WILFRID LAURIER UNIVERSITY
WATERLOO, ONTARIO

Fall Term, 2020

Course ID: ***MA222***

Course Title: ***Linear Algebra***

Instructors: ***K. Zhao***

Length of Examination: ***80 minutes***

Examination Aids Allowed: ***Non-programmable calculators.***

Total Value: *60 marks (30% weight for the final grade)*

Instructions: Non-programmable, non-graphing calculators are permitted. No other aids are allowed.
You should first login to Zoom Meeting for the regular course lectures, **keep your Zoom Meeting video on**, download this pdf file to your computer. During the exam period you can ask me questions by using Zoom Meeting. You have to submit your solution (one pdf file) to the "Midterm Test" folder of Dropbox on MyLS before 2:50pm, Nov.3, 2020. You should keep hard copy of your solutions yourself for at least 4 weeks.
Write your solutions in white papers (unlined). Show all of your work. Insufficient justification will result in a loss of marks.

[*10 marks*] 1. Find all complex numbers z so that $\frac{z^3+8}{z^3-8} = i$. Write your solutions in the standard form, and use square roots like $\frac{\sqrt{3}}{2}$.

[*8 marks*] 2. Find all complex zeros of the polynomial $f(x) = x^3 - 3x^2 + x + 5 \in \mathbb{R}[x]$.

3. Let u, v, and w be distinct vectors of a real vector space V.

[*6 marks*] (a). Show that span$\{u, v, w\}$=span$\{u + v - w, u + w - v, v + w - u\}$.

[*6 marks*] (b). Show that span$\{u, v, w\}$=span$\{u + v + 2w, v + w + 2u, w + u + 2v\}$.

[*6 marks*] (c). Show that $\{u + v + 2w, v + w + 2u, w + u + 2v\}$ is linearly independent iff $\{u + v - w, u + w - v, v + w - u\}$ is linearly independent.

4. Define a map $T : \mathbb{R}_3[x] \to \mathbb{R}^4$ by $T(f(x)) = (f(0), f(1), f(1) + f(-1), f(2))$ for $f(x) \in \mathbb{R}_3[x]$ where $\mathbb{R}_3[x]$ is the real vector space consisting all real polynomials in x with degree less than or equal to 3. Prove that

[*3 marks*] (a). T is linear;

[*5 marks*] (b). Find $N(T)$ (the null space);

[*2 marks*] (c). Find a basis for $N(T)$;

[*2 marks*] (d). Find a basis for $R(T)$.

5. Let V be a finite dimensional real vector space, and let $T : V \to V$ be a linear map with $T^2 = T$. Define $W = \{v \in V : T(v) = v\}$. Show that

[*3 marks*] (a). $W \leq V$;

[*6 marks*] (b). $V = W + N(T)$;

[*2 marks*] (c). $W \cap N(T) = \{0\}$;

[*1 marks*] (d). $V = W \oplus N(T)$.