

**CVG 3109  
SOIL MECHANICS - I  
FINAL EXAMINATION**

**Length of Examination: 3hrs**  
**Professor: Prof. Sai Vanapalli**

**19<sup>th</sup> Dec, 2017 (19:00 to 22:00) (FSS 2005)**  
**Page 1 of 23**

First Name: \_\_\_\_\_  
Last Name: \_\_\_\_\_  
Student Number: \_\_\_\_\_  
Signature \_\_\_\_\_

- i) This is a closed book exam. No textbooks are allowed
- ii) **Formula sheet** is available on last pages of this question paper
- iii) **If you do not understand a question, clearly state an assumption and proceed.**
- iv) Non programmable calculators are permitted
- v) Questions have the values shown next to the question.
- vi) **Marks will be taken out for missing units and labels.**
- vii) Answers should be succinct.

At the end of the exam, when time is up:

- Stop working and turn your exam upside down.
- Please remain silent.
- Do not move or speak until ALL exams have been picked up, and a TA or the Professor gives the go-ahead to leave.

<b><u>Question</u></b>	<b><u>Max Marks</u></b>	<b><u>Marks Awarded</u></b>
1	10 marks: Multiple choice questions	
2	15 marks: Multiple choice + Provide reason for your answer	
3	10 marks: Consolidation + Shear Strength	
4	12 marks: Stress Distribution Theory (Problem)	
5	15 marks: Consolidation (Problem)	
6	12 marks: Effective Stress + Seepage Analysis (Problem)	
7	26 marks: Shear Strength (Problem)	
<b>Total</b>	<b>100</b>	

**Question 1 (1 x 10 = 10 Marks)****(10 marks)**Circle **ONLY** the correct answer (i.e., **T** or **F**)

1	A shallow square foundation with similar dimensions was constructed to carry a load of 200 kN on two different clayey soils. The first soil, <b>A</b> , was an over consolidated clay and the second soil, <b>B</b> , was a normally consolidated clay. Both soils are saturated and have the same density and initial void ratio. For the above soil conditions and the chosen foundation; overconsolidated clay (i.e., <b>A</b> ) will typically settle less in comparison to normally consolidated clay (i.e., <b>B</b> ).	<b>T</b> <b>True</b>	<b>F</b>
2	The pore-water pressures measured in normally consolidated clay in a triaxial test can sometimes be negative, depending upon the stress applied on the specimen.	<b>T</b>	<b>F</b> <b>False</b>
3	Plasticity index ( $I_p$ ), plastic limit ( $PL$ ) and liquid limit ( $LL$ ) values for two soils; Soil <b>C</b> and Soil <b>D</b> are summarized below: Soil <b>C</b> : $I_p = 30\%$ ; $PL = 30\%$ ; $LL = 60\%$ Soil <b>D</b> : $I_p = 30\%$ ; $PL = 20\%$ ; $LL = 50\%$ Which one of the soils: Soil <b>C</b> or Soil <b>D</b> will have a higher Compression index, $C_c$	Soil <b>C</b> Answer <u>Soil C</u>	Soil <b>D</b>
4	Significant volume change occurs in a saturated clay soil when the total stress is increased rapidly.	<b>T</b>	<b>F</b> <b>False</b>
5	30% of the consolidation at a site subjected to a load of 100 kN/m <sup>2</sup> occurred in 3 months. If the site had been loaded to 300 kN/m <sup>2</sup> , 90% of the consolidation would occur in a period of 9 months.	<b>T</b>	<b>F</b> <b>False</b>
6	The effective cohesion, $c'$ value can never be a negative value irrespective of the soil being an over-consolidated or a normally consolidated clay.	<b>T</b> <b>True</b>	<b>F</b>
7	Compression index, $C_c$ for a normally consolidated clay is not a function of effective stress.	<b>T</b>	<b>F</b> <b>False</b>
8	The stress versus strain behavior of an overconsolidated clay is similar to that of a dense sand.	<b>T</b> <b>True</b>	<b>F</b>
9	The amount of volume change that will occur when a soil is subjected to load depends on the coefficient of permeability, $k$ of the soil.	<b>T</b> <b>True</b>	<b>F</b>
10	Sands and gravels rarely have angle of internal friction value, $\phi'$ which is less than 28°.	<b>T</b> <b>True</b>	<b>F</b>

**Question 2 (15 Marks) (1.5 x 10 = 15 Marks)**

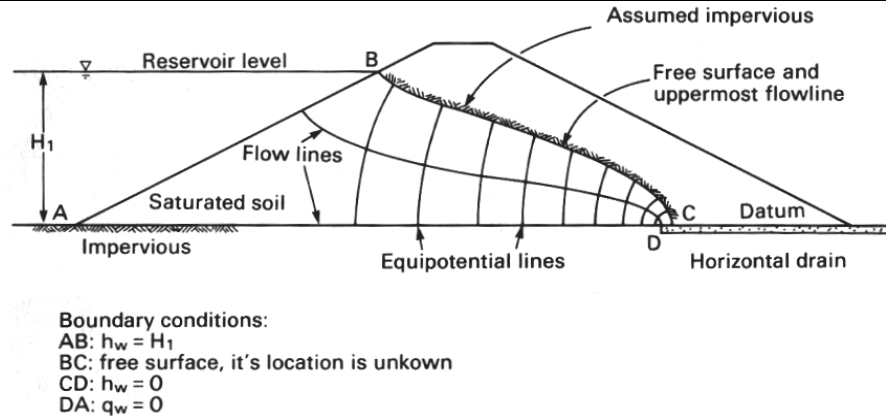
Select the correct answer (by underlining it) and explain the reason in the provided space in the box for your answer. **NO MARK WILL BE AWARDED IF REASON FOR YOUR ANSWER IS NOT PROVIDED**

**PS: Some questions are not multiple choice questions. You have to answer those questions in a line or two in the available space in the box.**

1	<p>Which one of the two samples: sample <b>A</b> (loose sand) or <b>B</b> (dense sand) will exhibit greater peak strength from consolidated drained test (i.e., <b>CD</b>) test on sand (i): Sample <b>A</b> (loose sand)      (ii) <u>Sample B (dense sand)</u></p> <p><b>Reason:</b> Sample B (dense sand) will exhibit greater peak strength from CD tests. This is because dense sand has more particles in comparison to loose sand. For this reason, dense sand has more particle contact points which offer resistance during shear loading and hence contribute to higher shear strength in comparison to loose sand.</p>
2	<p>Which one of the two samples: <b>C</b> (normally consolidated clay) or <b>D</b> (over consolidated clay with an <b>OCR</b> of 10) will exhibit greater peak strength for consolidated undrained tests (i.e., <b>CU</b>) test on saturated clay (i): Sample <b>C</b> (normally consolidated clay)      (ii) <u>Sample D (over consolidated clay with an OCR of 10)</u></p> <p><b>Reason:</b> Sample D, which is an over-consolidated clay with an OCR value of 10 has a high density and also has a high pre-consolidation pressure. Typically, normally consolidated clay has a low density and no pre-consolidation pressure. Sample C is normally consolidated and will develop positive pore-water pressure and hence the effective stress in the soil specimen would be low. However, in Sample D it is very likely negative pore-water pressure develop during shear which increase the effective stress and contribute to higher shear strength.,</p>
3	<p>Samples <b>E</b> and <b>F</b> are collected from the same site (i.e., they are identical samples – normally consolidated clay) and tested under <b>CU</b> and <b>CD</b> conditions in a triaxial test, respectively. Which sample; <b>E</b> or <b>F</b> will have higher shear strength when tested? Explain.</p> <p><b>Reason:</b> Effective stress in CU tests (Samples E) is lower than CD test (Samples F) (in CD test, pore-water pressure is zero). However, there will be positive pore-water pressure generated in CU tests for normally consolidated clay specimens. In other words, in sample F, there is more effective stress, which contributes to higher shear strength.</p>
4	<p>Will the pore pressure parameter, <b>A</b> at failure be positive, zero or negative for a clay sample with an <b>OCR</b> of about 4 in a <b>CU</b> test conducted with pore-water pressure measurement in a triaxial test apparatus (i.e., <math>\overline{CU}</math> tests).</p> <p><b>Reason:</b> The <b>A</b> parameter value, which is defined as <math>\Delta u / \Delta \sigma_d</math> (<b>pore-water pressure /deviator stress</b>) (see <b>Fig. 5 in Problem 7</b>) decreases with an increase in the OCR. Clayey soils with a higher pressure</p>

