CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Section(s)**
Mathematics	209	All EXCE
Examination	**Date**	**Pages**
Final	December 2016	3
Instructors		**Course Examiner**
ALL		B. Raphael

Special Instructions

- ▷ Ruled booklets to be used.
- ▷ Calculators <u>not</u> allowed.

MARKS

[6] **1.** (a) Find the following limits know:- → $\sqrt{\ }$, $|x|$, x^3 ...

(i) $\displaystyle\lim_{x\to 1} \frac{3x^2+2x-1}{x^2+3x+2}$

(ii) $\displaystyle\lim_{x\to 2} \frac{2x^2-3x-2}{x^2+x-6}$

(b) Where is the function $n(x) = \dfrac{x-3}{(x+2)(x+1)}$ continuous?

[5] **2.** Find the derivative $f'(x)$ of the functions $f(x)$: (Do not simplify)

(a) f(x)= $3x^5 - 7x^3 + x - 3$

(b) f(x)= $\dfrac{x^{-8}}{8} + \sqrt[3]{x}$

[9] **3.** Find $\dfrac{dy}{dx}$ (do not simplify):

(a) $y = \dfrac{x^2-5}{e^{2x}}$

(b) $y = ln(4x^3+5)$

(c) $y = (2x^2+1)^2(4x+6)^3$

(d) $y = (7 + x^2 lnx)^3$

[7] 4. Let $f(x) = 3x^4 - 3x^2 - 7$

(a) Find the slope of the tangent line to the curve when $x = 1$

(b) Find the equation of the tangent line to the curve when $x = 1$

[13] 5. Let $f(x) = (x-2)(x^2 - 4x - 8)$

Find

(a) the critical and inflection points of $f(x)$

(b) the intervals where $f(x)$ is increasing and where it is decreasing

(c) the intervals on which $f(x)$ is concave up and on which it is concave down

(d) use the above to sketch the graph

[8] 6. A student center sells 1600 cups of coffee per day at the price of \$2.40 per cup. A market survey shows that for every \$0.05 reduction in price per cup, 50 more cups of coffee will be sold.
How much should the student center charge for a cup of coffee in order to maximize revenue?

[7] 7. Find the absolute extrema of the function $f(x) = x^3 - 6x^2 + 9x - 6$ on the interval $[-1, 5]$.

[4] 8. If interest is compounded continuously and the annual nominal interest rate is 3.4%, how long will it take for money invested to double?

[10] 9. Find the equation(s) of the tangent line(s)to the graph of $y^2 - xy = 6$ at the point(s) with $x = 1$.

[11] 10. Compute these antiderivatives:

(a) $\int (4x^6 - 3x^3 - 8)\, dx$

(b) $\int \frac{e^{-5x}}{3 + e^{-5x}}\, dx$

(c) $\int \frac{x^2}{\sqrt{x-3}}\, dx$

[10] 11. Evaluate the integrals:

(a) $\int_0^1 (x^3 - 4)\, dx$

(b) $\int_6^{10} \frac{1}{x-5}\, dx$

(c) $\int_4^7 \sqrt{x-3}\, dx$

[10] 12. Find the area bounded by the graphs of $f(x) = x^2 - 1$ and $g(x) = x - 2$ over the interval $-2 \le x \le 1$.

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#1 (a) Find the following

(i) $\lim_{x \to 1} \dfrac{3x^2+2x-1}{x^2+3x+2} =$

(ii) $\lim_{x \to 2} \dfrac{2x^2-3x-2}{x^2+x-6}$

$\lim_{x \to 1} \dfrac{3(1)^2+2(1)-1}{(1)^2+3(1)+2} = \dfrac{4}{6} = \boxed{\dfrac{2}{3}}$

$\lim_{x \to 2} \dfrac{2(2)^2-3(2)-2}{(2)^2+(2)-6} = \boxed{\dfrac{0}{0}}$

(b) where is the fn $n(x) = \dfrac{x-3}{(x+2)(x+1)}$ continuous

$x=-2$, $x=-1$ continous on $x \in \mathbb{R} \setminus \{-2,-1\}$

or $x \in (-\infty,-2) \cup (-2,-1) \cup (-1,\infty)$

#2 Find the derivative $f'(x)$ of the fns

(a) $f(x) = 3x^5 - 7x^3 + x - 3$

$f'(x) = 15x^4 - 21x^2 + 1$

(B) $f(x) = \dfrac{x^{-8}}{8} + \sqrt[3]{x}$

$f'(x) = -x^{-9} + \frac{1}{3}x^{-2/3}$

#3 Find $\frac{dy}{dx}$ (do not simplify)

