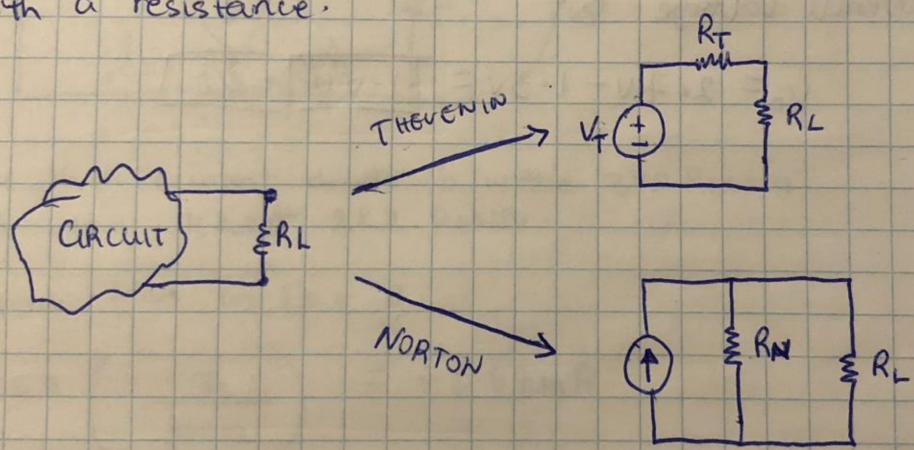
LECTURE 10

THEVENIN'S and NORTON'S THEOREMS

Often, we ~~use~~ use a relatively complex circuit to deliver power to a load.

Thevenin's theorem says that the entire circuit (exclusive of the load) can be replaced by an equivalent circuit containing only an independent voltage source in series with a resistor.

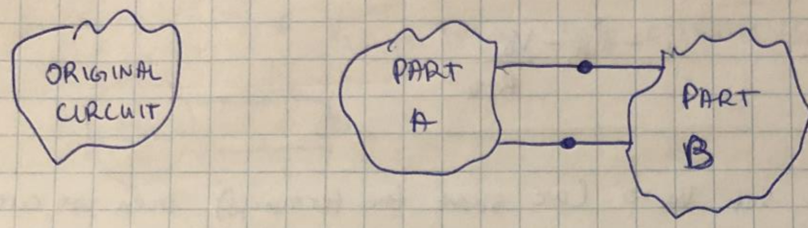
Norton's theorem says we can replace the driving circuit by an independent current source in parallel with a resistance.



In both cases the current-voltage relation at the load stays the same. We can change the load value and the equivalent will accurately model the actual circuit.

Consider the Thevenin equivalent first

Consider a complex circuit that can be split into two parts: A and B

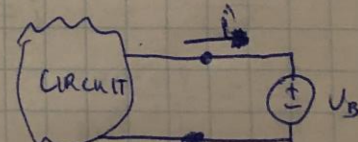


There is a current i between the circuits and a voltage V_B at the terminals. We could replace part B with a voltage source V_B without changing the voltage or current at the terminals.

If we were to apply superposition to the circuit, we c could find the current i_B due to the new source V_B, and the current with $V_B = 0$ (short circuit) due to other sources in circuit A i_A, then the total will be

$$i = i_A + i_B$$

Setting all sources in A to zero we find:



$$i_B = \frac{-V_B}{R_{TH}}$$

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Where R_{TH} would be the resistance between the terminals of circuit A with all independent sources $= 0$

therefore:
$$i = i_A - \frac{V_B}{R_{TH}}$$

Now if we set $V_B = 0$ (we short the terminals) then we are left with:

$$i = i_{sc} = i_A \qquad (I_{sc} = \text{short circuit current})$$

$$\Rightarrow \boxed{i = i_{sc} - \frac{V_B}{R_{TH}}} \quad — ①$$

Now say we open terminals, then $V_B = V_{oc}$ and the current is equal to zero so:

$$i \Rightarrow 0 = i_{sc} - \frac{V_{oc}}{R_{TH}} \qquad (V_{oc} = \text{open circuit voltage})$$

$$\boxed{V_{oc} = R_{TH} \cdot i_{sc}}$$

Thus the ratio of the open circuit voltage and the short current is related by the resistance of circuit A with all sources zeroed out! we can therefore rewrite ① as

