Final Exam
(One of the 2018 versions)

Examiner: Jeremy Balka

This exam is two hours in duration

Name:

ID:

Signature:

Please read the instructions:

1. Fill out your name and ID number above and on the computer answer sheet (the "bubble" sheet). (If you are an Open Learning student, and have a 9 digit ID number that won't fit on the bubble sheet, please enter only the last 7 digits of your ID number on the bubble sheet.)
2. When the examination starts, make sure your question paper is complete. You should have 26 multiple choice questions, along with 3 questions that require a written response. The multiple choice questions are worth 2 marks each, the written response questions are worth 5 marks each. There are 67 marks available in total. You should also have been given formula sheets and statistical tables.
3. Do all work on this paper.
4. You are allowed to bring in a calculator, and pens and pencils.
5. There is only **one** correct answer for each question. Fill in only one bubble for each question.
6. Fill out the computer answer sheet in pencil as you go. *There will be no extra time given at the end of the exam to fill in the sheet.*
7. The answers given in the exam are often rounded versions of the correct answer. Choose the closest value.

1. Consider the following sample of 5 observations:

$$-3.2, 1.1, 0.2, 9.8, 11.8$$

The value of the standard deviation is closest to which one of the following?

(a) 3.5
(b) 4.2
(c) 5.3
(d) 6.5
(e) 8.9

2. Which one of the following statements is FALSE? (Assume that the t distribution under discussion has finite degrees of freedom.)

(a) The median of the t distribution is equal to the median of the standard normal distribution.
(b) The variance of the t distribution is greater than the variance of the standard normal distribution.
(c) The t distribution has more area in the tails and a lower peak than the standard normal distribution.
(d) As the degrees of freedom increase, the t distribution tends toward the standard normal distribution.
(e) The t distribution is mathematically equivalent to the standard normal distribution if the degrees of freedom are at least 30.

3. A study found that the amount of protein in large white eggs is approximately normally distributed with a mean of 6.25 grams and a standard deviation of 0.45 grams. If 6 large white eggs are randomly selected, what is the probability that the mean amount of protein per egg is at least 6.16 grams? (Choose the closest value.)

   (a) 0.69
   (b) 0.74
   (c) 0.79
   (d) 0.84
   (e) 0.89

4. Two inspectors, A and B, work side by side at a factory. They both inspect the quality of each part produced, giving each part a pass or fail rating. Inspector A and Inspector B have both just started the job, and they have no idea what they are doing. They are friends and they decide that they will decide randomly between passing and failing each part. For each part, they will toss a single coin once. If the coin comes up heads, Inspector A will pass the part, and B will fail it. If the coin comes up tails, B will pass the part and A will fail it. Using this rule, Inspectors A and B both "inspect" the same part, A tosses the coin, and they use their rule to give the part a pass or fail rating. Consider the two events:

   A: Inspector A passes the part.
   B: Inspector B passes the part.

   Which one of the following is true?

   (a) A and B are mutually exclusive and independent.
   (b) A and B are mutually exclusive, but not independent.
   (c) A and B are not mutually exclusive, and not independent.
   (d) A and B are not mutually exclusive, but are independent.
   (e) None of the above.

5. Consider the following statements about confidence intervals for a population mean. Which one is true?

   (a) Standard errors are the estimated standard deviations of parameters in repeated sampling.
   (b) In repeated sampling, 95% of the 95% confidence intervals will contain the value of $\bar{X}$.
   (c) As the confidence level decreases (from 95% to 90%, say), the width of the interval increases.
   (d) In practical problems, σ is usually known and thus we usually use z procedures instead of t procedures.
   (e) None of the above.

6. Suppose researchers wish to carry out a test of H_0: $\mu = 12$ against a two-sided alternative hypothesis. The population is known to be normally distributed and the population standard deviation is known to equal 4.1 ($\sigma = 4.1$). They draw a random sample of 9 observations and find that the sample mean is 10.9. What is the p-value of the test? (Choose the closest value.)

