.FINAL EXAM-MAT 1300.
FALL TERM, 2015

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Please circle the name of your professor above.

First Name:____________________________**Last Name:**____________________________

I.D. Number____

Instructions-This final examination consists of 12 multiple choice questions worth 4 points each. Your answers to the multiple choice questions must be clearly marked in the squares below. There are also 4 long answer questions worth a total of 52 points. For the long answer questions, you must show your work **on the exam itself** and clearly display your answers. **Do not unstaple these pages.**

NO CALCULATORS. NO BOOKS. NO NOTES.
TURN OFF YOUR CELL PHONES AND PUT THEM AWAY.

Multiple Choice ANSWERS:

A	A	C	C
#1	#2	#3	#4
D	C	D	E
#5	#6	#7	#8
C	C	A	C
#9	#10	#11	#12

PLEASE READ THE FOLLOWING CAREFULLY AND SIGN BELOW

Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to immediately leave the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam or worse.

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Question 1- Consider the function:

$$f(x) = \frac{x+2}{x^2+3x+2} = \frac{(x+2)}{(x+2)(x+1)} \overset{x\neq -2}{=} \frac{1}{x+1}$$

Which of the following statements are correct?

$x=-1$ VA

$\lim_{x\to\infty} f(x) = 0$ $y=0$ H.A

A) The asymptotes of f occur at $y = 0, x = -1$.
B) The asymptotes of f occur at $y = 3, x = -2$.
C) $x = -2$ is the only asymptote of f.
D) $y = 0$ is the only asymptote of f.
E) The asymptotes of f occur at $y = 0, x = -1, x = -2$.

Question 2-Over what interval is the function $f(x) = \frac{x}{x-4}$ concave up?

A) $(4, \infty)$ B) $(-\infty, 4)$ C) $(-\infty, 0)$ D) $(0, 4)$ E) This function is always concave up.

$$f'(x) = \frac{x-4-x}{(x-4)^2} = \frac{-4}{(x-4)^2}$$

$$f''(x) = \frac{8}{(x-4)^3}$$

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Question 3- Suppose the demand function for a product is given by $p(x) = 120 - x^2$. Find the elasticity of demand when $x = 10$. Is demand elastic?

A) $-\frac{1}{10}$, elastic B) $-\frac{3}{20}$, elastic C) $-\frac{1}{10}$, inelastic D) $-\frac{3}{20}$, inelastic
E) $-\frac{3}{50}$, inelastic

$$\eta = \frac{20}{(-20)\,10} = \frac{-1}{10} < 1$$

$(|\eta| = \frac{1}{10} < 1)$

$P(10) = 100$

$P'(10) = -2(10) = -20.$

Question 4- For what interval or intervals is the function $f(x) = x^2 e^x$ decreasing?

A) $(-\infty, -2)$ B) $(-\infty, -2) \cup (0, \infty)$ C) $(-2, 0)$ D) $(0, 3)$ E) $(-\frac{1}{2}, \infty)$

$f'(x) = 2xe^x + x^2e^x$

$= e^x(2x + x^2) = 0$

$x(x+2) = 0$

$x = 0 \quad x = -2.$

$\nearrow \quad -2 \quad \searrow \quad 0 \quad \nearrow$

$+ \quad | \quad - \quad | \quad +$

Question 5-Calculate the following limit.

$$\lim_{x\to 0} \frac{\sqrt{1+x^2}-1}{x^2} \times \frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}+1} = \frac{\overbrace{(1+x^2)-1}^{x^2}}{x^2(\sqrt{1+x^2}+1)} =$$

