Problem 1:

A structural member is subjected to the following axial tensile forces:

Dead load:	mean value 100 kN, standard deviation 10 kN
Live load due to occupancy and use:	mean value 50 kN, standard deviation 20 kN

a) Find the total load value associated with a 1% probability of being exceeded.

b) Given that the target reliability index for the member is $\beta = 4.5$ and the coefficient of variation of the member resistance is 10%. Determine the required average resistance associated with the required target reliability index.

(Assume all variables to be normally distributed)

Solution for Problem 1:

(a) Define a total load function T=D+L

Determine the average and standard deviation for the total load function

$$\overline{T} = \overline{D} + \overline{L} = 100 + 50 = 150kN$$

$$\sigma_T = \sqrt{10^2 + 20^2} = 22.4kN$$

A 1% probability corresponds to $\beta = 2.3$

The total load value associated with a 1% probability of being exceeded

$$\beta = \frac{T_t - \overline{T}}{\sigma_t} \Rightarrow T_t = \overline{T} + \beta\sigma_t = 150 + 2.3 \times 22.4 = 201kN$$

(b) Define an excess resistance function

$$E = R - D - L$$

Average and standard deviation for the excess resistance function

$$\overline{E} = \overline{R} - \overline{D} - \overline{L}$$

$$\overline{E} = \overline{R} - 150$$

$$\sigma_E = \sqrt{\sigma_R{}^2 + 10^2 + 20^2}$$

$$\sigma_E = \sqrt{0.10^2 \overline{R}^2 + 10^2 + 20^2}$$

$$\beta = \frac{E_t - \overline{E}}{\sigma_E} \Rightarrow 4.5 = \frac{0 - \overline{E}}{\sigma_E}$$

$$\overline{R} - 150 = -4.5\sqrt{0.10^2 \overline{R}^2 + 10^2 + 20^2}$$

$$\overline{R}^2 - 300\overline{R} + 150^2 = (4.5)^2 0.10^2 \overline{R}^2 + (4.5)^2 10^2 + (4.5)^2 20^2$$

$$0.7975\overline{R}^2 - 300\overline{R} + 123,75 = 0$$

$$\overline{R} = \frac{300 \pm \sqrt{300^2 - 4(0.7975)(12,375)}}{2(0.7975)} = \frac{300 \pm 224}{1.595}$$

$$\overline{R} = 329kN, 47.6kN$$

The negative root should be rejected since an average resistance inferior to the average load is meaningless.

Problem 2:

A steel plate (300W material) is subject to a biaxial state of stresses. The stress is $\sigma_{xx} = 125 MPa$ (tensile). It is required to determine the stress σ_{yy} that would yield the steel. Clearly indicate whether the stress σ_{yy} is compressive or tensile.

Solution for Problem 2:

Starting with the Maximum Distortional Energy Density Yield criterion, and discarding all zero stresses, one obtains:

$$\sigma_{xx}{}^2 - \sigma_{xx}\sigma_{yy} + \sigma_{yy}{}^2 = F_y^2$$

$$125^2 - 125\sigma_{yy} + \sigma_{yy}{}^2 = 300^2$$

$$\sigma_{yy}{}^2 - 125\sigma_{yy} - 74,375 = 0$$

$$\sigma_{yy} = \frac{125 \pm \sqrt{125^2 + 4(74,375)}}{2} = \frac{125 \pm 559}{2}$$

$$= 342 MPa \quad tension,$$

$$or$$

$$= 217 MPa \quad compression$$

Here, both values are meaningful, i.e., a 342 MPa tensile stress for σ_{yy} will yield the plate and a 217 MPa compressive stress will also yield the plate

Problem 3:

The shown horizontal tension member consists of a W200x46 tension member (W350 Material). The member is connected to a vertical member through two gusset plates (300W material) as shown. The connection to each flange is done through six 20 mm bolts (all holes are punched), as shown.