consolidation pressure when they are sheared with application of a lower confining pressure which contributes to dilation resulting in negative pore-water pressure. The pore-water pressure is always positive when Normally consolidated clays are loaded. For this reason,  $A$  values are typically higher in normally consolidated clays or lightly over consolidated clays. However, as OCR increases,  $A$  values move from positive to negative soils. Clays with  $OCR = 4$ , will have  $A$  value equal to zero.

5



**Figure 1**

The pore-water pressure along the upper most flow line (i.e., phreatic surface, BC) in a homogeneous earth dam is equal to

- (i) negative value
- (ii) atmospheric pressure (i.e., pore-water pressure is equal to zero)**
- (iii) positive value

**Reason:**

It is the upper most flow line in a saturated soil which is connected to the free atmospheric pressure surface (line BC)

6

Assume you are swimming in a swimming pool at a depth of 1 m below from the top surface of water. One of the following statements is true with respect to pressure on your body:

- (i) The total stress is equal to pore-water pressure**
- (ii) The effective stress is equal to pore-water pressure
- (iii) There is no pore-water pressure as there are no pores in water.
- (iv) None of the above are correct
- (v) This is too confusing.. don't ask such questions in final exam 😊

**Reason:**

Both the total stress and pore pressure is generated by water and the effective stress is zero. Due to this reason, we can swim at any depth without the influence of over burden pressure.

7

There will be no pre-consolidation pressure for a clay slurry (i.e., clay prepared with a water content greater than the liquid limit value) soil sample. Explain in one or two sentences why the clay slurry would **NOT** have any pre-consolidation pressure.

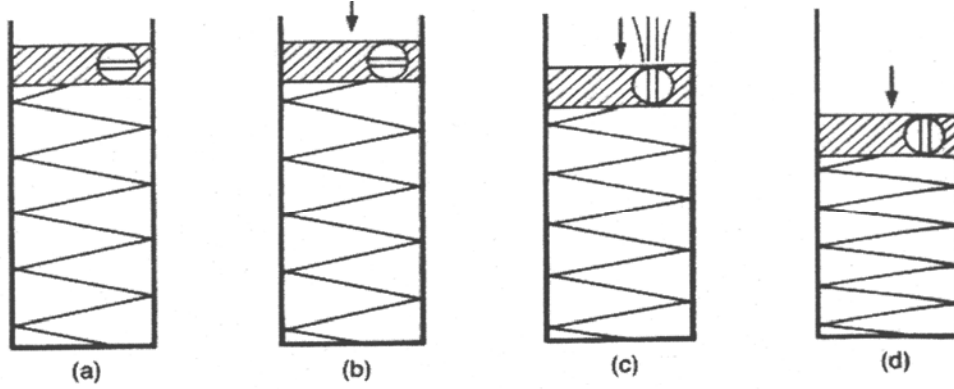
**Reason:**

Slurry is prepared by addition of a certain water content to a clayey soil (which is in a state of dry

	powder or dry soil) which is typically greater than the liquid limit value of the soil. There is no stress history for a soil that is prepared in this fashion. The slurry soil, which is freshly prepared does not have any stress history and hence no pre-consolidation pressure.
8	<p>The pore-pressure parameter, <b>B</b> value for an unsaturated soil is          (i) negative value      <b>(ii) between 0 to 0.95</b>      (iii) unity</p> <p><b>Reason:</b>          Skempton's pore pressure parameter <b>B</b> is defined as <math>\Delta u / \Delta \sigma_3</math>. (i.e., pore-water pressure / confining stress).. Under confined loading condition <b>B</b> value will be equal to 1, for saturated soils. However, over consolidated clays that are in a state of saturated condition can exhibit negative pore-water pressure value. For this reason, <b>B</b> values can be negative. However, unsaturated soils are 3-phase materials (with soil-water-water); such soils that can have <b>B</b> value between 0 to 1.</p>
9	<p>The effective shear strength parameters, <b>c'</b> and <b>φ'</b> values can be determined using one of the following triaxial tests          (i) UU tests      (ii) CU tests      (iii) <b>CD tests</b></p> <p><b>Reason:</b>          UU tests are Unconsolidated Undrained tests, are referred to as quick tests (i.e., Q tests) and are interpreted extending Total Stress Approach (TSA). In these tests pore-water pressure is not measured. They are used for evaluating short term (i.e. immediate) stability without considering pore-water pressure influence. Consolidated Undrained (CU) tests are useful for interpreting shear strength of soil which has been only already consolidated on which a new load is applied. These are referred to as R tests. <b>Consolidated Drained (CD) tests are useful for determining effective strength parameters;</b> namely <b>c'</b> and <b>φ'</b>/ These tests are useful for interpreting the long stability of soil structures, extending Effective Stress Approach (ESA). The pore-water pressure in the soil specimens under this loading is equal to zero.</p>
10	<p>In a <b>CD</b> test on saturated normally consolidated clay, the sample volume change during shear          (i) increases      (ii) remains unchanged      <b>(iii) reduces</b></p> <p><b>Reason:</b>          The present load (that is applied) is the maximum load acting on the normally consolidated clay. In other words, these clays do not have past stress history such as the over consolidated clays. For this reason, normal consolidated clays volume always decreases or reduces during shearing state under the applied load.</p>

**Question 3 (10 Marks)**

What type of tests and equipment do you recommend to determine the shear strength parameters for loadings shown in **Figure 2** below for **(b) and (d)**? Assume the spring in the Figure represents a normally consolidated clay. Note that the valve is closed in **(b)** and open in **(d)**. Also, **draw typical shear strength envelopes** for both these loadings. In addition, give a practical example for each of the tests where you use these shear strength tests in practice. **(5 marks +5 marks)**

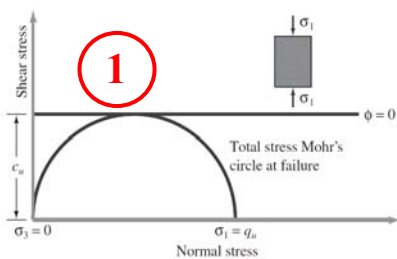


**Figure 2: Consolidation analogy model**

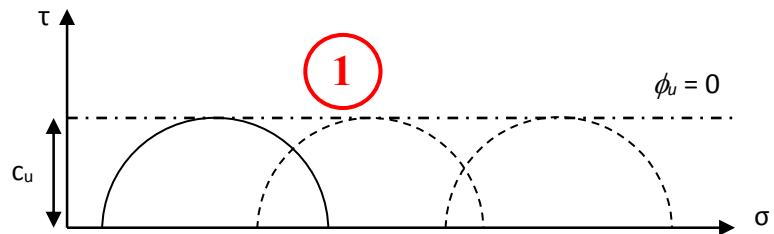
**Answer:**

**For (b): Undrained loading condition** is applied. **Triaxial device** can be used to determine the undrained shear strength parameters by conducting **Unconfined Compression test or Unconsolidated Undrained (UU) test**. **Vane shear test** also can be used in the field to determine the undrained shear strength for normally consolidated clay. 2

