Department of Electronics — Name: Solutions

Carleton University — ID:

ELEC 3908 Section – B1 (Mid-Term Exam) — Date: Feb. 27th, 2020

* Attempt all questions — Time allocated: 90 mins

**Question 1. (10 marks)**

a) Group III elements act as acceptor dopants whereas Group V elements act as donor dopants.

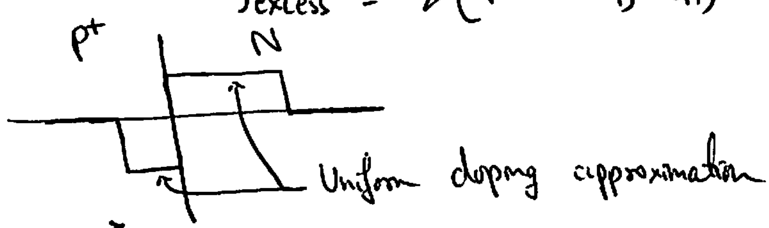
b) What is the dominant recombination mechanism for indirect bandgap semiconductors.

SRH recombination

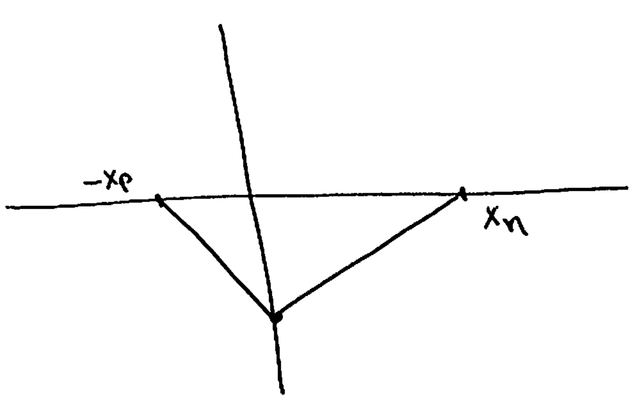
c) What is the uniform doping approximation and how does it simplify the excess charge density expression in the $p^+$ region of a $p^+n$ junction diode? Show in a linear-log plot of abs ($N_A - N_D$) vs x (depth in μm).

Uniform doping approximation assumes that dopants are uniformly distributed in the doped well/region. This allows us to simplify excess charge density expression in the depletion region.

$$\rho_{excess} = q(p - n + N_D - N_A) \approx \begin{cases} qN_D & (n\text{-side}) \\ -qN_A & (p\text{-side}) \end{cases}$$




d) Draw the excess charge density and the E-field plot of a $p^+n$ junction.



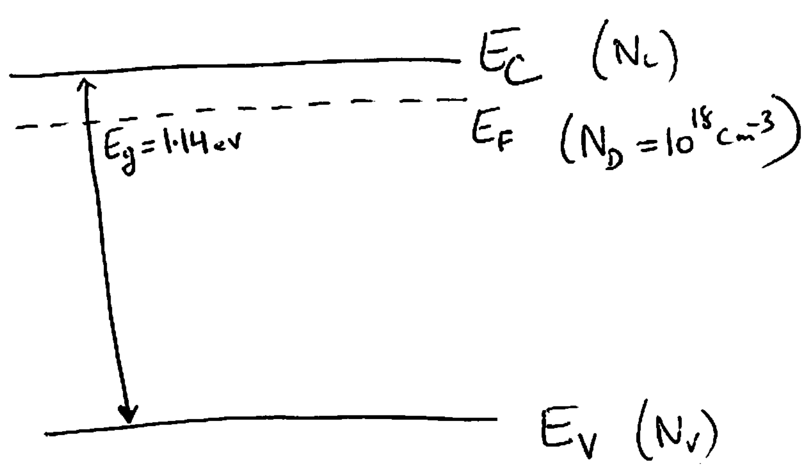
**Department of Electronics** **Name:**

**Carleton University** **ID:**

**ELEC 3908 Section – B1 (Mid-Term Exam)** **Date: Feb. 27th, 2020**

*** Attempt all questions** **Time allocated: 90 mins**

e) Draw energy band diagram of an N-type extrinsic semiconductor with Eg = 1.14 eV and a donor dopant conc., $N_D = 10^{18}$ $cm^{-3}$. Clearly label the valence and conduction bands, dopant atomic orbital and fermi energy levels.