$u = x^2 - 5 \quad v = e^{2x}$

$u' = 2x \quad v' = 2e^{2x}$

(a) $y = \dfrac{x^2-5}{e^{2x}}$

$y' = \dfrac{u'v - uv'}{v^2} = y' = \dfrac{(2x)(e^{2x}) - (x^2-5)(2e^{2x})}{(e^{2x})^2}$

(b) $y = \ln(4x^3+5)$

$y' = \dfrac{1}{4x^3+5} \cdot (12x^2)$

? (c) $y = (2x^2+1)^2(4x+6)^3$

$u = (2x^2+1)^2 \quad v = (4x+6)^3$

$u' = 2(2x^2+1)\cdot(4x) \quad v' = 3(4x+6)^2\cdot(4)$

$y' = u'v + uv' = 2(2x^2+1)(4x)\cdot(4x+6)^3 + (2x^2+1)\cdot 3(4x+6$

$= 4(2x^2+1)(4x+6)^2[2x(4x+6) + (2x^2+1)(3)]$

? (d) $y = (7 + x^2 \ln x)^3$

$u = x^2 \quad y = \ln x$

$u' = 2x \quad y' = \frac{1}{x}$

$y' = 3(7 + x^2 \ln x)^2 \cdot (0 + [2x(\ln x) + (x^2)(\frac{1}{x})])$

$u'y + uy'$

$3(7+x^2 \ln x)^2 \cdot (2x)(\ln x) + [x^2][\frac{1}{x}]$

#4 Let $f(x) = 3x^4 - 3x^2 - 7$

(a) Find the slope of the tangent line to the curve when $x = 1$

$$f(x) = 3x^4 - 3x^2 - 7$$
$$f'(x) = 12x^3 - 6x$$
$$12(1)^3 - 6(1) = 6$$

(b) Find the EQN of the tangent line to the curve when $x = 1$

$$y = m(x - x_1) + y_1 \rightarrow y = 6(x-1) - 7$$

$$f(1) = -7$$

#5 Let $f(x) = (x-2)(x^2 - 4x - 8)$

(a) the critical and inflection pts of $f(x)$
(b) the intervals where $f(x)$ is increasing and where it is decreasing
(c) the intervals on which $f(x)$ is concave up and on which it is conc down
(d) Use the graph to sketch.

(a) $(x-2)(x^2-4x-8) = x^3 - 4x^2 - 8x - 2x^2 + 8x + 16 = x^3 - 6x^2 + 16$

CP: $f' = 0 \Rightarrow 3x^2 - 12x = 0$

$3x(x-4) = 0$

$x = 0, \quad x = 4$

$f(0) = 0 + 0 + 16 = 16$

$f(4) = 4^3 - 6(4)^2 + 16 = -16$

CP = $(0,16)$, $(4,-16)$

IP: $f'' = 0 \Rightarrow 6x - 12$ $\quad f(2) = 0$

$6x = 12$

$x = 2$

IP = $(2,0)$

contd...

#5

(a) Find CPs and IPs
(b) the intervals where $f(x)$ is increasing and where it is decreasing
(c) the intervals on which $f(x)$ is concave up and on which it is concave down
(d) Use the above to sketch

CP and IP found in (a) previous page...

CP = $(0,16), (4,-16)$ IP = $(2,0)$

(b)
(c)