**Applications:** Rapid construction of earth embankment and cutting, placing a footing rapidly on clay deposit, sudden drawdown of water level in dam reservoirs, seismic or earthquake loading. 1



**Unconfined compression test**

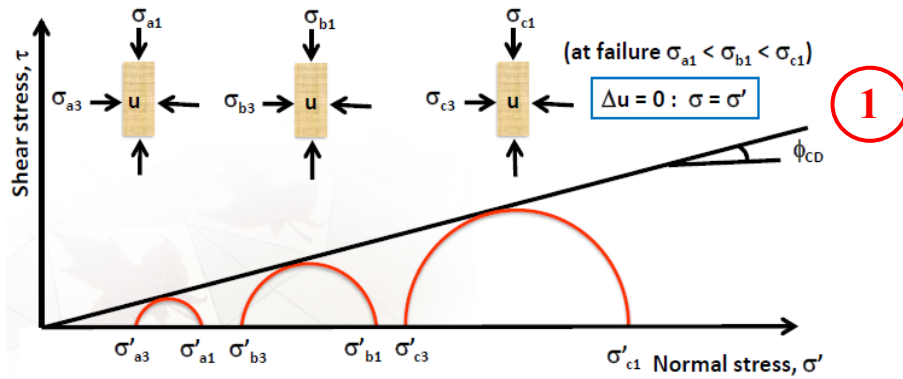


**UU test**

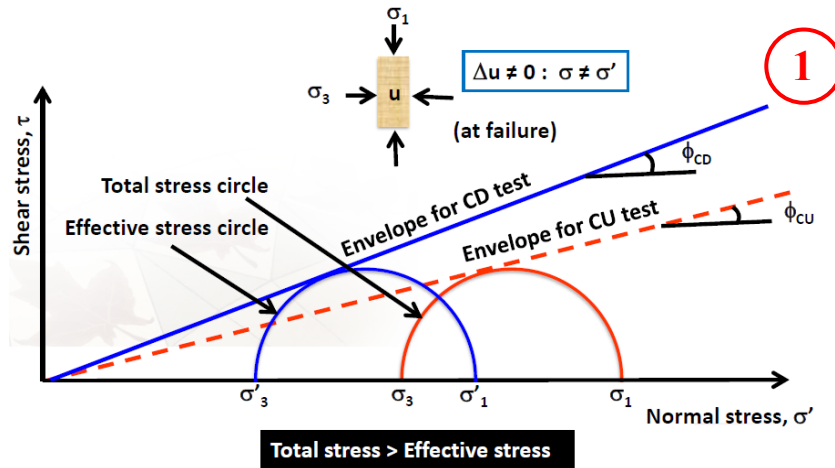
**For (d): Drained loading condition** is applied. **Triaxial device** can be used to determine the drained or effective shear strength parameters by conducting **Consolidated Drained (CD) test or Consolidated Undrained (CU) test with pore water pressure measurement**. **Direct shear test with very slow shearing rate** can be used to determine the drained shear strength parameters of normally consolidated clay. 2

**Applications:** Long term stability, steady state seepage, shallow foundations, slow rate of construction, and seepage control during construction.

1



CD test



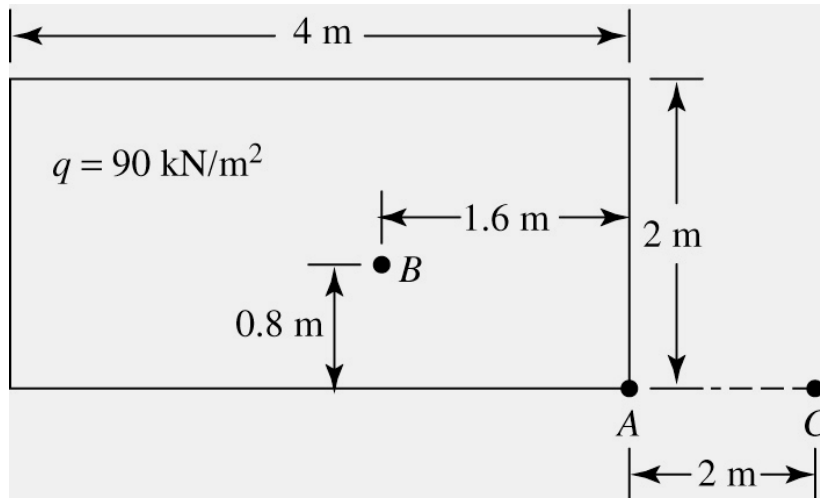
CU test

**Question 4 (12 Marks; Grading scheme shown along with the solution)**

The plan of a flexible rectangular loaded area is shown in **Figure 3** is uniformly distributed with an intensity of loading  $q = 90 \text{ kN/m}^2$ . Determine the vertical stress increase,  $\Delta\sigma_z$ , at a depth of  $z = 2 \text{ m}$

- (i) below at point **C** using *m and n* coefficients method, and
- (ii) below point **B** using Newmark's method.

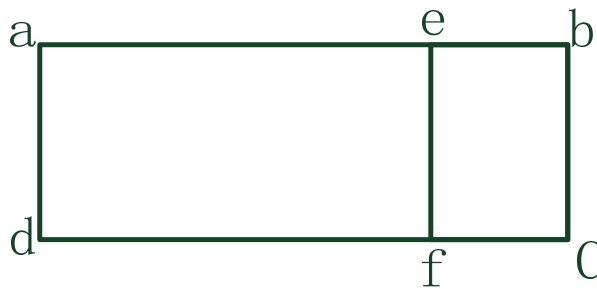
**(10 marks)**



**Figure 3**

**Answer:**

- (i) Stress increase below C at depth of  $z=2\text{m}$  (6 marks)**



①  $\Delta\sigma_z = qI_{qr}$ , where  $I_{qr} = I_{qr,abCd} - I_{qr,ebCf}$  (2 marks)

② For rectangle abCd

$m = \frac{L}{z} = \frac{6}{2} = 3$  (0.5 mark),  $n = \frac{B}{z} = \frac{2}{2} = 1$  (0.5 mark),  $I_{qr,abCd} = 0.203$  (0.5 mark)

Note: any value between 0.20 and 0.21 can be acceptable (*m* and *n* values are determined from Chart on page 10).

③ For rectangle ebCf

$$m = \frac{L}{z} = \frac{2}{2} = 1 \text{ (0.5 mark)}, n = \frac{B}{z} = \frac{2}{2} = 1 \text{ (0.5 mark)}, I_{qr,ebCf} = 0.175 \text{ (0.5 mark)}$$

Note: any value between 0.17 and 0.18 can be acceptable.

$$\textcircled{4} \Delta\sigma_z = q(I_{qr,abCd} - I_{qr,ebCf}) = 90 \times (0.203 - 0.175) = 2.52 \text{ kPa} \text{ (1 mark)}$$