A) 1 **B)** -1 **C)** 0 **D)** $\frac{1}{2}$ **E)** This limit does not exit

$$\lim_{x\to 0} \frac{1}{\sqrt{1+x^2}+1} = \frac{1}{2}$$

Question 6- Use implicit differentiation to find $\frac{dy}{dx}$ at $(0,1)$ when

$$x^3 + e^x y = y^2$$

A) $\frac{3}{2}$ **B)** $-\frac{1}{2}$ **C)** 1 **D)** -2 **E)** $e+1$

$$3x^2 + y + e^x y' = 2yy' \Big|_{(0,1)}$$

$$0 + 1 + y' = 2y' \Rightarrow y' = 1$$

Question 7- Calculate the following definite integral

$$\int_0^2 \frac{x}{\sqrt{2x^2+1}}dx = \int_1^9 u^{-1/2}\frac{du}{4} = \frac{2\sqrt{u}}{4}\Big|_1^9 =$$

$$\frac{\sqrt{u}}{2}\Big|_1^9 = \left(\frac{3}{2}-\frac{1}{2}\right) = 1$$

A) $\sqrt{2}+1$ B) $\ln(2)+2$ C) $\frac{1}{2}$ D) 1 E) 2

$$\begin{cases} u = 2x^2+1 \\ du = 4x\,dx \end{cases}$$

$x=0 \rightarrow u=1$

$x=2 \rightarrow u=9$

Question 8- Calculate:

$$\int_1^e 2x\ln(x)dx$$

A) $\frac{e^2}{2}$ B) $\frac{e^2-1}{2}$ C) e^2 D) $\frac{e+ln(2)}{2}$ E) $\frac{e^2+1}{2}$

$u = \ln x$

$u' = \frac{1}{x}$

$2x = v'$

$x^2 = v$

$$\ln x\, x^2\Big|_1^e - \int_1^e \underbrace{\frac{1}{x}(x^2)}_{x} dx$$

$$(\ln e)e^2 - \frac{x^2}{2}\Big|_1^e$$

$$e^2 - \left(\frac{e^2}{2}-\frac{1}{2}\right) = \frac{e^2}{2}+\frac{1}{2}.$$

$$\frac{e^2+1}{2}$$

Question 9- Find $f(-1)$, when $f'(x) = x^{-\frac{2}{3}} + x^2 + 3$ and $f(1) = 7$.

A) $\frac{-116}{3}$ B) $\frac{49}{3}$ C) $\frac{-17}{3}$ D) $\frac{3}{16}$ E) $\frac{13}{2}$

$f(x) = 3x^{1/3} + \frac{x^3}{3} + 3x + C$

$f(1) = 3 + \frac{1}{3} + 3 + C \qquad C = \frac{2}{3}$

$f(-1) = -3 - \frac{1}{3} - 3 + \frac{2}{3}$

$= -6 + \frac{1}{3} = \frac{-17}{3}$

Question 10- Suppose that for a certain product, the demand function is given by $D(x) = 8 - x^2$ and the supply function is given by $S(x) = x^2 + 4x + 2$. Calculate the consumer surplus.

A) 7 B) 12 C) $\frac{2}{3}$ D) $\frac{8}{3}$ E) 18

$8 - x^2 = x^2 + 4x + 2$

$2x^2 + 4x - 6 = 0$

$x^2 + 2x - 3 = 0 \qquad (x+3)(x-1) = 0$

$x = 1 \quad x = -3$

$D(1) = 7$

$\int_0^1 ((8 - x^2) - 7)\, dx = \int_0^1 (1 - x^2)\, dx$

$= x - \frac{x^3}{3} \Big|_0^1 = 2/3$

Question 11-How many critical points does the following function of 2 variables have?

$$g(x,y) = -3x^2 + 2y + x^3 + y^2 + 14$$

A) 2 B) 3 C) 4 D) 1 E) 0

$g_x = -6x + 3x^2 = 0$ $X = 0$ $X = 2$

$g_y = 2 + 2y = 0$ $y = -1$

$(0, -1)$ $(2, -1)$.

Question 12- Suppose $f(x,y) = e^{xy^2+1}$. Find $f_{yy}(1,1)$.

A) $5e^2$ B) $3e^2$ C) $6e^2$ D) $4e^2$ E) $2e^2$

$f_y = 2xy\,e^{xy^2+1}$

P.R $f_{yy} = 2x e^{xy^2+1} + (2xy)^2 e^{xy^2+1} \Big|_{(1,1)}$

$2e^2 + 4e^2 = 6e^2$

Long Answer Question 1 (8 points)

The number of bacteria in a certain culture is 25,000 at 6am. At 10am, the count is 75,000. Assuming the population is growing exponentially, determine the amount of time it takes for the population to reach 100,000.