$$\frac{iR_{TH} + V_B}{R_{TH}}$$

$$i = V_{oc}/R_{TH} - V_B/R_{TH}$$

$$\frac{V_{oc}}{I_{sc}} = \frac{R_{TH} \cdot I_{sc}}{i + V_B/R_{TH}}$$

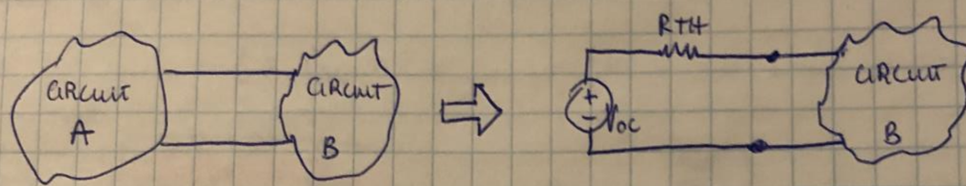
$$\frac{V_{oc}}{i_{sc}} = \frac{R_{TH} \cdot I_{sc}}{\frac{iR_{TH} + V_B}{R_{TH}}} = \frac{R_{TH}^2 \cdot I_{sc}}{iR_{TH} + V_B}$$

THEVENIN

R_{TH} = resistance looking into the terminals of driving circuit A with all sources = 0 ⟶ Thevenin resistance

V_{OC} = The voltage at the terminals of circuit A when the terminals are open circuit.

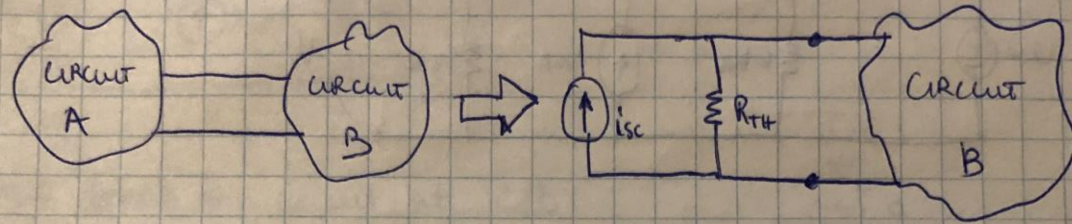
The circuit A can then be replaced by a source V_{OC} in series with a resistor R_{TH}:



THEVENIN'S THEOREM

NORTON'S THEOREM

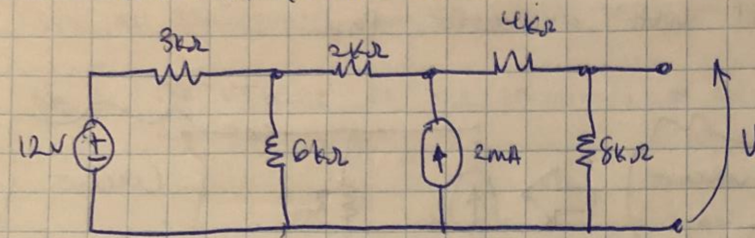
This indicates circuit A can also be replaced by a current source in parallel with a resistor R_{TH}.



Note: the two circuits equivalents are different representations of the same equations. Note they are equivalent only in terminal characteristics → Norton dissipates power oc.

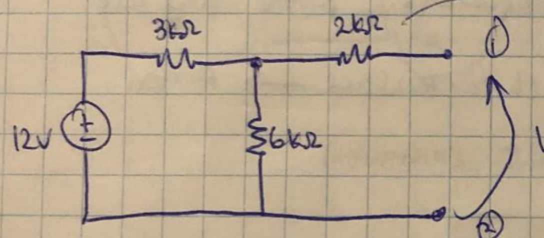
EX

use Thevinin to find V_o



We can do this in stages.

consider just the part left of the current source!