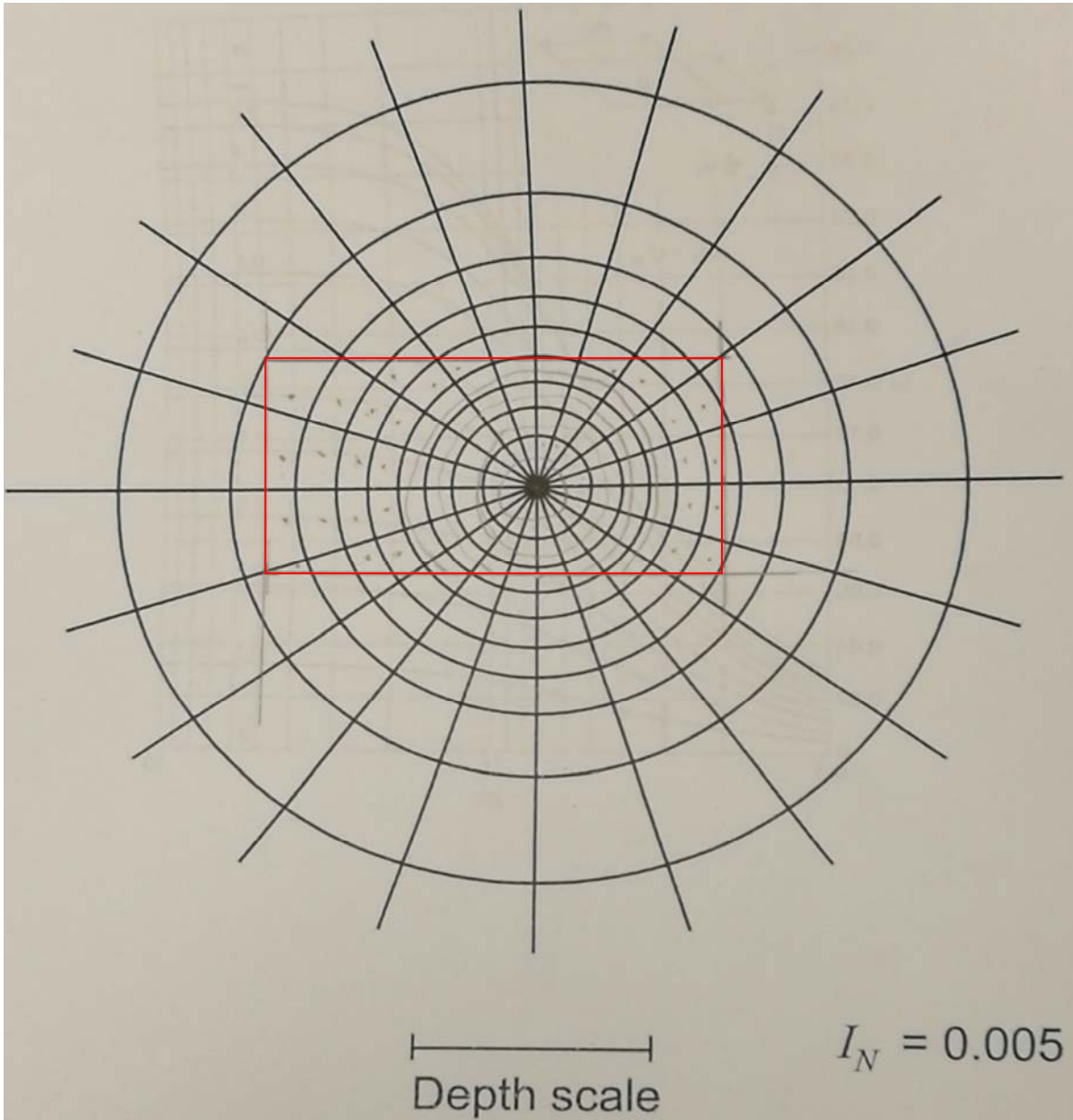
**(ii) Stress increase below B at depth of 2m (6 marks)**

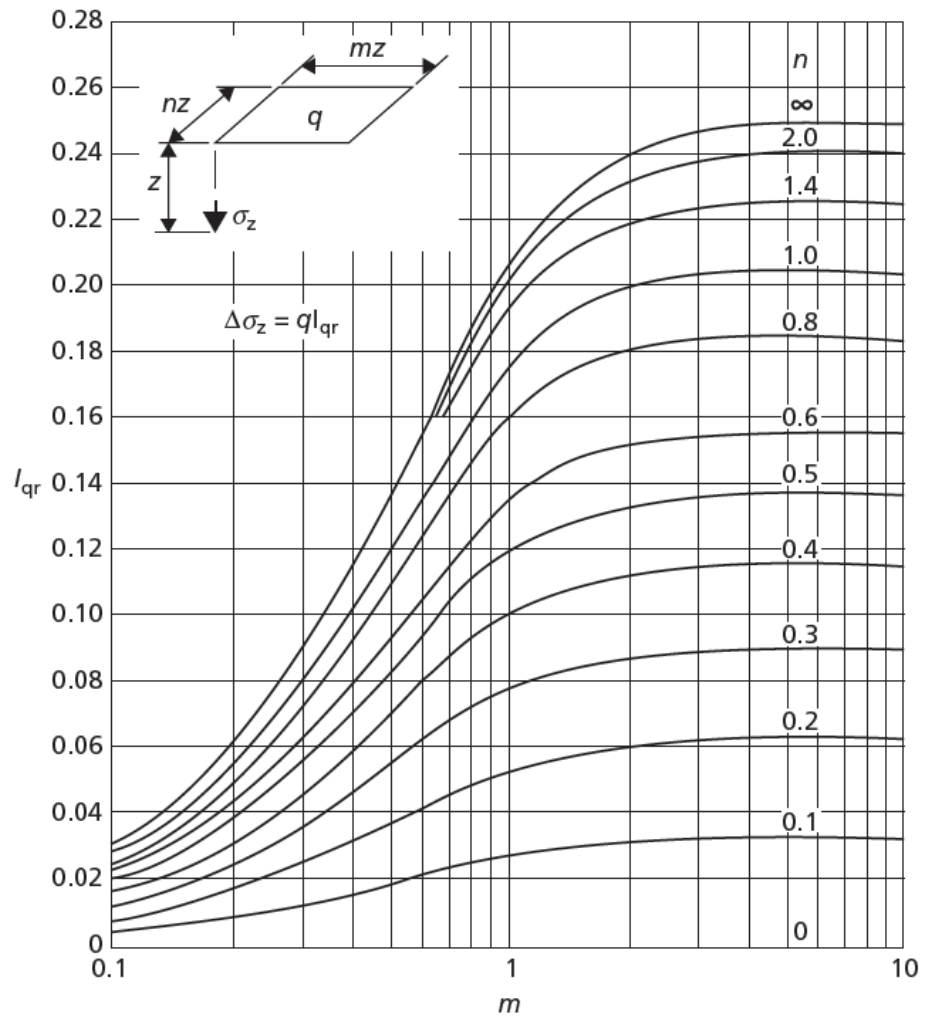
① Depth scale is 3.6 cm. This means 3.6cm in the chart represents 2m actual length. In other words, the scale is 1.8cm: 1m. (3 marks)

② The length of rectangle (4m) should be 7.2cm in the chart. The width (2m) should be 3.6 cm in the chart. The distance from B to right edge (1.6 m) should be 2.88 cm. The distance from B to bottom edge (0.8 m) should be 1.44 cm. B should be on the center of all the circles. (2 marks)

③ The number of the segments inside the rectangular area is about 94. Note any value between 90 and 105 can be acceptable.

$$\Delta\sigma_z = N \cdot I_N \cdot q = 94 \times 0.005 \times 90 \text{ kN/m}^2 = 42.3 \text{ kN/m}^2 \text{ (1 mark)}$$





**Question 5 (15 Marks)**

The following compression readings summarized in Table 1 were obtained from an oedometer test on a specimen of saturated clay ( $G_s = 2.73$ ).