$t=0 \quad P_0 = 25\times10^3$

$t=4 \quad P(4) = 75\times10^3$

$75\times10^3 = 25\times10^3\, e^{K(4)}$

$3 = e^{K4} \Rightarrow \ln 3 = 4K$

$\boxed{K = \frac{\ln 3}{4}}$

$100{,}000 = 25\times10^3\, e^{t\frac{\ln 3}{4}}$

$4 = e^{t\ln 3/4} \Rightarrow \ln 4 = t\frac{\ln 3}{4}$

$\boxed{\frac{4\ln 4}{\ln 3} = t}$

Long Answer Question 2 (14 points)
[a] Evaluate the following indefinite integral:

$$\int x(x+1)^{\frac{2}{3}}\,dx \quad =$$

$$x\,\frac{2}{5}(x+1)^{5/2} - \int \frac{3}{5}(x+1)^{5/3}\,dx$$

$u = x$

$u' = 1$

$v' = (x+1)^{2/3}$

$v = \frac{3}{5}(x+1)^{5/3}$

$$\frac{(x+1)^{2/3+1}}{2/3+1}$$

$$= \frac{3}{5}x(x+1)^{5/3} - \frac{3}{5}\cdot\frac{3}{8}(x+1)^{8/3} + C$$

$$= \frac{2}{5}x(x+1)^{5/2} - \frac{9}{40}(x+1)^{8/3} + C$$

[b] Evaluate the following improper integral:

$$\int_0^\infty xe^{-x^2}\,dx$$

$$\lim_{d\to\infty} \int_0^d xe^{-x^2}\,dx$$

$u = -x^2$

$du = -2x\,dx$

$x = 0 \to u = 0$

$x = d \to u = -d^2$.

$$= \lim_{d\to\infty}\left(\int_0^{-d^2} e^u \frac{du}{-2}\right) =$$

$$\frac{-1}{2}\lim_{d\to\infty}\left(e^{-d^2} - 1\right)$$

$$= 1/2$$

Convergent

or

$$\lim_{d\to\infty}\int_0^d e^u \frac{du}{-2}$$

$$= \lim_{d\to\infty} \frac{-1}{2} e^{-x^2}\Big|_0^d$$

$$= \lim_{d\to\infty} \frac{-1}{2}\left(e^{-d} - e^{0}\right)$$

$$= 1/2$$

Long Answer Question 3 (14 points)

Consider the two functions:

$f(x) = 4 - x^2$ and $g(x) = 3x$

(a) **(2 points)** Find the intersection points of the graphs of the two functions.

(b) **(4 points)** On the next page, graph these functions, and shade the region between the graphs of f and g for x such that $0 \leq x \leq 3$.

(c) **(8 points)** Find the area of the shaded region.

