MAT 1341 F13 Dias

1. If $u = (3,\ 0,\ 3)$, $v = (-5,\ 1,\ -8)$ and $w = (1,\ 4,\ 2)$, then $\|(2u+v)\times w\|$ is:

A. $10\sqrt{5}$
B. 25
C. $2\sqrt{10}$
D. $2\sqrt{5}$
E. $5\sqrt{5}$
F. $5\sqrt{2}$

$2u+v = (6,0,6) + (-5,1,-8) = (1,1,-2)$

$\therefore (2u+v)\times w = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & 4 & 2 \end{vmatrix} = (10,-4,3)$

Hence $\|(2u+v)\times w\|^2 = 100+16+9 = 125$

and so $\|(2u+v)\times w\| = \sqrt{125} = \sqrt{25.5} = 5\sqrt{5}$

2. Parametric equations of the line passing through $(1,\ 1,\ -1)$ and which is perpendicular to the plane $-2x - y + 3z = 6$ are:

A. $x = 1-2t,\ y = 1+t,\ z = -1+3t,\ t \in \mathbf{R}$
B. $x = 1+t,\ y = 1+t,\ z = -1-3t,\ t \in \mathbf{R}$
C. $x = 1+2t,\ y = 1-t,\ z = -1+3t,\ t \in \mathbf{R}$
D. $x = 1-t,\ y = 1+t,\ z = -1-6t,\ t \in \mathbf{R}$
E. $x = 1-2t,\ y = 1-t,\ z = -1+3t,\ t \in \mathbf{R}$
F. $x = 1-4t,\ y = 1-t,\ z = -1-3t,\ t \in \mathbf{R}$

A direction vector of the line is the normal to the plane, namely $(-2,-1,3)$. (or any non-zero multiple of this vector). There is only one response with this direction vector, namely E, and (for $t=0$) it does pass through $(1,1,-1)$

3. Which of the vectors below is perpendicular (orthogonal) to both $(2,\ 1,\ -1)$ and $(1,\ 1,\ 5)$?

A. (-4, 0, 3)
B. (3, 0, 2)
C. (1, 0, 1)
D. (2, -5, -1)
E. (1, -2, 0)
F. None of the above

Such a vector would be parallel to $(2,1,-1) \times (1,1,5)$, so we compute:

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 5 \end{vmatrix} = (6, -11, 1)$$

None of the above vectors is parallel to this vector.

4. Find the volume of the parallelepiped determined by the vectors $u = (1,\ 1,\ -1)$, $v = (3,\ 1,\ 0)$ and $w = (1,\ -1,\ 3)$.

A. 6
B. -6
C. 16
D. -2
E. 4
F. 2

The volume is $|u \cdot v \times w|$, so we compute

$$v \times w = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & 0 \\ 1 & -1 & 3 \end{vmatrix} = (3, -9, -4)$$

Hence $u \cdot v \times w = (1,1,-1) \cdot (3,-9,-4)$

$= 3 - 9 + 4 = -2.$

Thus the volume is 2.

5. An equation for the plane passing through the points (1, 2, -1) and (2, 3, 1), and parallel to the y-axis is:

A. $x + y - z = 4$
B. $-3x + 7y - 2z = 3$
C. $x - y = -1$
D. $2x - z = 3$
E. $2y - z = 5$
F. $x + y + z = 2$

Any plane parallel to the y-axis has a normal which is $\perp$ to the y-axis. There is only one plane here – D – with normal $\perp$ to the y-axis. Moreover, the plane with equation in D does contain both the above points.

6. Find an equation of the plane which passes through the point (1, 2, 3) and which is perpendicular to the line whose parametric equations are:

$$x = 2 + 2t,\ y = 7 - 4t,\ z = -3 + t;\ t \in \mathbf{R}.$$

A. $2x - 4y + z = -3$
B. $2x + 7y - 3z = 7$
C. $2x - 4y + z = -5$
D. $-4x + 2y + z = 3$
E. $2x - 4y + z = 0$
F. $-4x + 2y + z = 4$

Such a plane has normal a non-zero multiple of the direction vector of the above line, i.e. parallel to $(2, -4, 1)$. There are 3 candidates (A, C, E), so we check which one of these passes through $(1, 2, 3)$. Well, $2(1) - 4(2) + 1.(3) = -3$, hence A is correct.

7. If $u = (3,\ 3,\ 3)$ and $v = (4,\ 2,\ 6)$ then $\text{proj}_v u =$

$$\frac{(u \cdot v)}{\|v\|^2} v$$

A. $\frac{9}{7}(3,\ 3,\ 3)$
B. $\frac{12}{7}(3,\ 3,\ 3)$
C. $\frac{11}{7}(3,\ 3,\ 3)$
D. $\frac{9}{7}(2,\ 1,\ 3)$
E. $\frac{12}{7}(2,\ 1,\ 3)$
F. $\frac{11}{7}(2,\ 1,\ 3)$