**Table 1**

$\sigma'$ (kPa)	0	50	100	200	400	800	1600	3200	0
Dial Reading (mm)	5.00	4.75	4.5	4.10	3.45	2.60	1.65	0.75	1.45

The initial thickness of the specimen was 20.0 mm and at the end of the test the water content was 19.6%. Plot  $e - \log \sigma'$  curve and determine the pre-consolidation pressure. What is the value of compression,  $C_c$  for the latter increment?

**Answer:**

From equation  $wG_s = Se$ , the void ratio at the end of the test ( $e_1$ ) can be known,

$$e_1 = \frac{wG_s}{S} = \frac{0.196 * 2.73}{1} = 0.535 \quad \boxed{2 \text{ Points}}$$

From the dial gauge reading, we first calculate the change in the thickness of the sample ( $\Delta H$ ),

$\sigma'$ (kPa)	0	50	100	200	400	800	1600	3200	0
Dial gauge reading (mm)	5.00	4.75	4.5	4.10	3.45	2.60	1.65	0.75	1.45
Change in thickness, $\Delta H$ (mm)	0	0.25	0.5	0.9	1.55	2.4	3.35	4.25	3.55

From equation  $\frac{\Delta e}{1+e_0} = \frac{\Delta H}{H_0}$  and  $e_0 = e_1 + \Delta e$ , we know that,

$$\frac{\Delta e}{1+e_1 + \Delta e} = \frac{\Delta H}{H_0}, \text{ where } H_0 = 20\text{mm}, \Delta H = 3.55\text{mm},$$

Then, the  $\Delta e$  and  $e_0$  can be calculated as,

$$\Delta e = 0.331, e_0 = 0.866.$$

From  $e_0$  and equation  $\Delta e = \Delta H * \frac{1+e_0}{H_0}$ ,

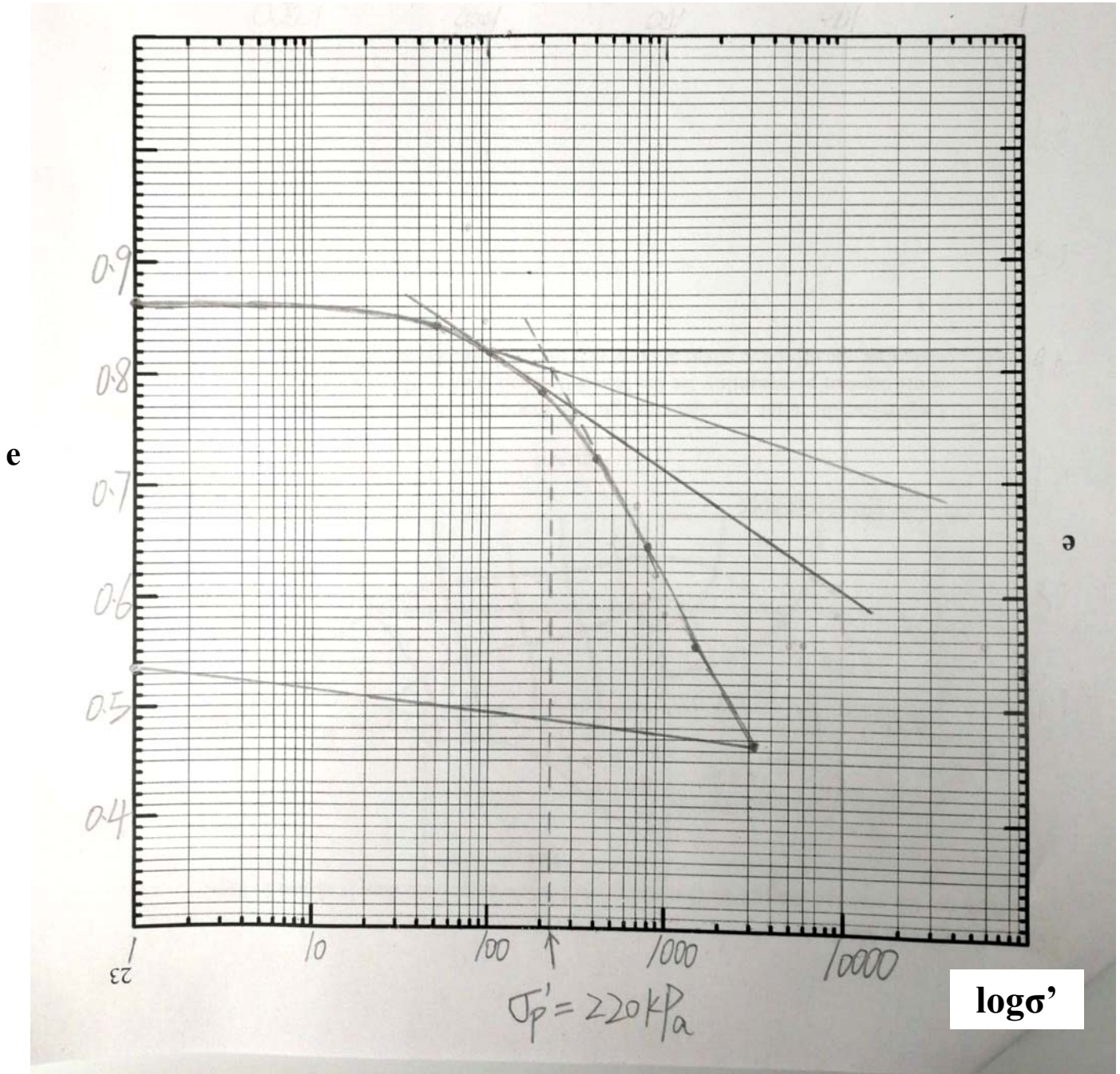
we can get the void ratio under each stress, see below,

**5 Points**

$\sigma'$ (kPa)	0	50	100	200	400	800	1600	3200	0
$\Delta H$		0.25	0.5	0.9	1.55	2.4	3.35	4.25	
Void ratio, $e$	0.866	0.843	0.819	0.782	0.721	0.642	0.553	0.469	0.535

Plot  $e - \log \sigma'$  curve, and determine the pre-consolidation pressure.

**4 Points**



**$e$  versus  $\log \sigma'$  relationship**

From the figure, the pre-consolidation pressure is, 220 kPa.

Note: the value of pre-consolidation pressure is based on locating maximum curvature point of the data plotted on the graph based on visual approximation. For this reason, pre-consolidation pressure may slightly vary from person to person.

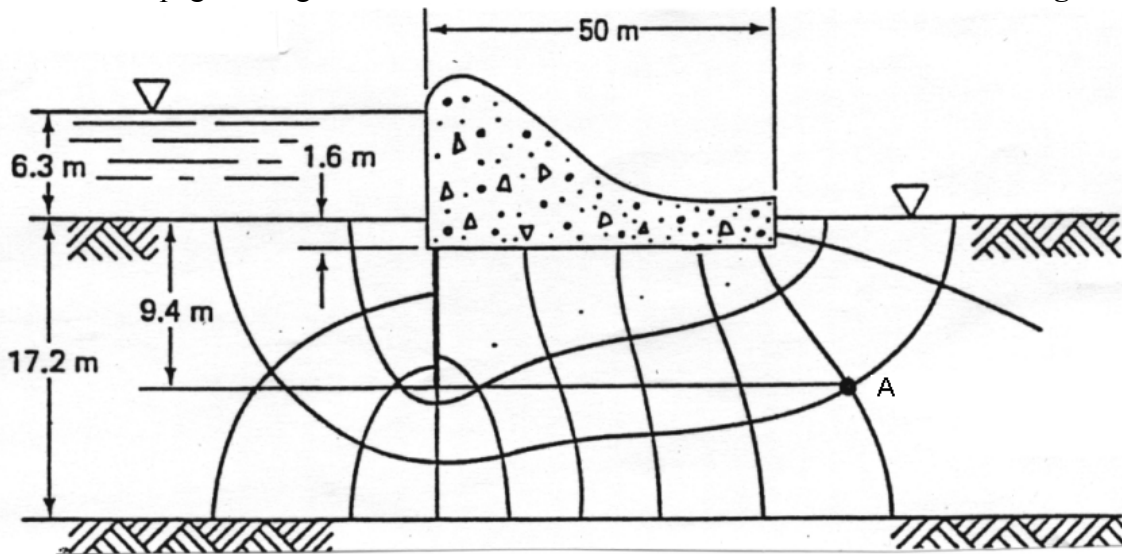
The compression index,  $C_c$ , can be determined by,

$$C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma_2}{\sigma_1}\right)} = \frac{0.553 - 0.469}{\log\left(\frac{3200}{1600}\right)} = 0.279$$