   (a) 0.26
   (b) 0.42
   (c) 0.45
   (d) 0.47
   (e) 0.51

7. A study of pregnant women in London, England investigated a possible association between parental smoking and the probability of a male birth. In 352 pregnancies in which both parents were heavy smokers during the pregnancy, at birth 145 of the 352 babies were male. Suppose these babies can be thought of as a random sample of babies born to heavy smokers in London. For babies born to heavy smokers in London, what is a 90% confidence interval for the true proportion that are male?

   (a) 0.41 ± 0.043
   (b) 0.41 ± 0.052
   (c) 0.41 ± 0.055
   (d) 0.41 ± 0.058
   (e) 0.41 ± 0.061

8. Consider again the information in the previous question. Suppose we wish to test the null hypothesis that half of babies born to heavy smokers in Manchester are male, and we feel that the appropriate alternative hypothesis is that less than half of are male. We decide to use a significance level of $\alpha = 0.05$. What would be the value of the appropriate z test statistic? (Choose the closest value.)

   (a) -6.12
   (b) -3.36
   (c) -3.30
   (d) -2.20
   (e) 2.25

9. Which one of the following statements about the normal distribution is **FALSE**?

   (a) If Z is a random variable with the standard normal distribution, then $P(Z = 0) = 0.5$.
   (b) The standard normal distribution is symmetric about 0.
   (c) The mean of a normally distributed random variable can be negative.
   (d) The probability a normally distributed random variable takes on a value within 10 standard deviations of the mean is greater than 0.999.
   (e) The 10th percentile of the standard normal distribution is negative.

10. A surgeon develops two new surgery methods, and they wish to study various characteristics of the different methods. In one part of the study, the surgeon wishes to investigate whether the two types of surgery differ in the proportion of patients that develop a post-operative infection. Of the following options, which one best represents the hypotheses of the appropriate test?

    (a) H_0: $p_1 = p_2 = 0.5$, H_a: $p_1 \neq p_2 \neq 0.5$.
    (b) H_0: $\hat{p}_1 = \hat{p}_2$, H_a: $\hat{p}_1 \neq \hat{p}_2$
    (c) H_0: $p_1 = p_2$, H_a: $p_1 \neq p_2$
    (d) H_0: $p_1 = p_2$, H_a: $p_1 > p_2$
    (e) H_0: $\mu_1 = \mu_2$, H_a: $\mu_1 \neq \mu_2$

11. A study investigated fatigued driving in shift workers. In one aspect of the study, the point of interest was in testing to see if there was a difference in the blink rate of drivers before and after a shift. 40 nurses were selected for the study. Each nurse had their blink-rate measured by technology as they drove their cars to their shift, and also when they drove home after their shift. If the researchers wish to compare the before-shift blink rate to the after-shift blink rate, which is the most appropriate analysis to use? (Assume normality where needed.)

    (a) A pooled-variance t test.
    (b) A Welch t test.
    (c) A paired-difference t test.
    (d) A simple linear regression with a t test on the intercept.
    (e) A central limit theorem based one-way manipulative analysis.

12. In a certain large population of raccoons, 8% have rabies. If raccoons from this population are randomly selected, what is the probability that the first raccoon with rabies occurs **on or before** the third raccoon sampled?

    (a) 0.07
    (b) 0.13
    (c) 0.17
    (d) 0.22
    (e) 0.26

13. Suppose we carry out a test of H_0: $\mu = 0.5$ against H_a: $\mu \neq 0.5$, and we find a p-value of 0.08. Which one of the following statements is true?

    (a) The null hypothesis would be rejected at $\alpha = 0.01$.
    (b) The null hypothesis would be not be rejected at $\alpha = 0.01$, but would be rejected at $\alpha = 0.05$.
    (c) The null hypothesis would be not rejected at $\alpha = 0.05$, but would be rejected at $\alpha = 0.10$.
    (d) The null hypothesis would be not be rejected at $\alpha = 0.10$.
    (e) Regardless of the value of α, we know that the null hypothesis is true.

14. Consider the simple linear regression model: $Y = \beta_0 + \beta_1 X + \epsilon$ that was discussed in the notes and videos. Which one of the following statements is true?

    (a) If $\beta_1 = 0$ then there is no linear relationship between X and Y.
    (b) The values of β_0 and β_1 are usually known, whereas the value of Y is usually unknown.
    (c) ϵ is assumed to have a uniform distribution.
    (d) For any given Y, X is assumed to have a uniform distribution.
    (e) For any given X, Y is assumed to have a uniform distribution.