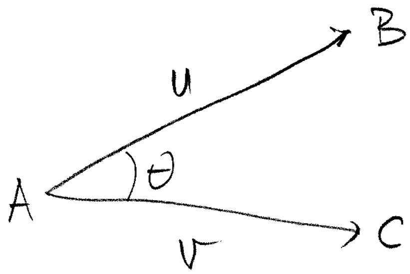
$$= \frac{(12+6+18)}{16+4+36} \cdot (4,2,6)$$

$$= \frac{36}{56} \cdot (4,2,6) = \frac{9}{14} \cdot (4,2,6)$$

$$= \frac{9}{7} \cdot (2,1,3)$$

8. If $A = (2,\ 4,\ 1)$, $B = (3,\ 0,\ 9)$ and $C = (1,\ 4,\ 0)$, find the angle $\angle BAC$.

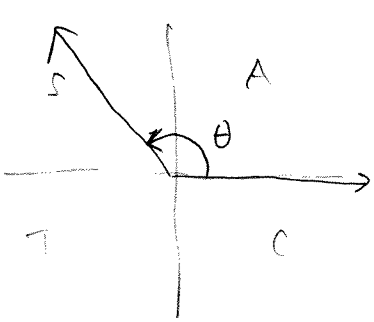
A. $\pi/2$
B. $3\pi/4$
C. $\pi/6$
D. $\pi/3$
E. $\pi/4$
F. $4\pi/3$



$u = B - A = (1,-4,8)$

$v = C - A = (-1,0,-1)$

We know $\cos\theta = \dfrac{u \cdot v}{\|u\|\ \|v\|} = \dfrac{(-1+0-8)}{\sqrt{(1+16+64)}\ \sqrt{2}}$



$$= \frac{-9}{9 \cdot \sqrt{2}} = -\frac{\sqrt{2}}{2}$$

Hence $\theta = 3\pi/4$.

9. Express the following complex numbers in the form $a + bi$ with a and b real.

$$z_1 = \frac{1}{1-i} = \frac{1+i}{2} = \frac{1}{2} + \frac{i}{2}$$

$$z_2 = (2+i)(1+i) = (2-1) + 3i$$

$$= 1 + 3i.$$

A. $z_1 = (1/2) - (1/2)i$; $z_2 = 1 - 3i$

B. $z_1 = (1/2) + (1/2)i$; $z_2 = 1 + 3i$

C. $z_1 = 2 - (1/4)i$; $z_2 = 6 - 2i$

D. $z_1 = 1 - i$; $z_2 = 4$

E. $z_1 = 1 - i$; $z_2 = 4 + 4i$

F. $z_1 = -1 + i$; $z_2 = 2 + 4i$

10. The point of intersection of the line (L) passing through (1, 1, 0) (A) and (0, 1, 0) (B) with the plane with equation $x + y - z = 1$ is:

A. (1/2, 1/2, 0)

B. (0, 1/2, −1/2)

C. (0, 1, 0)

D. (1/2, 0, −1/2)

E. (−1, 0, −1)

F. (1, 0, 0)

Parametric eqns for L are

$X = 0 + t, \quad y = 1, \quad z = 0$

(Since a direction vector is $A - B = (1, 0, 0)$, and L passes through $(0,1,0)$.)

Then, $X + y - z = t + 1 - 0 = t + 1 = 1$; hence $t = 0$, so $\underline{C}$ gives the point of intersection.

11. Find the polar form of

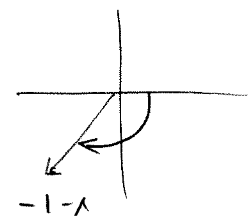
$$z = \frac{\sqrt{3}+i}{-1-i} = \frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}} = \left(\frac{r_1}{r_2}\right) e^{i(\theta_1-\theta_2)}$$

A. $\sqrt{2}(\cos(-\pi/12) + i\sin(-\pi/12))$
B. $\sqrt{2}(\cos(\pi/12) + i\sin(\pi/12))$
C. $\sqrt{2}(\cos(-5\pi/12) + i\sin(-5\pi/12))$
D. $\sqrt{2}(\cos(11\pi/12) + i\sin(11\pi/12))$
E. $\sqrt{2}(\cos(-7\pi/12) + i\sin(-7\pi/12))$
F. $\sqrt{2}(\cos(5\pi/12) + i\sin(5\pi/12))$

$r_1 = \sqrt{3+1} = 2$

$\cos\theta_1 = \frac{\sqrt{3}}{2}$, $\sin\theta_1 = \frac{1}{2}$ $\therefore \theta_1 = \pi/6$

Now $r_2 = \sqrt{1+1} = \sqrt{2}$ while



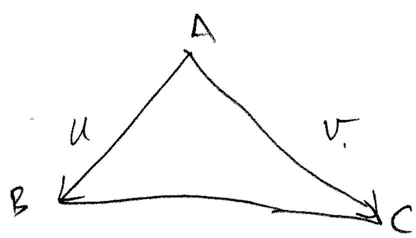
$\cos\theta_2 = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\sin\theta_2 = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\therefore \theta_2 = -\frac{3\pi}{4}$ or

Thus $z = \frac{2}{\sqrt{2}} \cdot e^{i\pi\left(\frac{1}{6}-\left(\frac{-3\pi}{4}\right)\right)} = \sqrt{2}\, e^{i\left(\frac{11}{12}\right)\pi}$ Thus D is correct

12. What is the area of the triangle with vertices $\overset{A}{(4,\ 1,\ -1)}, \overset{B}{(6,\ 3,\ 0)}$ and $\overset{C}{(6,\ 10,\ 1)}$?

A. 13
B. 15
C. 13/2
D. 15/2
E. 11
F. 17/2



The area is $\frac{1}{2}\|u \times v\|$.

Here, $u = B - A = (2,2,1)$ and $v = C - A = (2,9,2)$

$$\therefore u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 2 & 9 & 2 \end{vmatrix} = (-5, -2, 14)$$

$\therefore \|u \times v\|^2 = 25 + 4 + 196 = 225 \therefore \frac{1}{2}\|u \times v\| = \frac{15}{2}$