STAT249 (Fall 2020)
Assignment 3: Solutions

1. a. Using the probability distribution of X we calculate

$$E(X) = 0(0.1) + 1(0.2) + 3(0.4) + 4(0.2) + 5(0.1) = 2.7$$

b. We first calculate

$$E(X^2) = 0^2(0.1) + 1^2(0.2) + 3^2(0.4) + 4^2(0.2) + 5^2(0.1) = 9.5$$

then $V(X) = E(X^2) - E(X)^2 = 9.5 - (2.7)^2 = 2.21$

c. We can express the total cost $Y = X(1000) + 500$ from which we get

$$E(Y) = 1000E(X) + 500 = 3200, \quad V(Y) = 1000^2 V(X) + 0 = 2210000$$

2. If we draw fair coin the chance of drawing 0 heads is $(0.5)^2$, of drawing 1 head is $2(0.5)^2$ and of drawing 2 heads is again $(0.5)^2$. If we draw unfair coin these probabilities become: for drawing 0 heads is $(0.1)^2$, for drawing 1 head is $2(0.1)(0.9)$, for drawing 2 heads it is $(0.9)^2$.

a. We calculate probabilities for Y by using the Law of Total Probabilities after we first condition on which coin we drew and then adding the two possible contributions:

$$P(Y = 0) = \frac{1}{2}(0.5)^2 + \frac{1}{2}(0.1)^2 = 0.13$$
$$P(Y = 1) = \frac{1}{2}2(0.5)^2 + \frac{1}{2}2(0.1)(0.9) = 0.34$$
$$P(Y = 2) = \frac{1}{2}(0.5)^2 + \frac{1}{2}(0.9)^2 = 0.53$$

b. From the above we calculate

$$E(Y) = 0(0.17) + 1(0.34) + 2(0.53) = 1.4$$

and

$$E(Y) = 0(0.17) + 1(0.34) + 4(0.53) = 2.46, \Rightarrow V(Y) = 2.46 - (1.4)^2 = 0.5$$

3. The sequence of days is a Bernoulli experiment with having an accident considered a "success".

a. In 5 days the number of accidents ("successes") is a Binomial variable X with $n = 5, p = 0.0.5$. Its possible values are $y = 0, 1, 2, ..., 5$ and its distribution is

$$P(Y = y) = \binom{5}{y}(0.0.5)^y(1 - 0.0.5)^{5-y}, \;\; y = 0, ...5$$

b. For Binomial(n, p) variable we have that $E(Y) = np = 5(0.05) = 0.25$

c. For Binomial(n, p) variable we have that $V(Y) = np(1 - p) = 5(0.05)(1 - 0.05) = 0.2375$

4. Using machine in a sequence is a Bernoulli experiment with it breaking down considered a "success", the chance of which is given as $p = 1/10 = 0.1$

a The probability of break down ("success") on the first use is =0.1.

b The probability of the first break down ("success") on the third use is =(1-0.1)(1-0.1)(0.1)=0.081

c The first break down ("success") occurs on or after the third use if on the first use and the second use there is no break down (no "success"). The probability of no "success" on the first two uses is

$$= (1 - 0.1)(1 - 0.1) = 0.81$$

5. The true chance of finding a product in any store is $p = 0.6$. Visiting stores is a Bernoulli experiment with finding product in a store considered a "success".

a. If we visit 3 stores we have $n = 3$ trials, so the number of products found ("successes") is a Binomial variable Y with $n = 3, p = 0.6$, so the chance of exactly 1 "successes" is

$$P(Y = 1) = \binom{3}{1}(0.6)(1 - 0.6)^{3-1} = 0.288$$

b. If we need to find product/have 2 "successes" in 2 different stores then the number of stores/trials needed to do that is a Negative Binomial variable Y with $r = 2, p = 0.6$, so the chance this number of trials is exactly 4 is

$$P(Y = 4) = \binom{3}{2}(0.6)^2(1 - 0.6)^{4-2} = 0.1728$$