	$-\infty$	-1	0	1	4	10	∞
f'		$+$		$-$		$+$	
f		↗		↘		↗	

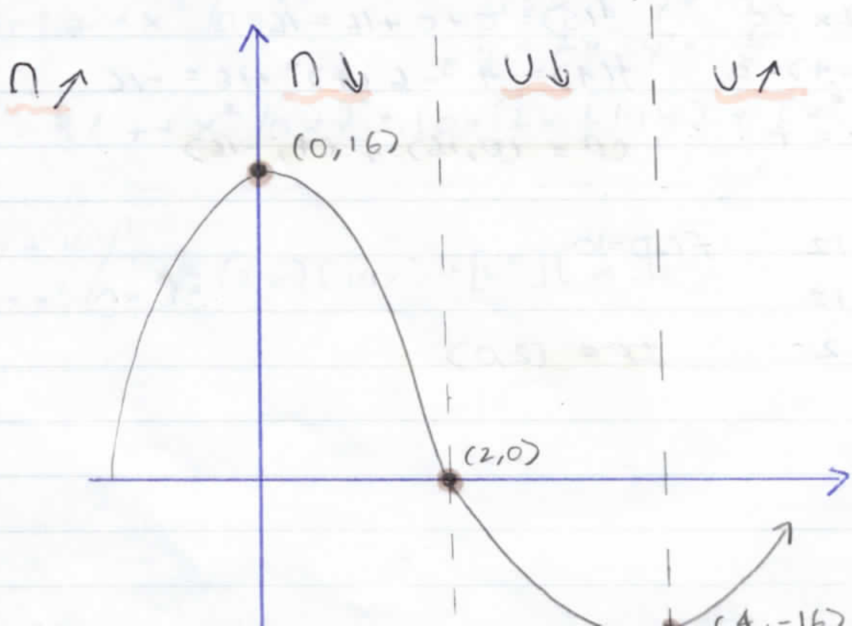
?

	$-\infty$	0	2	5	∞
f''		$-$		$+$	
f		$\cap$		$\cup$	

Interval of inc ↑: $x \in (-\infty, 0) \cup (4, \infty)$
Interval of dec ↓: $x \in (0, 4)$
Local Max: $f(0) = 16$
Local Min: $f(4) = -16$

Concave up: $x \in (2, \infty)$
Concave down: $x \in (-\infty, 2)$

(d)



A student call center sells 1600 cups of coffee per day at the price of \$2.40 / cup. A market survey shows that for every \$0.05 reduction in price per cup, 50 more cups of coffee will be sold.

#6 Q: How much should the student center charge for a cup of coffee in order to maximize revenue?

units price

Step ①, ②

$R = (1600 + 50x)(2.40 - 0.05x)$

$R = 3840 - 80x + 120x - 2.5x^2$

$R = 3840 + 40x - 2.5x^2$

Foil
Like terms
Step ①, ② done

Step ③ $= 3840 + 40x - 2.5x^2$

$0 + 40 - 5x = 0$

$-5 = -40$

$x = \frac{-40}{5} = x = 8$

Step ④ $p = 2.40 - 0.05x$

$p = 2.40 - 0.05(8)$

$p = \$2$

Student should charge \$2 per coffee

#7 Find the absolute extrema of the $fn = x^3 - 6x^2 + 9x - 6$ on interval $[-1, 5]$

Step ①: $f' = 0$; $3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3$, $x = 1$ ✓ $[-1, 1, 3, 5]$

$f(-1) = (-1)^3 - 6(-1)^2 + 9(-1) - 6 = -22$

$f(1) = (1)^3 - 6(1)^2 + 9(1) - 6 = -2$

$f(3) = (3)^3 - 6(3)^2 + 9(3) - 6 = -6$

$f(5) = (5)^3 - 6(5)^2 + 9(5) - 6 = 14$

Abs max $= f(5) = 14$
Abs min $= f(-1) = -22$

$(x^3 - 6x + 9x - 6)$

#8 If interest is compounded continuously and the annual nominal interest rate is 3.4%, how long will it take for money invested to double?

$A = Pe^{rt}$

$2 = e^{0.034t}$

$\frac{\ln 2}{0.034} = \frac{0.034t}{0.034} = 20.39$ years

#9 Find the EQN of the tangent line to the graph of $y^2 - xy = 6$ at the point with $x = 1$

Points: $y^2 - xy = 6$

$y^2 - (1)y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3$, $y = -2$

Finding intervals

Slope: $y^2 - (xy) = 6$ $\quad u = x \quad v = y \quad u' = 1 \quad v' = y'$

meth ①

$2yy' - (y + xy') = 0$

$2yy' - xy' = y$

$\frac{y'(2y-x)}{(2y-x)} = \frac{y}{2y-x} \rightarrow y' = \frac{y}{2y-x}$

$(1,-2) = \frac{-2}{2(-2)-1} = \frac{2}{5}$

$(1,3) = \frac{3}{2(3)-1} = \frac{3}{5}$

meth ②

(1, found) $(1,3)$: $2(3)y' - (3 + y') = 0$

$6y' - 3 - y' = 0$

$5y' = 3$

$y' = 3/5$

EQN: $y = \frac{3}{5}(x-1) + 3$

(1, found) $(1,-2)$: $2(-2)y' - (-2 + y') = 0$

$-4y' + 2 - y' = 0$

$-5y' = -2$

EQN: $y = \frac{2}{5}(x-1) - 2$

#10 Compute these anti derivatives:

(a) $\int (4x^6 - 3x^3 - 8)\, dx =$

$= \frac{4x^7}{7} - \frac{3x^4}{4} - 8x + C$

(b) $\int \frac{e^{-5x}}{3+e^{-5x}}\, dx$

$= \int \frac{e^{-5x}}{u} \cdot \frac{du}{-5e^{-5x}}$

$u = 3 + e^{-5x}$
$dx = \frac{du}{-5e^{-5x}}$

$= -\frac{1}{5} \int \frac{1}{u} \cdot du$

$= -\frac{1}{5} \ln |u| + c =$

$= -\frac{1}{5} \ln |3 + e^{-5x}| + C$

(c) $\int \frac{x^2}{\sqrt{x-3}}\, dx$ → u is always inside $\sqrt{u}$

$u = x - 3$ → to elimate x: $u + 3 = x$

$dx = \frac{du}{1}$

$= \int \frac{x^2}{\sqrt{u}} \cdot du =$

$= \int \frac{(u+3)^2}{u^{1/2}} \cdot du =$

$= \int \left(\frac{u^2}{u^{1/2}} + \frac{6u}{u^{1/2}} + \frac{9}{u^{1/2}} \right) \cdot du$

$= \int \left(u^{3/2} + 6u^{1/2} + 9u^{-1/2} \right) \cdot du$

Integrate

$= \frac{2u^{5/2}}{5} + 9(2)u^{1/2} + C = \frac{2}{5}(x-3)^{5/2} + 4(x-3)^{3/2} + 18(x-3)^{1/2} + C$

#11 Evaluate the integrelas:

(a) $\int_0^1 (x^3 - 4)\,dx$

$= \left[\frac{x^4}{4} - 4x\right]_0^1$

$= \left(\frac{1}{4} - 4\right) - (0 - 0)$

$\boxed{= -15/4}$

(B) $\int_6^{10} \frac{1}{x-5}\,dx$

$= \int_1^5 \frac{1}{u} \cdot du =$

$u = x - 5$
$dx = du$

$x = 10 \to u = 5$
change bounds $x \to u$!!
$x = 6 \to u = 1$

$= \left[\ln|u|\right]_1^5 = \ln 5 - \ln 1 = \ln 5$

$= \left[\ln|x-5|\right]_6^{10} = \ln 5 - \ln 1 = \ln 5$

(c) $\int_4^7 \sqrt{x-3}\,dx$

$u = x - 3$
$dx = du$

$x = 7 \longrightarrow u = 4$
$x = 4 \longrightarrow u = 1$

$= \int_1^4 u^{1/2}\,du$

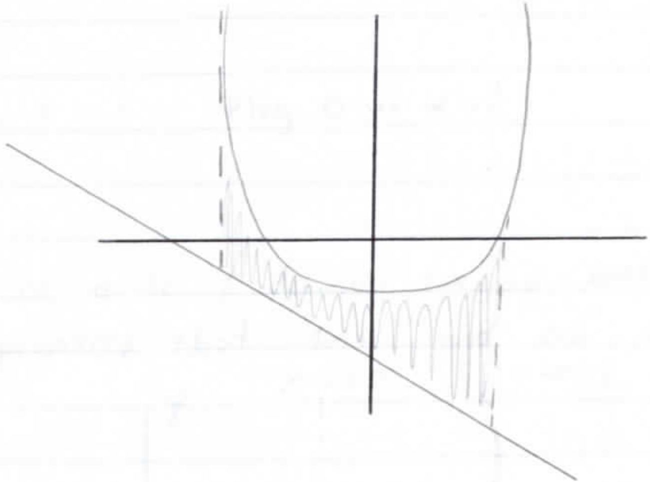
$= \left[\frac{2u^{3/2}}{3}\right]_1^4 - \frac{2}{3} = \frac{14}{3}$

#12 Find the area bounded by the graphs of $f(x) = x^2 - 1$ and $g(x) = x - 2$ over the interval $-2 \leq x \leq 1$

$f(x) = g(x)$ $\quad x^2 - 1 = x - 2$

$x^2 - x + 1 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



$$\frac{1 \pm \sqrt{1 - 4(1)(1)}}{2(1)}$$

$$= \frac{1 + \sqrt{-3}}{2} \longrightarrow \text{DNE}$$

No pts of intersection