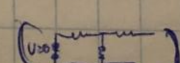


No current, No voltage

If terminals ① & ② are open circuited then there is no current flow through the 2kΩ Resistor

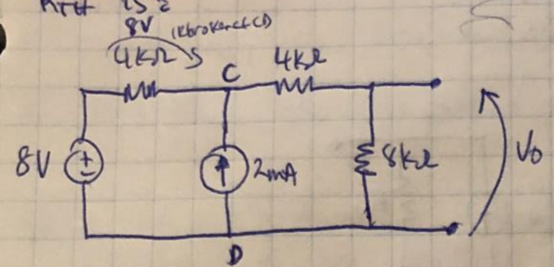
∴ using a voltage devider:

$$V_{TH} = V_{oc} = 12V \times \frac{6K}{(6+3)\,K} = 8V$$

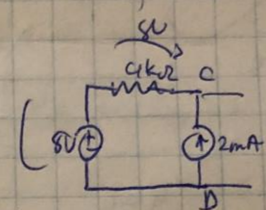
R_{TH} → find Req with sources =0 

$R_{TH} = 2k + 6k // 3k = 4 k\Omega$

Our new circuit with the left side replaced with V_{TH} & R_{TH} is:



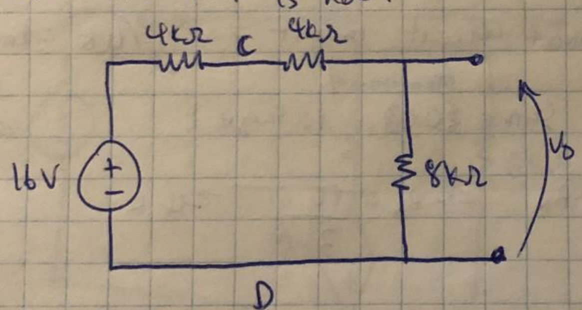
Now break the circuit at C/D:



The new $V_{TH} = 8V + 2mA \times 4k = 16V$

New $R_{TH} = 4k$ (current source open / voltage source short)

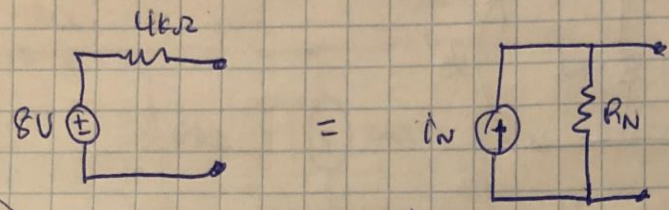
Our equivalent is now:



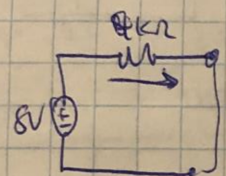
And $V_0 = 16V \times \frac{8k}{(8k+4+4)k} = 8V$

$R_{TH} = 8k \parallel (4+4) = 4k$ (Voltage source again a short circuit

The equivalent could also be found using ~~Equivalent of~~ Superposition of 2 sources.



Equivalent



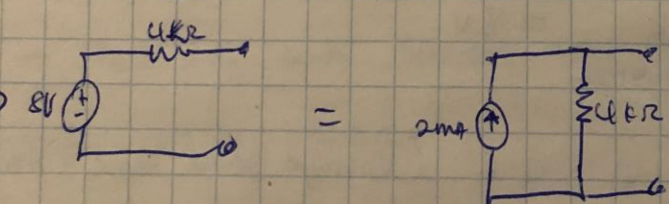
Short circuit to find current i_N

i_N would be the short circuit current = 8V/4k = 2mA

& R_N would provide the same {OC} (open circuit) voltage:

$$8V = i_N \cdot R_N \Rightarrow R_N = \frac{8V}{2mA} = 4k$$

i.e. $R_N = R_{TH}$ → source transformation

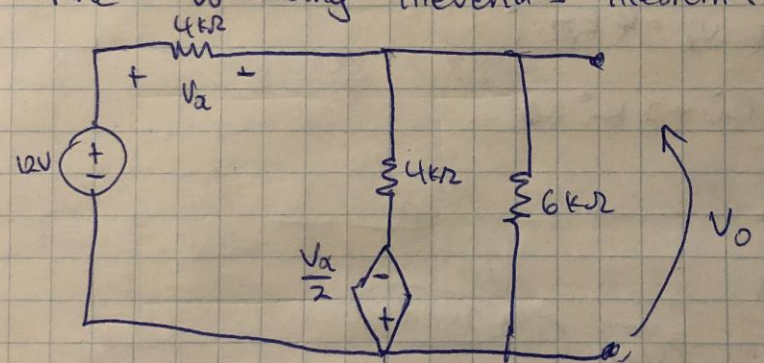
⇒ 

Thevenin Steps

1. Remove load; find OC voltage
2. Find R_{eq} b/w the output terminals with independent sources set to zero
3. If there are dependent sources present, R_{TH} may not be obvious so a 'test source' is connected to the output and R_{TH} is found from V_{test}/i_{test} (Ohm's law)
4. Re-attach the load & complete the analysis

