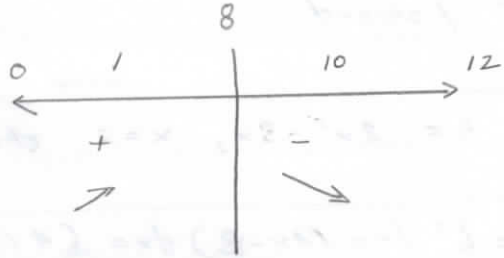
Inelastic: $p \in (0, 20)$

Elastic: $p \in (20, 40)$

#4 $N(x) = 4x - 0.25x^4 + 500$

(a) Rate of change of sales: $12x^2 - x^3$

(b)



$N'' = 24x - 3x^2 = 0$

$3x(8 - x) = 0$

$x = 0$, $x = 8$

(c) point of diminshing:

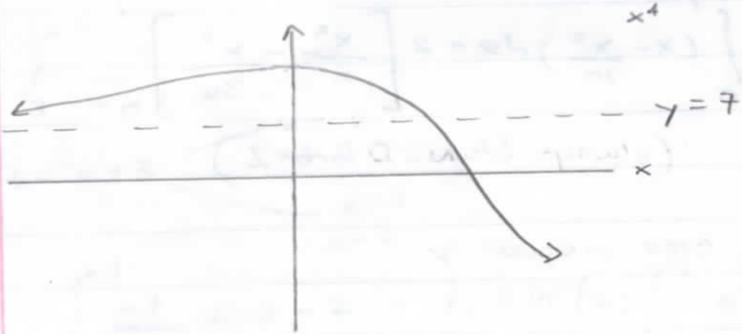
$x = 8$

N' is inc: $x \in [0, 8)$

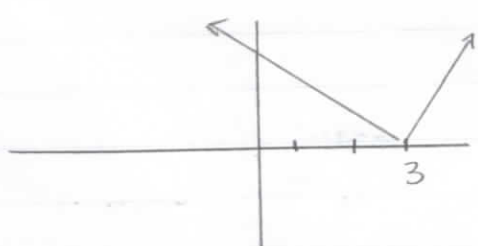
N' is dec: $x \in (8, 12]$

#5 (a) Find $\lim_{x \to -\infty} \dfrac{7 - x^2}{x^4 + 13x} = \lim_{x \to -\infty} \dfrac{\frac{7}{x^4} - \frac{x^2}{x^4}}{\frac{x^4}{x^4} + \frac{13x}{x^4}} = \lim_{x \to -\infty} \dfrac{\cancel{\frac{7}{x^4}}^{0} - \cancel{\frac{1}{x^2}}^{0}}{1 + \cancel{\frac{13x}{x^4}}} = 1$

(b)



(c) Give an example of a function that is continuos but not differentiable at $x=3$



#6 A point is moving on the graph of $x^2 y = 12$ when the point is at $(-2, 3)$, its x-coordinate is increasing by 7 units / second. How fast is the y-coordinate changing at the moment?

S3: $u = x^2$ $\quad v = y$ $\qquad 2xx'y + x^2 y' = 0$

$u' = 2xx'$ $\quad v' = y'$

S4: $2(-2)(7)(3) + (-2)^2 y' = 0$

21 units / second

#7 differential if $h = 2x^2 - 3x$, $x = 2$ change in x is 0.1

(a) $dh = h'\,dx = (4x-3)\,dx = (4(2)-3)(0.1) = 0.5$ (approx)

side Q (b) Find $\Delta h = h(x + \Delta x) - h(x) = h(2.1) - h(2)$

$= 2.52 - 2$

$= 0.52$ (exact)

#8 Lorenz curve $y = \dfrac{x^2}{10}$. Find Gini index

$$GI = 2\int_0^1 [x - f(x)]\,dx = 2\int_0^1 \left(x - \frac{x^2}{10}\right) dx = 2\left[\frac{x^2}{2} - \frac{x^3}{30}\right]_0^1$$

$$= 2\left[\frac{1}{2} - \frac{1}{30}\right] = \frac{14}{15} = 0.933$$ (always btwn 0 and 1)

CONCLUSION: Close to 1, = large inequality

#9 For $f(x) = 12x - x^3$ find the abs max and min, if either exists, on the interval $[-3, 3]$

$f' = 0 \qquad 12 - 3x^2 = 0$

$3(4 - x^2) = 0$

$3(2 - x)(2 + x) = 0$

$x = 2, \quad x = -2$ on interval $[-3, 3]$

$f(-3) = 12(-3) - (-3)^3 = -9$

$f(-2) = 12(-2) - (-2)^3 = -16$ min

$f(2) = 12(2) - (2)^3 = 16$ max

$f(3) = 12(3) - (3)^3 = 9$

Abs max $= f(2) = 16$, abs min $f(-2) = -16$

#10 $\int \left(\frac{2}{\sqrt{x}} - \frac{1}{x^4} \right) dx = \int \left(2x^{-1/2} - x^{-4} \right) dx =$

(a)

$\frac{2x^{1/2}}{1/2} - \frac{x^{-3}}{-3} + C = 4x^{1/2} + \frac{x^{-3}}{3} + C$

(b) $\int_3^6 \frac{-7}{x+3}\, dx =$

$u = x + 3 \quad x = 6,\ u = 9 \quad x = 3,\ u = 6$

$\int_6^9 \frac{-7}{u}\, du = -7 = \left[-7 \ln |u| \right]_6^9 = -7 \ln 9 + 7 \ln 6 = 7(-\ln 9 + \ln 6)$

$= 7(\ln 6 - \ln 9)$

$= 7 \ln \frac{6}{9} = 7 \ln \frac{2}{3}$

#11 Find the area bounded by $f(x) = 6 - x^2$ and $g(x) = x$

$$6 - x^2 = x$$
$$0 = x^2 + x - 6$$
$$0 = (x+3)(x-2)$$
$$x = -3, \quad x = 2$$

$$A = \int_{-3}^{2} (6 - x^2 - x)\, dx = \left[6x - \frac{x^3}{3} - \frac{x^2}{2} \right]_{-3}^{2}$$

-3 $\quad$ 2

$$= \left[12 - \frac{8}{3} - 2 \right] - \left[-18 + 9 - \frac{9}{2} \right] = \frac{125}{6}$$