

Assignment 1 Part 2

Due Sept 25 23:59 EST

Marks for each part of each question are indicated in parentheses in the questions below.

Please see course outline for Part 2 Marking Scheme for penalties. There are no bonus marks for this assignment.

Total marks: 12. This assignment is worth 5% of the course mark. The mark on this assignment will be multiplied by 5/12 to calculate the course mark /100.

Q1. First some definitions. Trucks entering Canada are scanned using x-rays in order to find illegal weapons. If the scanner fails to find an illegal weapon when one is actually present in the truck we call this a “false negative”. The test was “negative”, because it said there was no illegal weapon. It was “false” because it should have said there was an illegal weapon. Similarly a “false positive” is when the scanner says there is an illegal weapon when in fact there is not. Before using such a scanner it is important to know how accurate it is. We test the scanner on 100 trucks known to have illegal weapons and note how many times it fails to detect them. If it fails to detect illegal weapons in 5 of the 100 trucks, the “false negative rate” is 5%. Similarly if the scanner alerts us to the presence of illegal weapons in 3 trucks out of 100 known not to have illegal weapons, the “false positive rate” is 3%. These “rates” can be used to estimate the “false negative probability” = $P(\text{scanner says no weapon} \mid \text{weapon present}) = 0.05$ and “false positive probability” $P(\text{scanner says weapon} \mid \text{no weapon present}) = 0.03$ of the scanner making these errors when it is in operation scanning trucks entering Canada. The “rate” is expressed as a %; the probability is expressed as a decimal.

Q1 (6 marks) Covid-19 Testing. When SARS-Cov2 (the virus that causes Covid-19) infects a patient, it goes through 3 stages.

1. It enters the patient’s cells and lies dormant there for a number of days that varies from patient to patient. The patient usually shows no symptoms of illness.
2. It then uses the patient’s cellular machinery to replicate itself and some copies of the virus enter the bloodstream. The patient usually starts to show respiratory symptoms.
3. This causes a response from the patient’s immune system which may fight against the virus, or it may cause excessive inflammation that could kill the patient. The patient shows respiratory symptoms and may be very sick.

One of the tests for Covid-19 is a genetic test, the aim of which is to detect viral RNA. If this test is applied during Stage 1, it will not detect the virus since the virus is hidden away inside the patient’s cells. 2% of patients with respiratory symptoms whose test indicates the presence of the SARS-Cov2, do not in fact have the SARS-Cov2, due to errors in the way the test is conducted. If this test is applied during Stage 2, it should detect SARS-Cov2 but 3% of these tests fail to detect SARS-Cov2 also due to errors in the way the test is conducted.

- a) (2 marks) Write the 2% and 3% probabilities using conditional probability notation, e.g. $P(X|Y)=0.02$.
- b) (2 marks) Are the 2% and 3% an example of a false positive rate or a false negative rate or neither?
- c) (1 mark) Is it possible to estimate the false negative probability of the genetic test by applying the test to patients known to be in Stage 3 of Covid-19?
- d) (1 mark) Is it possible to estimate the false positive probability by applying the test to patients known to be in Stage 3 of Covid-19?

Q2 (6 marks) Alberta Tar Sands. The price at which the crude oil extracted from Alberta's tar sands can be sold is determined by the "Brent Crude" price at which crude oil is traded. At a Brent Crude price of \$60/barrel 42% of the oil in the tar sands can be extracted profitably.

(a) A market research company forecasts the Brent Crude price will be over \$60/barrel next year with a probability of 0.9 and that it will be over \$60/barrel the following year with a probability of 0.8. These two forecasts are independent of each other.

(i) (1 mark) What is the probability that the Brent Crude price will be over \$60/barrel *both* next year *and also* the following year?

(ii) (1 mark) What is the probability that the Brent Crude price will be over \$60/barrel *either* next year *or* the following year or both years?

(b) Ignore the forecast in part (a). A market research company forecasts the Brent Crude price will be over \$60/barrel next year with a probability of 0.8. The forecast for the following year depends on whether the price next year is in fact over \$60/barrel. If it is, then the probability of being over \$60/barrel the following year is 0.7. If the actual Brent Crude price is not over \$60/barrel next year then the probability of being over \$60/barrel the following year is 0.4.

(i) (2 marks) What is the probability that the Brent Crude price will be over \$60/barrel next year and under \$60/barrel the following year.

(ii) (2 marks) What is the probability that the Brent Crude price will be over \$60/barrel in exactly one of the next two years?