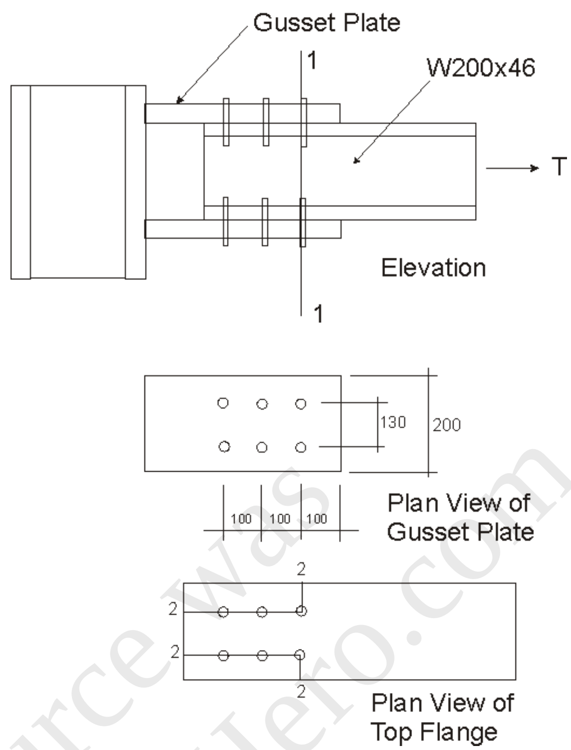
1) Knowing that the end distance for flanges is 100mm, determine the capacity of the tension member

a) based on a yield limit state
b) based on mode failure along plane 1-1 (elevation view)
c) based on mode failure along plane 2-2 (plan view)

1) Determine the optimum thickness (rounded to the millimeter) of the gusset plates necessary to develop the full capacity of the tension member.

Limit to calculations on the

a) yield strength of the gusset plates
b) rupture strength of gusset plate (clearly identify the plane of failure on a sketch)
c) A shear-tension block mode of failure (clearly show the plane of failure on a sketch)
d) Determine the optimum thickness of the plate



Solution for Problem 3

d =203 mm, b=203mm, t=11mm, w=7.2mm, A=5,820 mm2

1. Capacity of tension member

a) Yield limit state

$$T_{r1} = \phi A F_y = 0.90 \times 5{,}860 \times 350 \times 10^{-3} = 1{,}846 kN$$

b) Mode of failure 1-1

Since the mode of failure intercepts the unconnected element (the web), a shear lag reduction factor needs to be applied.

$$A_n = A_g - 4\phi t = 5{,}860 - 4 \times 24 \times 11 = 4{,}804 mm^2$$

$$A_{ne} = 0.90 A_n = 0.90 \times 4{,}804 = 4{,}324 mm^2$$

$$T_r = 0.85 \phi A_{ne} F_u = 0.85 \times 0.90 \times 4{,}324 mm^2 \times 450 \times 10^{-3} = 1{,}489 kN$$

c) Mode of failure 2-2

The shown mode of failure is a shear tension block failure.
Since the mode of failure does not intercept the unconnected element (the web), no reduction for shear lag needs to be applied. A free body diagram for the left part right before failure shows that the total force at the plane of failure is equal to the full tension force T

$T_r + V_r =$ Lesser of $\phi A_{nt} F_u + 0.6\phi A_{gv} F_y$ and

$\phi A_{nt} F_u + 0.6\phi A_{nv} F_u$

We have four such planes (two on each flange). Thus, in the following, we multiply all areas by 4

$$A_{nt} = 4(36.5 - 12) \times 11 = 1,078 mm^2$$

$$A_{gv} = 4 \times 300 \times 11 = 13,200 mm^2$$

$$A_{nv} = 4 \times (300 - 2.5 \times 24) \times 11 = 10,560 mm^2$$

Substituting:

$$(T_r + V_r)_1 = \phi A_{nt} F_u + 0.6\phi A_{gv} F_y$$
$$= 0.90(1,078)(0.450) + 0.6(0.90)(13,200)(0.350)$$
$$= 437 + 2,495 = 2,932 kN$$

$$(T_r + V_r)_2 = \phi A_{nt} F_u + 0.6\phi A_{gv} F_y$$
$$= 0.90(1,078)(0.450) + 0.6(0.90)(10,560)(0.450)$$
$$= 437 + 2,566 = 3,003 kN$$

Thus, take $T_r + V_r = \min(2932, 3003) = 2,932 KN$

Capacity is governed by mode 1-1 and is 1,489 *kN*

2 Tensile capacity of gusset plates (all calculations are functions of the unknown plate thickness *t*)

a. Strength based on Yield Strength

One plate: $T_{r1} = \phi A F_y = 0.90 \times (200) t \times 300 \times 10^{-3} = (54t) kN$

Two plates: $T_{r1} = \phi A F_y = 0.90 \times 2 \times (200) t \times 300 \times 10^{-3} = (108t) kN$

b. Strength based on Rupture

One plate: $T_{r1} = 0.85\phi A_n F_u = 0.85 \times 0.90 \times (200 - 2 \times 24) t \times 450 \times 10^{-3} = (52.3t) kN$

Two plates: $T_{r1} = 0.85\phi A_n F_u = 0.85 \times 0.90 \times 2(200 - 2 \times 24) t \times 450 \times 10^{-3} = (104.6t) kN$

c. Strength based on Shear tension block failure

Modes of failures in Figs. 1 an 2 are shear-tension block failure modes and need to be checked. Both failure modes have the same shear area. However, the tension area of mode 1 is smaller than that of mode 2. One can conclude that mode 2 will not govern the design.