**Department of Electronics** | **Name:**

**Carleton University** | **ID:**

**ELEC 3908 Section – B1 (<u>Mid-Term Exam</u>)** | **Date: Feb. 27th, 2020**

*** Attempt all questions** | **Time allocated: 90 mins**

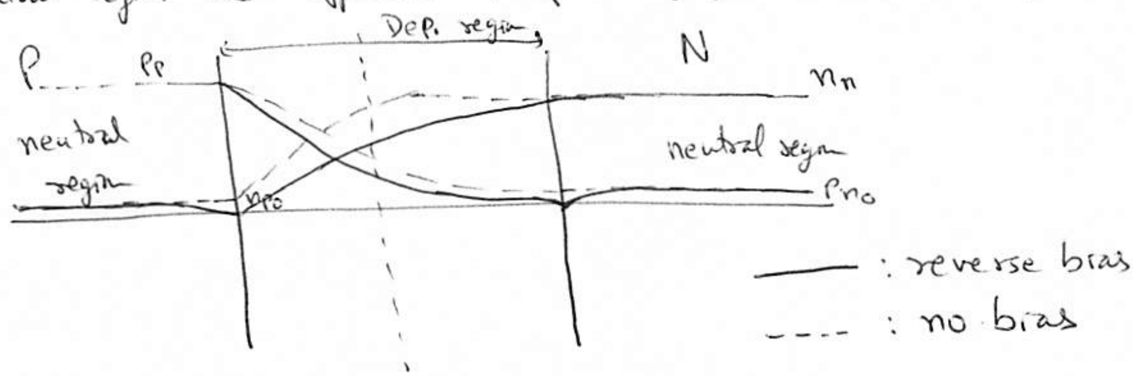
## Question 2. (10 marks)

**a)** What causes the reverse bias breakdown in a diode?

Impact ionization under large reverse bias voltage condition ~~leads to~~ results from increased majority carrier conc. in the depletion region. This results in very high $\mathcal{E}_{max}$ leading to critical material failure.

**b)** What happens to minority carrier densities in the depletion and neutral region under reverse bias? Show on graph.

Minority carriers are depleted in the depletion regions under reverse bias. Neutral region are affected in proximity of the depletion region.




**c)** If a diode with $N_A = N_D = 10^{17}$ cm$^{-3}$ at T = 300 K, has a reverse bias breakdown voltage of -8 V (i.e. $p_{ii} = 1$ at $V_D$ = -8 V), what is the magnitude of the critical electric field, $\xi_{crit}$?

$$V_{bi} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} = 0.02586 \ln\left(\frac{10^{34}}{2.1\times10^{20}}\right) = 0.814\ V$$

$$\Rightarrow V_{bi} \cong 0.8\ V \quad \text{and} \quad V_D = -8V$$

$$\mathcal{E}_{crit} = \sqrt{\frac{2q}{\epsilon_{Si}}\left(\frac{N_A N_D}{N_A+N_D}\right)(V_{bi}-V_D)} = \sqrt{\frac{2\times1.6\times10^{-19}}{11.7\times8.854\times10^{-14}}\left(\frac{10^{34}}{2\times10^{17}}\right)\times 8.8}\ \text{V/cm}$$

$$= \sqrt{0.1359\times10^{12}} \cong 3.67\times10^{5}\ \text{V/cm}$$

**Department of Electronics** **Name:**

**Carleton University** **ID:**

**ELEC 3908 Section – B1 (<u>Mid-Term Exam</u>)** **Date: Feb. 27th, 2020**

*** Attempt all questions** **Time allocated: 90 mins**

**d)** What is the depletion width, W, of the device at the breakdown voltage?

$$W = \sqrt{\frac{2\epsilon_{Si}}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(V_{bi}-V_D)}$$

$$= \sqrt{\frac{2\times 11.7\times 8.854\times 10^{-14}}{1.6\times 10^{-19}}\times\frac{2}{10^{17}}\times 8.8} \quad cm$$