**4 Points**

**Question 6 (12 Marks)**

The flow net for seepage through the foundation soil below a concrete dam is shown in **Figure 4** below



**Figure 4**

- (i) Determine the total seepage through the foundation soil in cubic meters per day, if the coefficient of permeability for the foundation soil is  $25 \times 10^{-6} \text{ m/s}$  (Total: 8 points)

$$q = kh_w \left( \frac{N_f}{N_d} \right) (\text{width})$$

$$q = 25 \times 10^{-6} \text{ m/s} \times 6.3 \text{ m} \left( \frac{3}{10} \right) \times 1 \text{ m} = 47.25 \times 10^{-5} \text{ m}^3/\text{s}$$

$$q = 40.824 \text{ m}^3/\text{day}$$

Use the correct equation: 1point;

$N_f=3$ : 2 points;

$N_d=10$ : 2 points;

$h_w=6.3$ : 2points;

Correct calculation/ with units: 1point;

- (ii) Calculate the effective stress at point A if the total unit weight of soil is  $20 \text{ kN/m}^3$ . (Total 4 points)

Total stress at A:  $\sigma_A = 20 \times 9.4 = 188 \text{ kPa}$

Total head at A:  $6.3/10 = 0.63 \text{ m}$ , head at A:  $h_p = 0.63 \times 2 - (-9.4) = 10.66 \text{ m}$ ,

$u_w = 9.8 \times 10.66 = 104.468 \text{ kPa}$

Effective stress at A:  $\sigma_A' = 188 - 104.468 = 83.532 \text{ kPa}$

$\sigma_A = 20 \times 9.4 = 188 \text{ kPa}$ : 1point;

Total head at A:  $6.3/10 = 0.63 \text{ m}$ : 1point;

$h_p = 0.63 \times 2 - (-9.4) = 10.66 \text{ m}$ ; 1 points;

$\sigma_A' = 188 - 104.468 = 83.532 \text{ kPa}$ ; 1point.

**Question 7 (26 Marks)**

The results summarized below in Table 2 were obtained at failure conditions from a series of Consolidated-Undrained triaxial tests with pore-water pressure measurements on fully saturated clay specimens.

**Table 2**

Specimen	Confining pressure $\sigma_3$ (kPa)	Deviator stress ( $\sigma_1 - \sigma_3$ ) kPa	Pore-water stress u (kPa)
A	150	103	82
B	300	202	169
C	450	305	252

- i) Determine the effective shear strength parameters for the tested soil (i.e.,  $c'$  and  $\phi'$ ) using graphical method ONLY. (10 Marks)

**Answer:****Table A (4 marks)**

Specimen	$\sigma_3$ (kPa)	( $\sigma_1 - \sigma_3$ ) (kPa)	u (kPa)	$\sigma_1$ (kPa)	$\sigma_3'$ (kPa)	$\sigma_1'$ (kPa)	$p'$ (kPa)	$q'$ (kPa)
A	150	103	82	253	68	171	119.5	51.5
B	300	202	169	502	131	333	232.0	101.0
C	450	305	252	755	198	503	350.5	152.5