15. Suppose we are about to randomly select a Canadian adult. Let A be the event that the person is at least 52 years old. Let B be the event that the person is a resident of a retirement home. Which one of the following statements is true?

    (a) $P(A|B) > P(A)$.
    (b) $P(B|A) < P(B)$.
    (c) A and B are mutually exclusive.
    (d) A and B are independent.
    (e) $P(A) = P(B)$.

16. Researchers wish to investigate the effect of 4 newly developed diets on the longevity of rats. (They wish to see whether the mean lifetime of rats differs between the diets.) They have 200 rats available to use in the study. Which one of the following best describes this situation? (Suppose that any necessary assumptions (such as normality) are reasonable.)

    (a) This study should be carried out as an experiment, and analyzed with one-way ANOVA.
    (b) This study should be carried out as an experiment, and analyzed using a one-sample t test.
    (c) This study should be carried out as a survey, and analyzed using simple linear regression.
    (d) This study should be carried out as an observational study, and analyzed with the pooled-variance t procedure.
    (e) This study should be carried out as an observational study, and analyzed with one-way ANOVA.

17. A random sample of 100 University of Guelph students had their blood tested. The following table lists their ABO blood type.

Blood Type	O	A	AB	B
Count	21	32	30	17

Suppose we wish to test the null hypothesis that, for University of Guelph students, the four ABO blood types (O, A, AB, B) are equally likely. What is the value of the appropriate χ^2 test statistic? (Choose the closest value.)

(a) 6.2
(b) 7.1
(c) 12.1
(d) 18.0
(e) 27.8

18. Daly and Wilson (1988) investigated a possible relationship between the age of the child and the sex of the child in murders committed by their father. The following table summarizes the counts for Canadian murders from 1974-1983.

	Age of child victim				
	0-1	2-5	6-10	11-16	≥ 17
Male child	24	21	21	29	104
Female child	17	27	10	14	47

The researchers are interested in investigating a possible relationship between age and sex of the murder victims. They use an appropriate χ^2 test, find $\chi^2 = 10.9067$ with a corresponding p-value of 0.028.

Of the following options, which one is the most appropriate conclusion at $\alpha = 0.01$?

(a) There is not significant evidence that sex of the child and age of the child are independent.
(b) There is not significant evidence that sex of the child and age of the child are not independent.
(c) There is not significant evidence that sex of the child and age of the child are equal.
(d) Fathers are significantly more likely to murder female children than male children.
(e) There is significant evidence that sex of the child and age of the child are not independent.

19. Researchers are interested in investigating a possible effect of 4 different diets on the growth rate of tumours in rats. 60 rats are available for an experiment. The rats are given an injection of cancerous cells to induce tumour growth. The rats are then randomly assigned to the 4 different diets (15 rats to each diet). After a certain amount of time, the rats are sacrificed and the diameter (mm) of the tumour is measured. The researchers feel a one-way ANOVA is appropriate. You have enough information to complete the following ANOVA table.

Source	DF	SS	MS	F
Treatments		12.0		
Error				
Total		262.0		

The value of mean square error is closest to which one of the following?

(a) 0.05
(b) 0.90
(c) 1.98
(d) 4.46
(e) 6.19

20. In a related (but different!) experiment by the same researchers, the effect of 4 different exercise programs on the growth of tumours was investigated. (The 4 types of exercise were: Forced high level of exercise; forced low level of exercise; voluntary exercise; sedentary.) 100 rats were randomly assigned to the different exercises programs (25 rats to each exercise program). After a certain amount of time, the rats were sacrificed and the diameter (mm) of the tumour was measured. The resulting ANOVA table was:

```
          Df Sum Sq Mean Sq F value   Pr(>F)
exercise   3 1.2617  0.4206  6.5104 0.000468 ***
Residuals 96 6.2016  0.0646
```

Of the following options, which one of the most appropriate conclusion to the F test in the ANOVA table?

(a) There is strong evidence that the sample means are not all equal.
(b) There is strong evidence that the population means are all equal.
(c) There is strong evidence that the exercise programs all have the same effect on tumour size.
(d) There is strong evidence that the exercise programs do not all have the same effect on tumour size.
(e) We can be certain that the population means are not all equal.