$$= \sqrt{2.28\times 10^{-9}} \quad cm = 4.77\times 10^{-5} \quad cm$$

$$= \underline{\underline{0.48\ \mu m}}$$

**Department of Electronics** — **Name:**

**Carleton University** — **ID:**

**ELEC 3908 Section – B1 (Mid-Term Exam)** — **Date: Feb. 27th, 2020**

*** Attempt all questions** — **Time allocated: 90 mins**

## Question 3. (10 marks)

A **n+-p** junction is at zero bias. Given $N_A = 10^{16}$ cm$^{-3}$ and $N_D = 10^{17}$ cm$^{-3}$, and assuming full depletion approximation and uniform doping approximation,

a) Determine analytical equation for net excess charge density, ρ(x) and the electric field, ξ(x) for x from $-2x_p$ to $+2x_n$. Assume that the junction is oriented such that the metallurgical junction is at x=0 and the n-type material covers x <0.

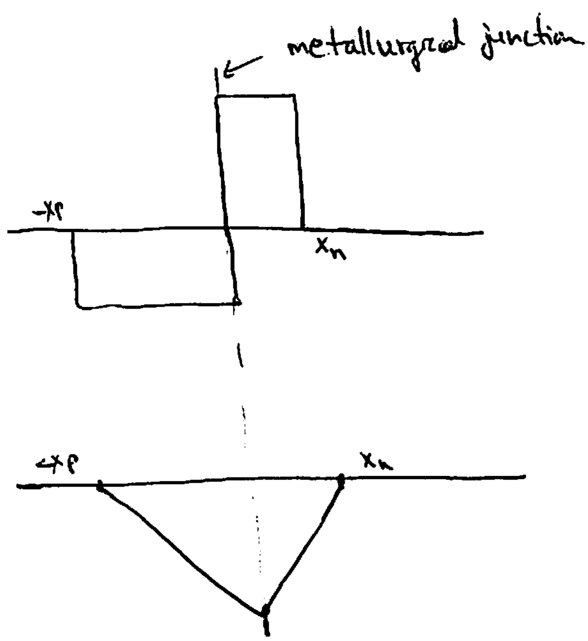
$$\rho(x) = q\left(p(x) - n(x) + N_D^+ - N_A^-\right)$$

$$= \begin{cases} -qN_A & \text{for } -2x_p \le x < 0 \\ qN_D & \text{for } 0 \le x \le 2x_n \end{cases}$$

$$E = \int_{-2x_p}^{2x_n} \frac{\rho}{\epsilon}\, dx = \frac{-qN_A}{\epsilon_{Si}} x \quad ; \text{ from } -2x_p \text{ to } 0$$

$$\frac{qN_D}{\epsilon_{Si}} x \quad ; \text{ from } 0 \text{ to } 2x_n$$

b) Plot ρ(x) and ξ(x) for the given p-n junction axis orientation.



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|---|---|
| **Department of Electronics** | **Name:** |
| **Carleton University** | **ID:** |
| **ELEC 3908 Section – B1 (Mid-Term Exam)** | **Date: Feb. 27th, 2020** |
| *** Attempt all questions** | **Time allocated: 90 mins** |

c) What is the unbiased depletion width, W of the diode at equilibrium?

$$W = \sqrt{\frac{2\epsilon_{Si}}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_{bi}} \quad ; \quad V_{bi} = 0.02586 \ln \frac{N_A N_D}{n_i^2}$$

$$= 0.02586 \ln \frac{10^{33}}{2.10} = 0.75V$$

$$= \sqrt{\frac{2 \times 11.7 \times 8.854 \times 10^{-14}}{1.6 \times 10^{-19}}\left(\frac{1}{10^{16}}+\frac{1}{10^{17}}\right) \times 0.75}$$

$$= \sqrt{106.83 \times 10^{-11}} \text{ cm} = 3.268 \times 10^{-5} \text{ cm}$$

$$W = 0.327 \mu m$$

d) How far does the depletion region extend into the n-side of the diode?

$$x_n = W \frac{N_A}{N_A + N_D} = 0.327 \mu m \times \frac{10^{16}}{1.1 \times 10^{17}} \simeq 0.03 \mu m$$

