Lec 12 mini review.

Substitution: $\int F'(g(x))g'(x)dx = \int F'(u)du = F(u) + C = F(g(x)) + C$

Integrals with Even Symmetry: If $f(-x) = f(x)$, then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

Integrals with Odd Symmetry: If $f(-x) = -f(x)$, then $\int_{-a}^{a} f(x)dx = 0$

Lec 13 mini review.

Integration by Parts: $\boxed{\int uv' = uv - \int u'v}$ or $\boxed{\int u\,dv = uv - \int v\,du}$

useful trig identities for some trig integrals:

$$\cos^2(x) + \sin^2(x) = 1$$
$$1 + \tan^2(x) = \sec^2(x)$$
$$\cos^2(x) = \tfrac{1}{2}(1 + \cos(2x))$$
$$\sin^2(x) = \tfrac{1}{2}(1 - \cos(2x))$$

Lec 14 mini review.

useful trig identities:

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$

$\sin(2x) = 2\sin(x)\cos(x)$
$\cos(2x) = \cos^2(x) - \sin^2(x)$

expression:	**identity**:	**substitution**:
$\sqrt{1-x^2}$	$1 - \sin^2\theta = \cos^2\theta$	$x = \sin\theta$ $(-\frac{\pi}{2} \le \theta \le \frac{\pi}{2})$
$\sqrt{1+x^2}$	$1 + \tan^2\theta = \sec^2\theta$	$x = \tan\theta$ $(-\frac{\pi}{2} < \theta < \frac{\pi}{2})$
$\sqrt{x^2-1}$	$\sec^2\theta - 1 = \tan^2\theta$	$x = \sec\theta$ $(0 \le \theta < \frac{\pi}{2},\quad \pi \le \theta < \frac{3\pi}{2})$

Lec 15 mini review.

- ⋄ **integrating rational functions:** $\int \frac{1}{ax+b}dx = \frac{1}{a}\ln|ax+b|$ $\quad \int \frac{1}{x^2+1}dx = \tan^{-1}(x)$
- ⋄ use long division to obtain a proper fraction
- ⋄ factor denominator into product of linear and irreducible quadratic factors
- ⋄ decompose integrand into its partial fractions

partial fractions: denominator factors into product of

- distinct linear factors: *partial fractions expression*

$$\frac{N(x)}{D(x)} = \frac{N(x)}{(a_1x+b_1)\dots(a_kx+b_k)} = \frac{A_1}{a_1x+b_1} + \dots + \frac{A_k}{a_kx+b_k}$$

- distinct irreducible quadratic factors: *partial fractions expression*

$$\frac{N(x)}{D(x)} = \frac{N(x)}{(a_1x^2+b_1+c_1)\dots(a_kx^2+b_kx+c_k)} = \frac{A_1x+B_1}{a_1x^2+b_1x+c_1} + \dots + \frac{A_kx+B_k}{a_kx^2+b_kx+c_k}$$

- if a linear factor is repeated, say $(ax+b)^r$ appears in factorization of denominator, we use

$$\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \dots + \frac{A_r}{(ax+b)^r} \quad \text{intead of the single term} \quad \frac{A}{ax+b}$$

- if an irreducible quadratic factor is repeated, say $(ax^2+bx+c)^r$ appears in factorization of denominator, we use

$$\frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \dots + \frac{A_rx+B_r}{(ax^2+bx+c)^r} \quad \text{intead of the single term} \quad \frac{Ax+B}{ax^2+bx+c}$$

Lec 16 mini review.

- **Midpoint Rule**: $\boxed{M_n = \sum_{i=1}^{n} f(\overline{x_i})\Delta x}$ $\quad (\overline{x_i} = \frac{1}{2}(x_{i-1}+x_i))$
- **Trapezoidal Rule**: $\boxed{T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)\right]}$
- **Simpson's Rule (n even)**:

$$\boxed{S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]}$$

Error Bounds:

- If $|f''(x)| \le K$ for all $a \le x \le b$, then $\left|T_n - \int_a^b f(x)dx\right| \le \frac{K(b-a)^3}{12n^2}$ and $\left|M_n - \int_a^b f(x)dx\right| \le \frac{K(b-a)^3}{24n^2}$
- If $|f^{(4)}| \le K$ for all $a \le x \le b$, then $\left|S_n - \int_a^b f(x)dx\right| \le \frac{K(b-a)^5}{180n^4}$