**Calculation for Table A:**

$$\sigma_1 = \sigma_3 + (\sigma_1 - \sigma_3)$$

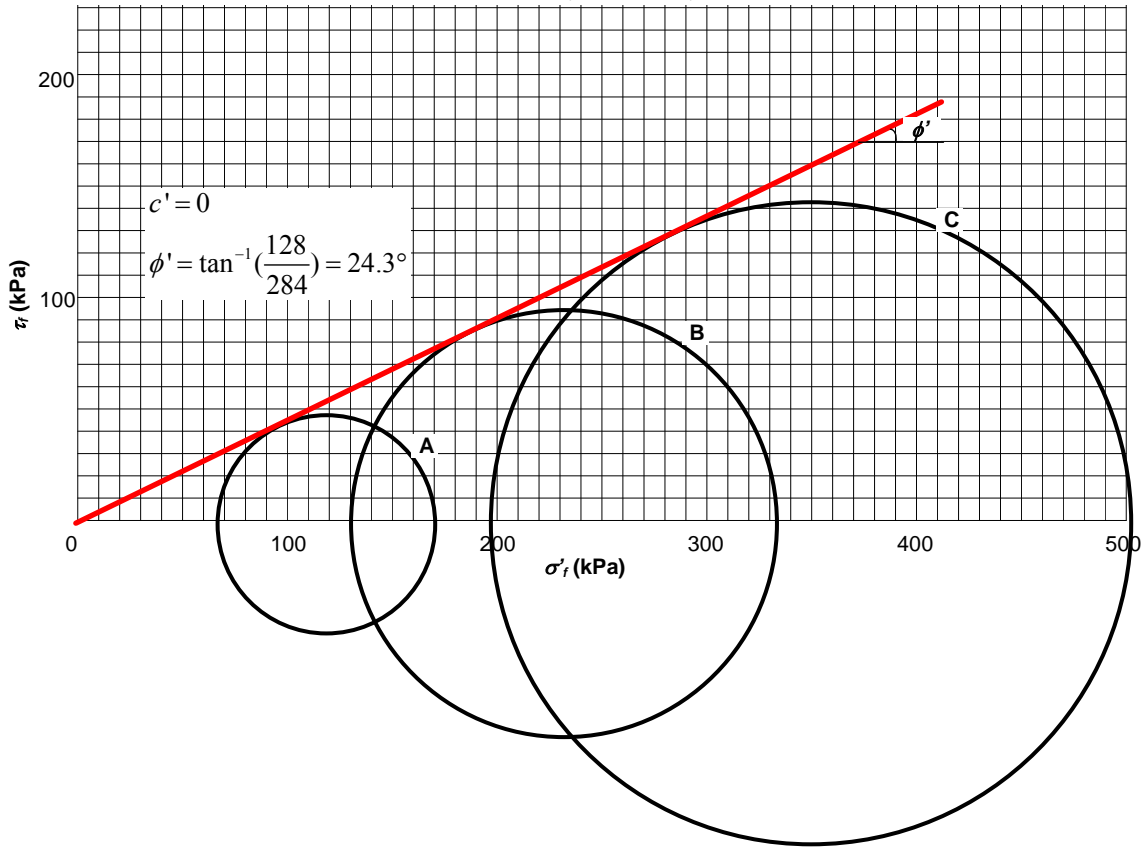
$$\sigma_3' = \sigma_3 - u$$

$$\sigma_1' = \sigma_1 - u$$

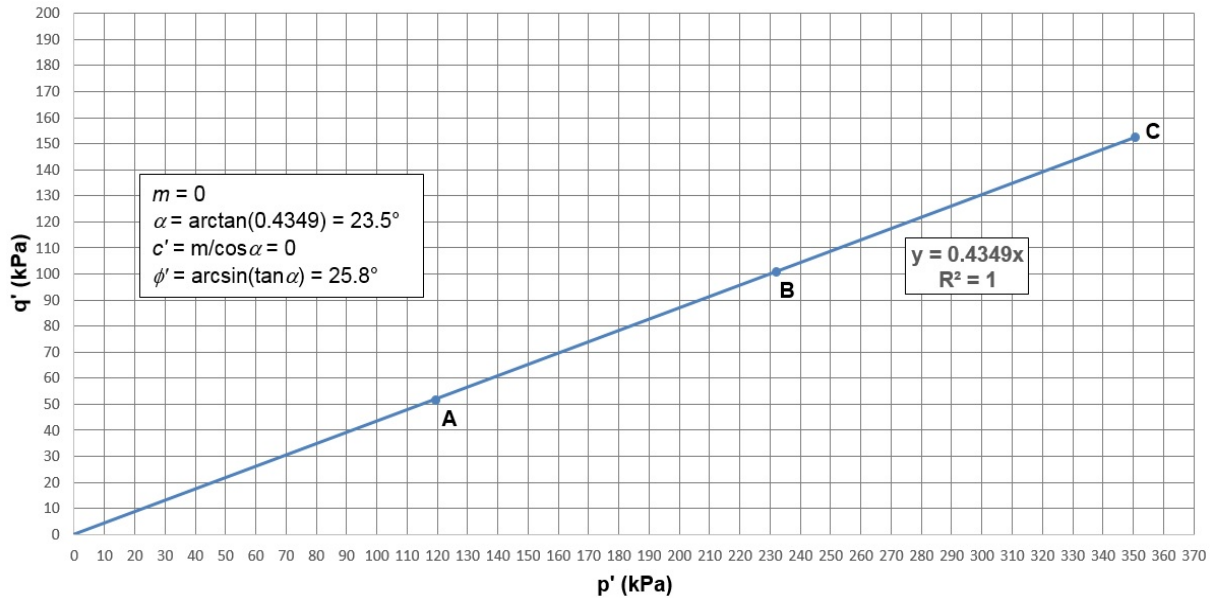
$$p' = \frac{\sigma_1' + \sigma_3'}{2}$$

$$q' = \frac{\sigma_1' - \sigma_3'}{2}$$

**Approach 1: Mohr-Coulomb failure envelope,  $\tau_f = c' + \sigma_f' \tan \phi'$ , Figure a: (6 marks: 2, 2, 2)**



**Approach 2: Modified failure envelope:  $q' = m + p' \tan \alpha$ , Figure b: (6 marks)**



- ii) Calculate the Skempton's  $A_f$  value for this clay. Is the clay normally consolidated or over consolidated? Give reasons. Hint: Use Figure 5 may be useful for this question (6 Marks)

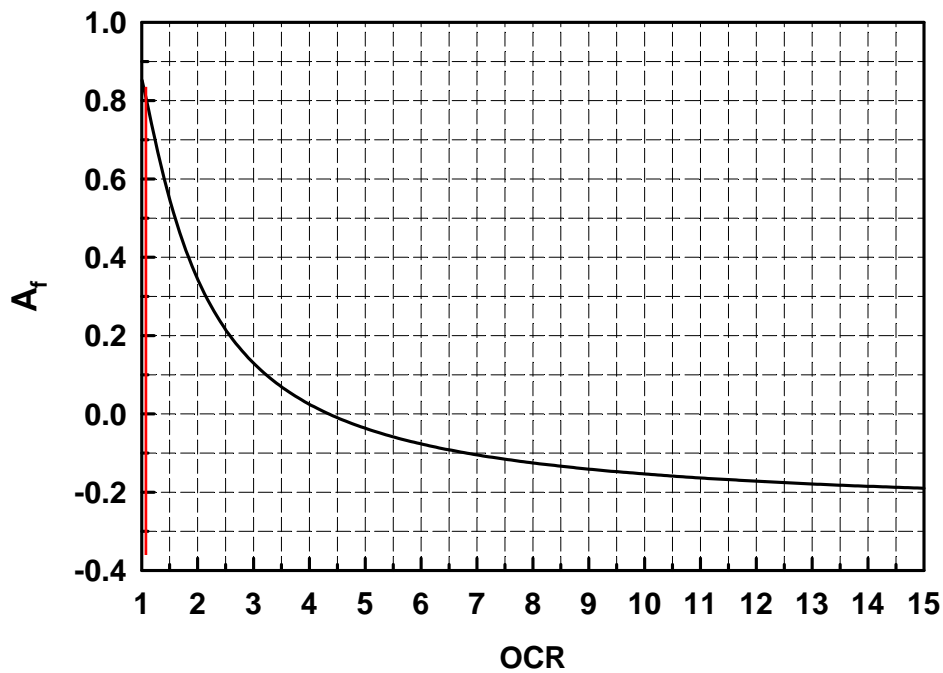


Figure 5. OCR versus  $A_f$  relationship

**Answer:**

Skempton's pore pressure parameter,

$$A = \frac{\Delta u_{(deviator)}}{(\Delta \sigma_1 - \Delta \sigma_3)}$$

For specimen A,  $A_f = 82 / 103 = 0.80$ ;

For specimen B,  $A_f = 169 / 202 = 0.84$ ;

For specimen C,  $A_f = 252 / 305 = 0.83$ ;

Then the average  $A_f = (0.80 + 0.84 + 0.83) / 3 = 0.82$

From Figure 5, we have  $OCR \approx 1.0$ , therefore, the clay is **normally consolidated**.

- b) In a series of Unconsolidated-Undrained triaxial tests on specimens of fully saturated clay, the following results (Table 3) were obtained at failure conditions. Determine the shear strength parameters,  $c_u$  and  $\phi_u$ . **(10 Marks)**

**Table 3**

Confining pressure, $\sigma_3$ (kPa)	200	400	600
Deviator stress, $\sigma_d$ (kPa)	222	218	220

**Answer:**

**Table B (3 marks)**

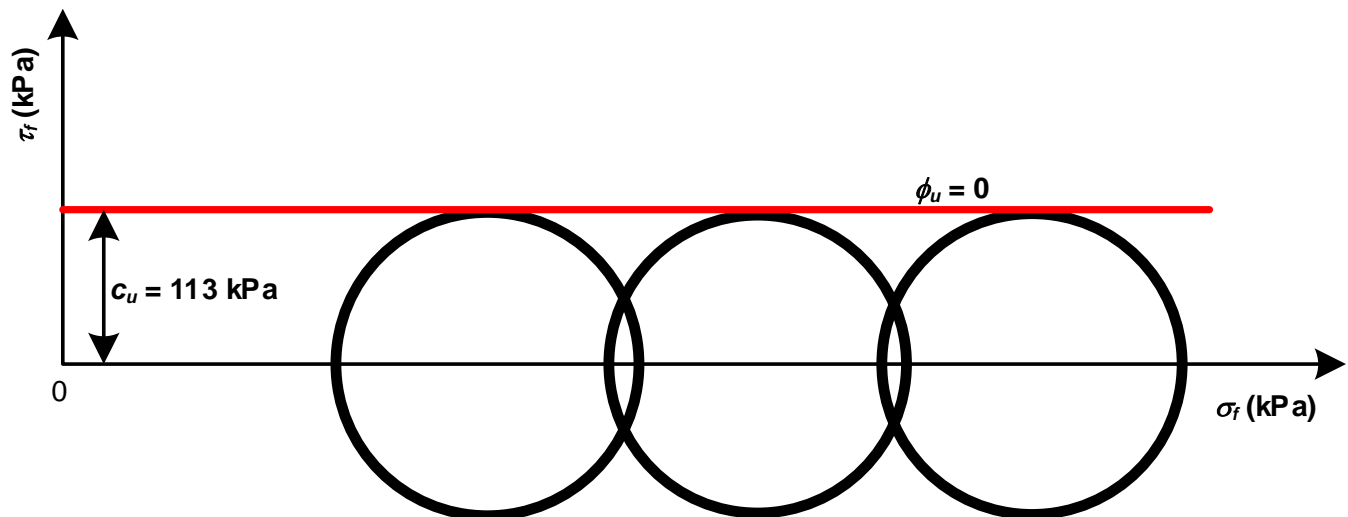
Confining pressure, $\sigma_3$ (kPa)	200	400	600
Deviator stress, $\sigma_d$ (kPa)	222	218	220
Major principal stress, $\sigma_1$ (kPa)	422	618	820