**Department of Electronics** **Name:**

**Carleton University** **ID:**

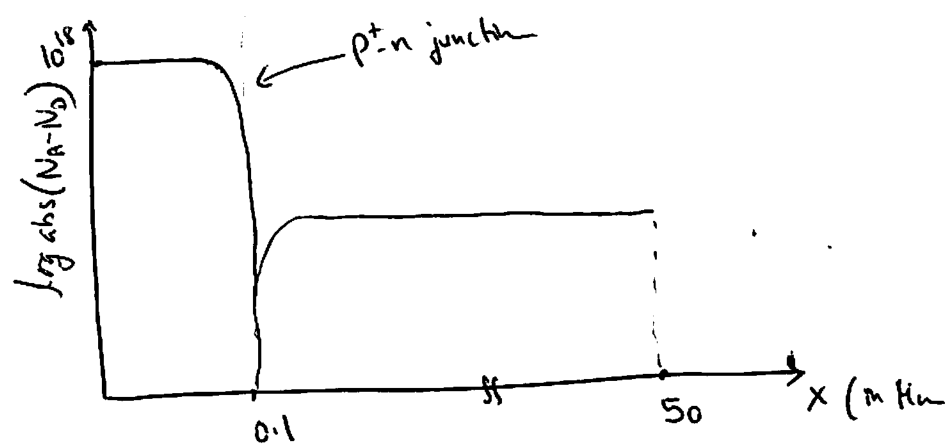
**ELEC 3908 Section – B1 (<u>Mid-Term Exam</u>)** **Date: Feb. 27th, 2020**

*** Attempt all questions** **Time allocated: 90 mins**

## Question 4. (10 marks)

A p⁺n substrate diode is fabricated using a 50 μm thick n-type silicon substate with $N_D = 10^{16}$ $cm^{-3}$ and $N_A = 10^{18}$ $cm^{-3}$. Thickness of the doped p⁺ layer is 0.1 μm. Assume $\tau_o$ = 0.5 μs.

a) Draw the doping profile showing log abs ($N_D$-$N_A$) vs x (depth in μm).




b) What is the saturation current density, $J_s$ in μA/cm²?

$$J_s = \frac{q D_n n_{p_o}}{W_p} + \frac{q D_p p_{n_o}}{L_p}$$

Use: $D_n = 34.9$ cm²/s, $D_p = 12.1$ cm²/s

$L_p = 34.8$ μm $= 3.48\times10^{-3}$ cm

$W_p = 0.1$ μm

$$J_s = 1.6\times10^{-19}\left(\frac{34.9\times2.1\times10^{2}}{0.1\times10^{-4}} + \frac{12.1\times2.1\times10^{4}}{3.48\times10^{-3}}\right) A/cm^2$$

$$= 1.6\times80.592\times10^{-12}\ A/cm^2$$

$$J_s \approx 84.97\times10^{-6}\ \mu A/cm^2$$

$n_{p_o} = \frac{2.1\times10^{20}}{10^{18}} = 2.1\times10^{2}\ cm^{-3}$

$p_{n_o} = \frac{2.1\times10^{20}}{10^{16}} = 2.1\times10^{4}\ cm^{-3}$

c) If the area of the diode well is 0.1 cm², what is the net parasitic resistance for the diode?

Ignore the p⁺ depth for $R_{S_{net}}$ estimation

$$R_S = \frac{\rho_{sub}\, t_{sub}}{A_{well}} \quad ; \quad \rho_{sub} = \frac{1}{q N_D \mu_n} = \frac{1}{1.6\times10^{-19}\times10^{16}\times1.35\times10^{3}} = 0.463\ \Omega\text{-cm}$$

$$= \frac{0.463\times50\times10^{-4}}{0.1} = 2.32\times10^{-2}\ \Omega$$

or

$23.2\ m\Omega$

**Department of Electronics** **Name:**

**Carleton University** **ID:**

**ELEC 3908 Section – B1 (Mid-Term Exam)** **Date: Feb. 27th, 2020**

*** Attempt all questions** **Time allocated: 90 mins**

## Question 5. (10 marks)

should be $3\times10^{16}$

A p-n junction has $N_A = 10^{16}$ $cm^{-3}$ and $N_D = 10^{16}$ $cm^{-3}$ and a minority lifetime, $\tau_o = 0.2$ µs. At certain depth, x in the n-type material outside the depletion region, $n = 3 \times 10^{16}$ $cm^{-3}$ and $p = 10^9$ $cm^{-3}$.