21. Consider again the information in the previous question (involving the different exercise programs). Suppose the researchers want to use the LSD method to construct 95% confidence intervals for the pairwise differences in the means of tumour sizes. What is the appropriate margin of error of the intervals? (Choose the closest value.)

(a) 0.141
(b) 0.143
(c) 0.151
(d) 0.153
(e) 0.228

Crab researchers often investigate factors affecting the closing force of the crab claw. One possible explanatory variable is the claw height. In one study, researchers investigated a possible relationship between an "Index of closing force" and claw height (mm) for a species of crab (*Lophopanopeus bellus*). The following scatterplot and output from R is from a simple linear regression analysis of the data from a sample of 12 crabs.

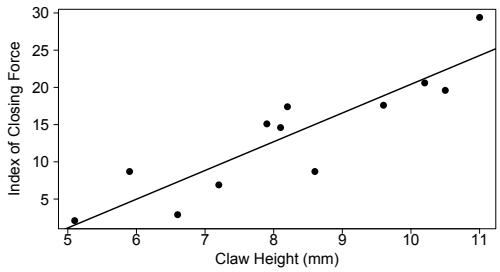


Figure 1: Index of closing force vs claw height.

```
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -18.1767     5.2226   -3.48  0.00592 **
Claw Height   3.8597     0.6195    6.23  9.75e-05 ***
Residual standard error: 3.808 on 10 degrees of freedom
Multiple R-squared: 0.7952,Adjusted R-squared: 0.7747
```

22. Based on the regression equation, what is the predicted index of closing force for a claw height of 8.1 mm?

(a) 12.5
(b) 12.7
(c) 12.8
(d) 12.9
(e) 13.1

23. Consider the information in the previous question. Only one of the following statements is reasonable. Which one?

(a) There is not strong evidence (p-value $= 9.75 \times 10^{-5}$) of a relationship between closing force and claw height.
(b) There is strong evidence (p-value $= 0.00592$) that closing force and claw height have the same population mean.
(c) There is strong evidence (p-value $= 0.00592$) of a relationship between closing force and claw height.
(d) There is not strong evidence (p-value $= 0.00592$) of a relationship between closing force and claw height.
(e) There is strong evidence (p-value $= 9.75 \times 10^{-5}$) of a relationship between closing force and claw height.

24. In adults, chronically high values of C-reactive protein (CRP) have been linked to an increased risk of cardiovascular disease. Researchers were interested in estimating the mean CRP levels in the blood of healthy male Canadians. The researchers drew a sample of 70 Canadian males. Assume that this sample can be thought of as a random sample of healthy adult male Canadians. The researchers measured CRP levels on the 70 subjects and, using the methods discussed in our course, calculated a 95% confidence interval for μ of 6.3 to 6.6 mg/l. Of the following options, which one is the most appropriate interpretation of that interval?

(a) We can be 95% confident that the mean CRP level of the 70 subjects in the study lies between 6.3 and 6.6 mg/l.
(b) In repeated sampling, 95% of the confidence intervals calculated in this manner would contain the mean CRP level of the 70 subjects in the study.
(c) 95% of healthy male Canadians have CRP levels that lie between 6.3 and 6.6 mg/l.
(d) We can be 95% confident that individuals with a CRP level above 6.6 mg/l have an increased risk of cardiovascular disease.
(e) We can be 95% confident that the true mean CRP level of healthy male Canadians lies between 6.3 and 6.6 mg/l.

25. Which one of the following statements is true?

(a) A Poisson random variable is a continuous random variable.
(b) A Poisson random variable can take on negative values.
(c) The mean and variance of a Poisson random variable are always equal.
(d) If X has a Poisson distribution, then $P(X = 0) < P(X = 1)$.
(e) Every random variable that represents a count is a Poisson random variable.

26. A researcher wants to test the null hypothesis H_0: $\mu = 29$ against the alternative that μ is different from 29. She forgets how to do the test, but remembers how to calculate confidence intervals. She calculates a 95% confidence interval of $(23.20, 28.80)$, and a 99% confidence interval of (22.32, 29.68). Which one of the following could be the p-value of the appropriate test?