**Calculation for Table B:**

$$\sigma_d = \sigma_1 - \sigma_3$$

$$\sigma_1 = \sigma_3 + \sigma_d$$

The three Mohr circles and the failure envelope are drawn in the below figure, from which the shear strength parameters can be obtained as  $\phi_u = 0$  (undrained);  $c_u = 113$  kPa. **(Graph 3: Calculation and Result: 2 for  $c_u$  and 2 for  $\phi_u$  values)**



## Formulae Sheet

### Phase relationships:

Unit weight of soil:

$$\gamma = \frac{W}{V} = \frac{(Se + G_s)\gamma_w}{1 + e}$$

Submerged unit weight:  $\gamma_{sub} = \frac{(G_s - 1)}{1 + e}\gamma_w$

Dry unit weight:

$$\gamma_d = \frac{W_s}{V} = \frac{G_s}{1 + e}\gamma_w = \frac{\gamma}{1 + w}$$

$$Se = wG_s$$

### Elastic Theory (Stress Distribution theory)

Boussinesq's equation for determining vertical stress due to a point load

$$\sigma_z = \frac{3Q}{2\pi z^2} \left\{ \frac{1}{1 + \left(\frac{r}{z}\right)^2} \right\}^{5/2}$$

Determination of vertical stress due to a rectangular loading:  $\sigma_z = qI_c$  (Charts also available)

$$I_c = \frac{1}{4\pi} \left[ \frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + m^2n^2 + 1} \left( \frac{m^2 + n^2 + 2}{m^2 + n^2 + 1} \right) + \tan^{-1} \frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + m^2n^2 + 1} \right]$$

$m = B/z$  and  $n = L/z$  (both  $m$  and  $n$  are interchangeable)

Approximate method to determine vertical stress,

$$\sigma_z = \frac{qBL}{(B+z)(L+z)}$$

Vertical stress determination using Newmarks Chart

$$\sigma_z = qI_c \text{ (No. of sectors)}$$

### Permeability & Effective Stress

Total head = Pressure head + Elevation head + Velocity head

$$h = \frac{u}{\gamma_w} + \frac{v^2}{2g} + z$$

Hydraulic gradient:  $i = \frac{\Delta h}{L}$

$$v = ki$$

Darcy's law:  $q = vA$

$$Q = kiAt = k \cdot \frac{\Delta h}{L} \cdot At$$

Equivalent hydraulic conductivity:

$$k_{H(eq)} = \frac{k_{H_1}H_1 + \dots + k_{H_n}H_n}{H_1 + \dots + H_n}$$

$$k_{V(eq)} = \frac{H_1 + \dots + H_n}{\left(\frac{H_1}{k_{V_1}}\right) + \dots + \left(\frac{H_n}{k_{V_n}}\right)}$$

Seepage in a flow net:

$$q = k \cdot h_w \cdot \frac{N_f}{N_d}$$

Pore-water pressure (kPa):

$$u_p = \gamma_w [h_p - (-z_p)] = \gamma_w (h_p + z_p)$$

Total seepage force =  $i\gamma_w V$

Seepage per unit volume =  $i\gamma_w$

Effective stress:  $\sigma' = \sigma - u_w$

Effective stress on downward seepage:

$$\sigma' = \gamma_{sub}z + jz = \gamma_{sub}z + iz\gamma_w$$

Effective stress on upwards seepage:

$$\sigma' = \gamma_{sub}z - jz = \gamma_{sub}z - iz\gamma_w$$

Critical hydraulic gradient:

$$\sigma' = 0 = \gamma_{sub}z - iz\gamma_w \Leftrightarrow i_c = \frac{\gamma_{sub}}{\gamma_w}$$

$$i_c = \frac{(G_s - 1)\gamma_w}{1 + e} \cdot \frac{1}{\gamma_w} = \frac{(G_s - 1)}{1 + e}$$

### Shear strength:

$$\tau_f = c' + (\sigma - u_w) \tan \phi'$$

$$\sigma'_1 = \sigma'_3 \tan^2 \left( 45^\circ + \frac{\phi'}{2} \right) + 2c' \tan \left( 45^\circ + \frac{\phi'}{2} \right)$$

$$\tau_f = \frac{1}{2} (\sigma'_1 - \sigma'_3) \sin 2\theta$$

$$\sigma_f = \frac{1}{2} (\sigma'_1 + \sigma'_3) + \frac{1}{2} (\sigma'_1 - \sigma'_3) \cos 2\theta$$

$$B = \frac{\Delta u_{(Confining)}}{\Delta \sigma_3}$$

$$A = \frac{\Delta u_{(Deviator)}}{(\Delta \sigma_1 - \Delta \sigma_3)}$$

$$q' = m + p' \cdot \tan \alpha$$

$$\phi' = \sin^{-1}(\tan \alpha)$$

$$c' = \frac{m}{\cos \alpha}$$

### Consolidation:

Compression index

$$C_c = \frac{e_o - e_1}{\log \left( \frac{\sigma'_1}{\sigma'_0} \right)} \quad (\sigma'_1 > \sigma'_0)$$

$$C_c = 0.009 [LL(\%) - 10] \text{ for undisturbed clay}$$

$$C_c = 0.007 [LL(\%) - 10] \text{ for disturbed clay}$$

$$OCR = \frac{\sigma'_p}{\sigma'_0}$$

Swelling index,  $C_s$  : Slope of swelling path

$$m_v = \frac{\Delta e}{1 + e_o} \left( \frac{1}{\Delta \sigma'} \right) = \frac{1}{1 + e_o} \left( \frac{e_o - e_1}{\sigma'_1 - \sigma'_0} \right)$$

$$s_c = \int_0^H \frac{e_o - e_1}{1 + e_o} dz = \frac{\Delta e}{1 + e_o} H$$

$$s_c = \int_0^H m_v \Delta \sigma' dz = m_v \Delta \sigma' H$$

$$s_c = \frac{C_c}{1 + e_o} H \log \left( \frac{\sigma'_0 + \Delta \sigma}{\sigma'_0} \right)$$

$$s_c = \frac{C_s}{1 + e_o} H \log \left( \frac{\sigma'_p}{\sigma'_0} \right) + \frac{C_c}{1 + e_o} H \log \left( \frac{\sigma'_p + \Delta \sigma}{\sigma'_p} \right)$$

$$T_v = \frac{c_v t}{H_{dr}^2}$$

$$U = \frac{e_1 - e}{e_1 - e_2}$$

$$U = \frac{u_i - u}{u_i} = 1 - \frac{u}{u_i}$$

$$U = \frac{\delta}{\delta_c}$$

for  $U < 60\%$ ,

$$T_v = \frac{\pi}{4} \left( \frac{U\%}{100} \right)^2$$

for  $U \geq 60\%$ ,

$$T_v = 1.781 - 0.933 \log(100 - U\%)$$