**a)** Is the device forward biased, reverse biased or un-biased. Explain your answer.

Outside the depletion region, $n \times p = 3\times10^{25}$ $cm^{-3}$ $> n_i^2 (= 2.10\times10^{20} cm^{-3})$

Since $n > N_D$ and $np > n_i^2$, the device is certainly biased.

n-side

$np > n_i^2$ indicates forward bias.

**b)** Calculate the net recombination rate, U using the Shockley-Reed-Hall equation. Justify any assumptions.

$$U = \frac{n(x)p(x) - n_i^2}{\tau_0 \left( n(x) + p(x) + 2n_i \right)}$$

$$= \frac{3\times10^{25} - 2.1\times10^{20}}{(0.2\times10^{-6} s)(3\times10^{16} + 2.9\times10^{10})}$$

$$\simeq \frac{3\times10^{25}}{6\times10^{9}} \quad cm^3/sec$$

$$= 5\times10^{15} \quad cm^3/sec$$

**Department of Electronics** | **Name:**

**Carleton University** | **ID:**

**ELEC 3908 Section – B1 (Mid-Term Exam)** | **Date: Feb. 27th, 2020**

*** Attempt all questions** | **Time allocated: 90 mins**

# Equations

Ideal Diode: $I_D = I_S\left(e^{qV_D/kT} - 1\right)$

with depletion region GR: $I_D = I_S\left(e^{qV_D/nkT} - 1\right)$

with series resistance: $I_D = I_S\left(e^{q(V_{Dt} - I_D R_s)/kT} - 1\right)$ where $V_{Dt} = V_D + I_D R_s$

with both $R_s$ and GR: $I_D = I_S\left(e^{q(V_{Dt} - I_D R_s)/nkT} - 1\right)$ where $V_{Dt} = V_D + I_D R_s$

Shockley-Reed-Hall: $U = \dfrac{n(x)p(x) - n_i^2}{\tau_0\left(n(x) + p(x) + 2n_i\right)}$

Einstein Relations: $D_n = \dfrac{kT}{q}\mu_n,\quad D_p = \dfrac{kT}{q}\mu_p$

Diffusion Length: $L_n = \sqrt{D_n \tau_0},\quad L_p = \sqrt{D_p \tau_0}$

Saturation Current Density:

$J_S = \dfrac{qD_n n_{p0}}{L_n} + \dfrac{qD_p p_{n0}}{L_p}$, for $pn$ junction (thick p and n)

$J_S = \dfrac{qD_n n_{p0}}{w_p} + \dfrac{qD_p p_{n0}}{L_p}$, for $p^+n$ junction (thin $p^+$)

$J_S = \dfrac{qD_p p_{n0}}{w_n} + \dfrac{qD_n n_{p0}}{L_n}$, for $n^+p$ junction (thin $n^+$)

Resistivity: $\rho = \left(\dfrac{1}{\rho_n} + \dfrac{1}{\rho_p}\right)^{-1} = \left(\sigma_n + \sigma_p\right)^{-1} = \left(qn\mu_n + qp\mu_p\right)^{-1}.\quad R = \dfrac{\rho \cdot l}{A}$

Least Squares: $\bar{y} = m\bar{x} + b.\quad m = \dfrac{n\sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} = \dfrac{\overline{(x \cdot y)} - \bar{x}\cdot\bar{y}}{\overline{x^2} - \bar{x}^2}$

Poisson's Equation: $-\dfrac{d^2\psi(x)}{dx^2} = \dfrac{d\mathcal{E}(x)}{dx} = \dfrac{\rho(x)}{\varepsilon_{Si}}$

Excess Charge Density in a Diode: $\rho(x) = q\left[p(x) - n(x) + N_D - N_A\right]$