(a) 0.0022.
(b) 0.0358.
(c) 0.0517.
(d) 0.0812.
(e) 1.8143.

The next 3 questions require a written response, and are worth 5 marks each. The multiple choice questions above are worth 2 marks each. You do not need to be long winded; one or two well-written sentences can result in perfect marks. The 5 marks have nothing at all to do with me looking for 5 different points; I am simply marking them out of 5.

27. A study investigated several aspects of the ratio of the lengths of the index finger to the ring finger in women who visited a sexual health clinic in Manchester, UK. (The ratio of the lengths of these fingers is called the 2D:4D ratio.) The distribution of the 2D:4D ratio depends on several factors, including the ethnic background of the individuals. The following table summarizes the results of the measurements of the 2D:4D ratio on the left hand for white and black women in the study.

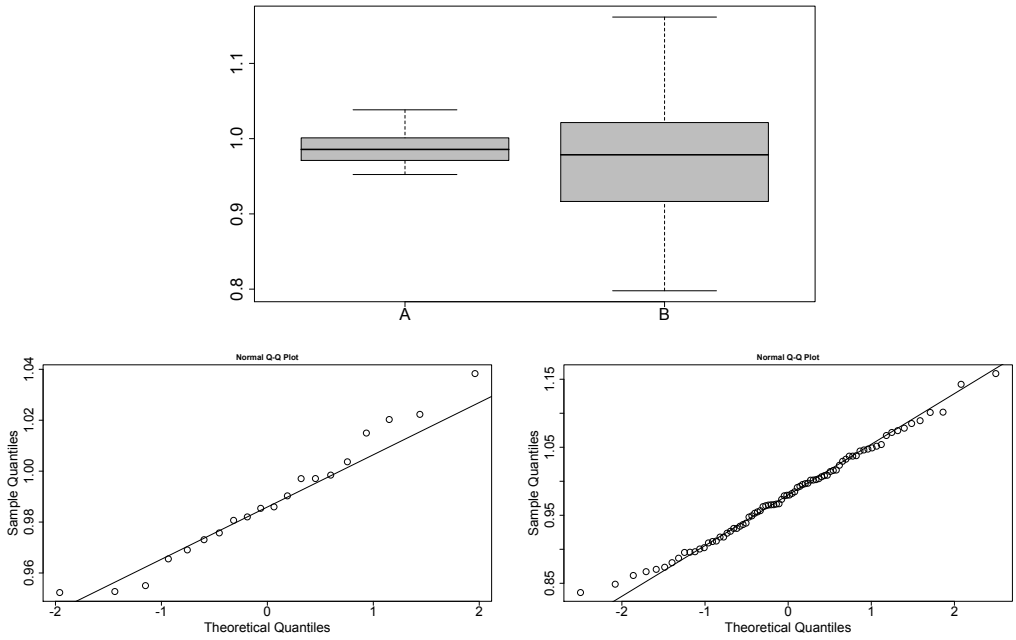
White women	$\bar{X}_1 = 0.994$	$s_1 = 0.035$	$n_1 = 246$
Black women	$\bar{X}_2 = 0.963$	$s_2 = 0.034$	$n_2 = 46$

The output from the statistical software R for the pooled-variance procedure is:

```
        Two Sample t-test
data:  white and black
t = 5.538, df = 290, p-value = 6.859e-08
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.01998280 0.04201720
sample estimates:
mean of x mean of y
    0.994     0.963
```

Give a summary of the results of the analysis (including the results of the hypothesis test and the confidence interval).

28. A researcher drew random samples of individuals, 20 individuals from Group A, and 80 from Group B. The researcher measured a certain characteristic on all of the individuals, and wanted to test to see if there is evidence of a difference in population means. The following figures give boxplots and normal quantile-quantile plots of the observations.



(a) Group A. (b) Group B

What would be the best statistical inference procedure for the researcher to use? Justify your response.

29. What is the central limit theorem and why it is important in statistics?

    (There are formal definitions of the central limit theorem, but here I'm looking for a brief, accurate description, along with a concise explanation why it is important in statistics.)

This page does not have a question. You may use this page to finish any of the written questions for which you need a little more space. If you use this page, please make it clear what question you are answering, and point to it at the original question.