Example with Dependent & independent sources

Find V_o using Thevenin's Theorem:

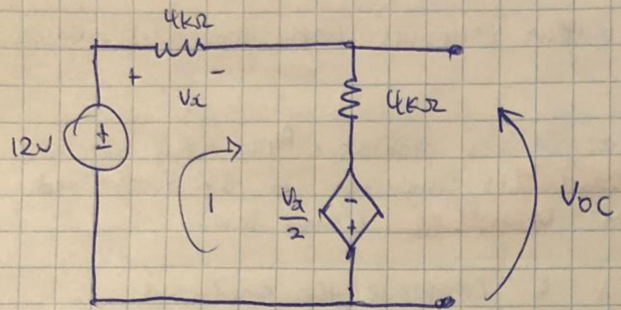


FIRST

Remove load = 6kΩ

First

Remove load = 6 kΩ



using KVL:

$$12V - V_x - 4k\Omega \cdot i + \frac{V_x}{2} = 0$$

we also note that $V_x = 4k\Omega \cdot i$

$$\Rightarrow \quad 12V - 4k i - 4k \cdot i + 2k \cdot i = 0$$

$$\boxed{i = 2mA}$$

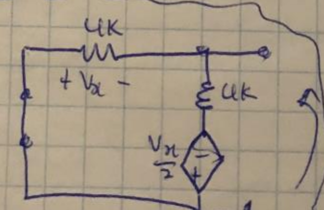
we know $i = 2mA$

$$V_{oc} = 4k\Omega \cdot i - \frac{V_x}{2} \rightarrow 2k\Omega \cdot i$$

$$V_{oc} = 2k\Omega \cdot i = 2k\Omega \cdot 2mA = 4V$$

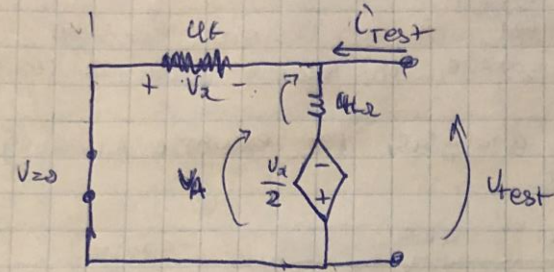
2

$R_{eq} = R_{TH} =$



② find $R_{eq} = R_{TH}$

we apply a test voltage to find a test current! after zeroing all dependent sources



$V_{Test} + V_x = 0$

$\Rightarrow \boxed{V_x = -V_{Test}}$

$\Rightarrow \boxed{\frac{V_x}{2} = \frac{-V_{Test}}{2}}$

$V_A = -\frac{V_x}{2} = +\frac{V_{test}}{2}$

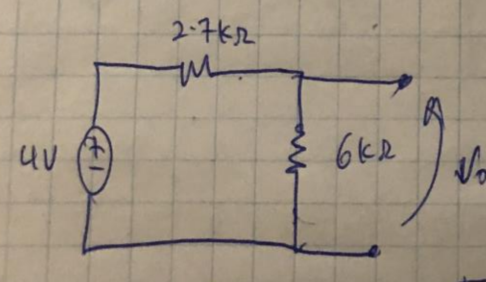
$i_{Test} = \overset{i_1}{\frac{V_{test}}{4k}} + \overset{i_2}{\frac{(V_{Test}/2)}{4k}}$

$\frac{V_{test}}{4k} + \frac{V_{test}}{8k} = \frac{2V_{test} + V_{test}}{8k} = \frac{3V_{test}}{8k}$

$i_{test} = \frac{3}{8} V_{test}$

$\frac{V_{TEST}}{I_{Test}} = \left(\frac{8\,k\Omega}{3}\right) = \boxed{2.7\,k\Omega}$

Now we can replace the 6k resistor with our simple model to give



$V_o = 4V \times \frac{6K}{(2.7+6)k} = \boxed{2.75V}$