Built-In Voltage: $V_{bi} = \dfrac{kT}{q}\ln\left(\dfrac{N_A N_D}{n_i^2}\right)$

Depletion Width: $W = \sqrt{\dfrac{2\varepsilon_{Si}}{q}\left(\dfrac{1}{N_A} + \dfrac{1}{N_D}\right)\left(V_{bi} - V_D\right)}.\quad x_n = W\left(\dfrac{N_A}{N_A + N_D}\right).\quad x_p = W\left(\dfrac{N_D}{N_A + N_D}\right)$

Maximum Electric Field: $\left|\mathcal{E}_{depl\ max}\right| = \sqrt{\dfrac{2q}{\varepsilon_{Si}}\left(\dfrac{N_A N_D}{N_A + N_D}\right)\left(V_{bi} - V_D\right)}$

Avalanche Multiplication Factor: $M = \dfrac{1}{1 - p_{ii}}$

Impact Ionization Probability: $p_{ii} = \left(\dfrac{\left|\mathcal{E}_{depl\ max}\right|}{\mathcal{E}_{crit}}\right)^{\nu},\quad 3 < \nu < 6$

Diode Junction Conductance: $g_D = \dfrac{dI_D}{dV_D} = \dfrac{q}{nkT} I_S e^{qV_D/nkT} \approx \dfrac{q}{nkT} I_D$

Diode Junction Capacitance (per area): $\acute{C}_{depl}(V_D) = \dfrac{d\acute{Q}_{depl}}{dV_D} = \dfrac{d\acute{Q}_{depl}}{dW}\dfrac{dW}{dV_D} = \left[-q\dfrac{N_A N_D}{N_A + N_D}\right]\left[-\dfrac{\varepsilon_{Si}}{Wq}\dfrac{N_A + N_D}{N_A N_D}\right] = \dfrac{\varepsilon_{Si}}{W(V_D)}$

Department of Electronics | Name:

Carleton University | ID:

ELEC 3908 Section – B1 (<u>Mid-Term Exam</u>) | Date: Feb. 27th, 2020

* Attempt all questions | Time allocated: 90 mins

# Physical Constants and Material Properties

| Quantity | Symbol | Value |
|---|---|---|
| Micron | μm | $10^{-4}$ cm = $10^{-6}$ m |
| Angstrom Unit | Å | $10^{-8}$ cm = $10^{-10}$ m |
| Boltzmann's Constant | $k$ | $8.62\times10^{-5}$ eV/K |
| | | $1.381\times10^{-23}$ J/K |
| Electronic Charge | $q$ | $1.602\times10^{-19}$ C |
| Electron Volt | eV | $1.602\times10^{-19}$ J |
| Electron Rest Mass | $m_o$ | $9.11\times10^{-31}$ kg |
| Free Space Permittivity | $\varepsilon_o$ | $8.854\times10^{-14}$ F/cm |
| Plank's Constant | $h$ | $6.626\times10^{-34}$ J-s |
| | | $4.14\times10^{-15}$ eV-s |
| Thermal Voltage at 300K | $kT/q$ | 0.0259 V |

**Properties of Silicon at 300K**

| Quantity | Symbol | Value |
|---|---|---|
| Intrinsic Carrier Concentration | $n_i$ | $1.45\times10^{10}$ cm$^{-3}$ |
| Effective Densities of States | $N_v$ | $1.08\times10^{19}$ cm$^{-3}$ |
| | $N_c$ | $2.8\times10^{19}$ cm$^{-3}$ |
| Electron Affinity | $\chi^{Si}$ | 4.05 eV |
| Energy Gap | $E_g$ | 1.08 eV |
| Bulk Electron Mobility | $\mu_n$ | 1350 cm$^2$/V-s |
| Bulk Hole Mobility | $\mu_p$ | 470 cm$^2$/V-s |
| Surface Electron Mobility | $\overline{\mu_n}$ | 520 cm$^2$/V-s |
| Permittivity | $\varepsilon_{Si}$ | $11.7\varepsilon_o$ |

**Properties of Silicon Dioxide**

| Quantity | Symbol | Value |
|---|---|---|
| Permittivity | $\varepsilon_{ox}$ | $3.9\varepsilon_o$ |