$f(x) = g(x)$

$4 - x^2 = 3x$

$x^2 + 3x - 4 = 0$

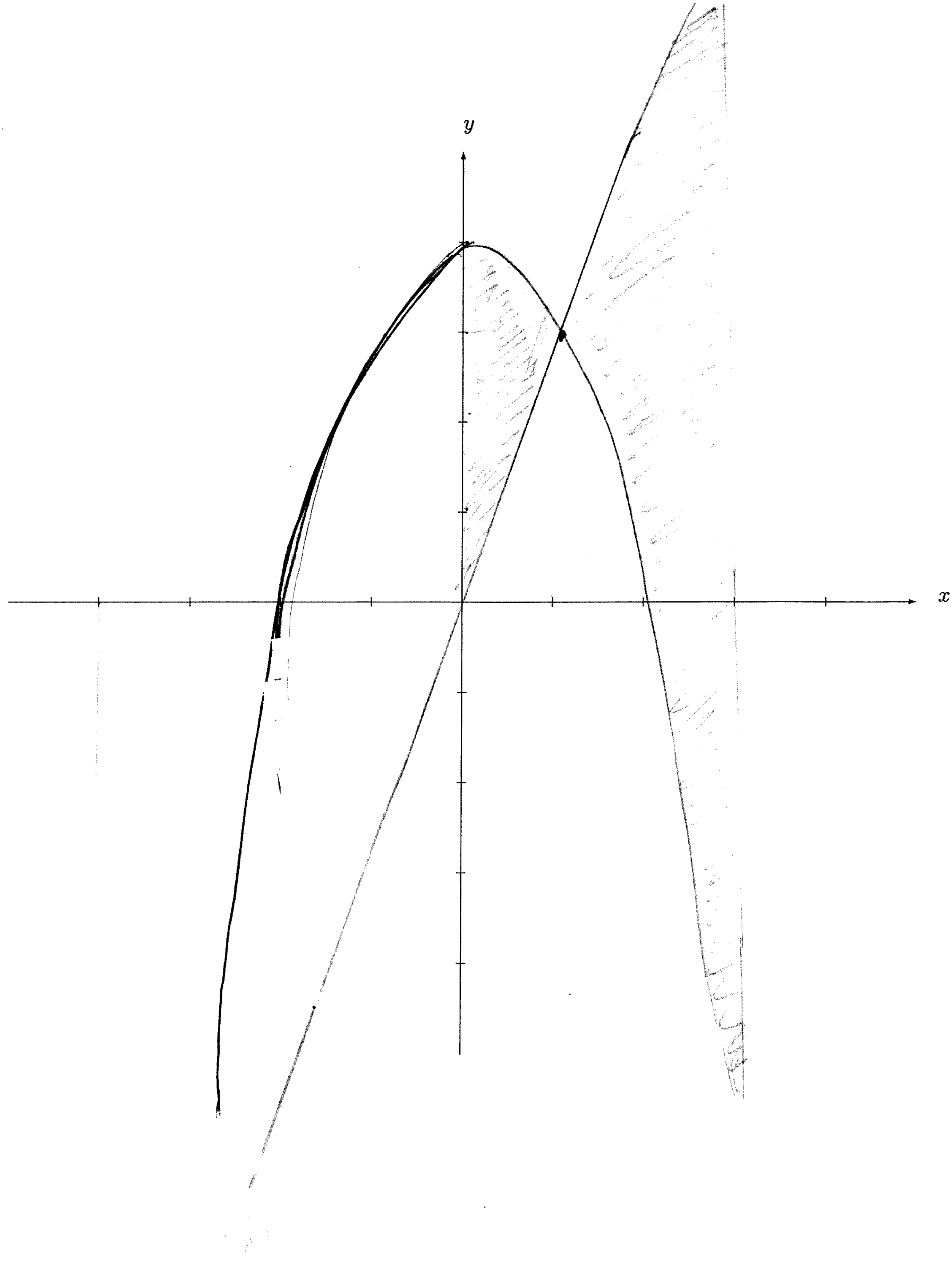
$(x+4)(x-1) = 0$

$x = 1 \qquad x = -4.$

$A_1 = \int_0^1 \left[(4-x^2) - 3x\right] dx = \frac{13}{6}$

$A_2 = \int_1^3 \left[3x - (4-x^2)\right] dx = \frac{38}{3}$

$A = \frac{13}{6} + \frac{38}{3} = \frac{13+76}{6} = \frac{89}{6}$


y
x

Long Answer Question 4 (16 points)
Consider the function of two variables

$$f(x, y) = x^3 - 3x^2 + 3y^2 + 6xy + 12$$

(a) **(4 points)** Calculate the first-order partial derivatives.

(b) **(6 points)** Find all critical points.

(c) **(6 points)** Identify what type of critical points they are (local max, local min or saddle point).

$$\begin{cases} f_x = 3x^2 - 6x + 6y = 0 \\ f_y = 6y + 6x = 0 \end{cases} \Rightarrow x = -y$$

$3x^2 - 6x - 6x = 0$

$3x^2 - 12x = 0$ $\quad 3x(x-4) = 0$

$x = 0 \quad (0,0)$

$x = 4 \quad (4,-4)$

$f_{xx} = 6x - 6 \qquad f_{yy} = 6$

$f_{xy} = 6 \qquad f_{yx} = 6$

$D(x,y) = (6x-6)6 - 36 = 36x - 72$

$D(0,0) < 0 \Rightarrow (0,0)$ saddle point.

$D(4,-4) = 72 > 0 \rightarrow f_{xx}(4,-4) = 24 - 6 > 0$

$(4,-4)$ local min.