For mode of failure 1

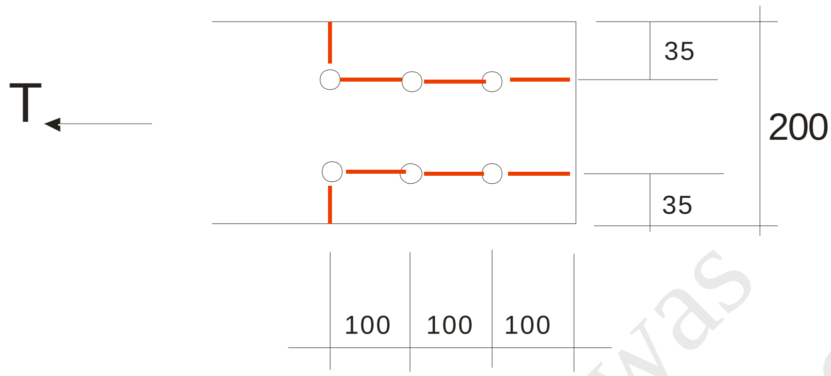


Fig. 1 Mode of Failure 1

For a single plate:

$T_r + V_r$ = Lesser of $\phi A_{nt} F_u + 0.6\phi A_{gv} F_y$ and

$$\phi A_{nt} F_u + 0.6\phi A_{nv} F_u$$

$$A_{nt} = (70 - 24) \times t = 46t\ mm^2$$

$$A_{gv} = 2 \times 300 \times t = 600t\ mm^2$$

$$A_{nv} = 2 \times (300 - 2.5 \times 24) \times t = 480t\ mm^2$$

Substituting:

$$(T_r + V_r)_1 = \phi A_{nt} F_u + 0.6\phi A_{gv} F_y$$
$$= 0.90(46t)(0.450) + 0.6(0.90)(600t)(0.300)$$
$$= 18.6t + 97.2t = 116t$$

$$(T_r + V_r)_2 = \phi A_{nt} F_u + 0.6\phi A_{gv} F_y$$
$$= 0.90(46t)(0.450) + 0.6(0.90)(480t)(0.450)$$
$$= 18.6t + 116.6t = 135t$$

Thus, take $T_r + V_r = \min(116t, 135t) = 116tKN$

For two plates, capacity $= T_r + V_r = 2 \times 116tKN = 232t\ kN$

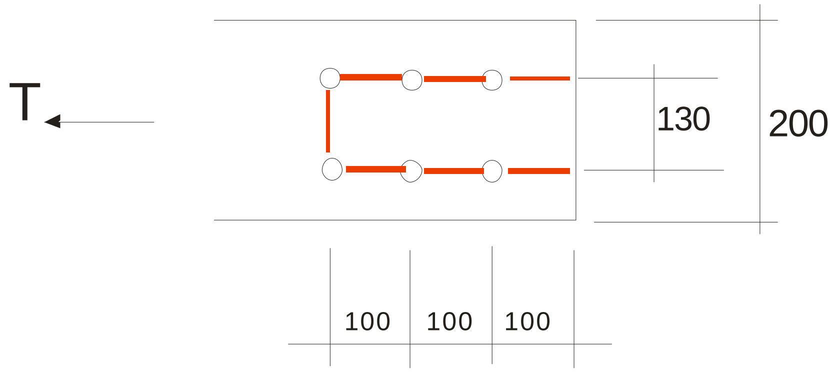


Figure 2

d. Optimum value for thickness t

Based on the previous analysis, the rupture mode of failure governs the resistance of the gusset plate as it has the lowest resistance of 104.6t kN.

Equate the resistance of the gusset plate to that of the tension member for an optimum thickness for the gusset plate

$$104.6t = 1,489kN \Rightarrow t = 14.2mm$$

Take t=15mm