Engineering Economics
ECO 1192

Topic 3: Single sums; Annuities; Payback Methods

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Lecture Objectives

- Study summary measures for decision-making
  - Single sums
    - Present worth and future worth
  - Annual equivalent method (annuity)
  - Payback Methods
  - Comparisons
  - Assumptions, pros/cons & results
- Examples

Recommended reading

- Fraser et al.** chapters 3 and 4
  - Newnan et al. chapters 5 and 6
  - Park chapters 5 and 6

** N.M. Fraser and E.M. Jewkes, Engineering Economics, 5th edition, Pearson Education, Toronto, Ontario, 2013.

Subscription to a scientific journal

Options:

1. One year: $100
2. Two years: $180
3. Four years: $340

Which option is best if

- Money will earn 10% in the next 10 years
- Your life expectancy exceeds the longest subscription option

Major trip or purchase?

- A newly minted engineer dreams of purchasing a very high quality home sound and video system for which he plans to pay cash.
- Without the necessary cash ($10,000), he will deposit an equal amount at the end of each of the three years to raise sufficient funds.
- Remember that there is no inflation (the price of the system remains at $10,000)
- If the local bank pays 10% interest, how much should the engineer deposit at the end of each year?

Apartment building: maximum purchase price?

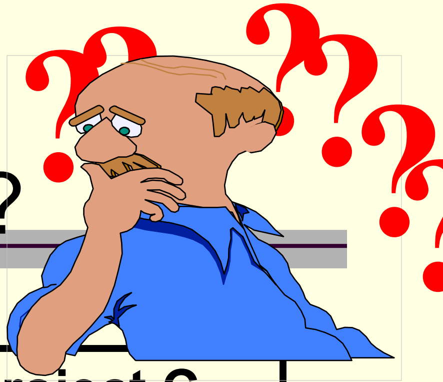
1. Annual rental income = $120,000 (no risk)
2. Annual operating costs = $40,000 (no risk)
3. Annual maintenance costs = $10,000 (no risk)
4. Expected period of ownership = 20 years
5. Expected resale value = $1,000,000
6. Asking price = $1.2 million
7. Current (and future) interest rate = 10%
8. Inflation or deflation = 0%.

What's your BEST offer?

Summary Measures

- The result of converting all relevant project information to an **equivalent** indicator or measure.
- Relevant project information (for now):
  - Initial or first cost: P
  - Duration (life): N
  - Revenues (AOR or AR) and costs (AOC or AC)
    - AOR = Annual operating revenues
    - AOC = Annual operating costs
  - Salvage value: SV
  - Discount factor: MARR

Which projects are valid?
Which project (if any) is best?



Project parameters	Project A	Project B	Project C
First cost ($)	3,000	5,000	8,000
Annual cost ($)	600	900	1300
Annual revenues ($)	1500	1750	2000
Salvage value ($)	0	-200	1000
Duration (years)	5	10	20
Interest rate (10%)	10	10	10

Reminder: symbols

r = nominal interest rate
i = effective interest rate
n = number of years
m = number of within-year compounding periods
PW = present value or principal
NPW = Net present worth
NPW = PW (Cash Inflows) – PW(Cash Outflows)
FW or = Future value or amount
NFW = Net future worth or value

Summary measures

1. **Single Sums**
   - Present Worth (PW): project cash flows are converted to a single equivalent value now (today).
   - Future Worth (FW): project cash flows are converted to a single future value (usually at the end of a project's life).
2. **Annual Equivalent Worth (AEW)**: project information is converted to an annuity (a series of equivalent and equidistant cash flows).

Other summary measures (2)

3. **Payback**
   - **Simple**: number of time periods (usually years) required to recover a project's first cost assuming that MARR =0%
     - A project's unrecovered initial cost has no opportunity cost (money has no time value).
   - **Discounted:** number of time periods (usually years) required to recover a project's first cost assuming that MARR ≠ 0%
     - A project's unrecovered initial cost has an opportunity cost since the unrecovered cost could have been invested elsewhere at MARR.

Other summary measures (1)

4. <u>Rates of return</u>

- **Internal (IRR)**: a rate of return that equates the present or future worth, or annual equivalent worth of the cash inflows and outflows.
- **External (ERR)**: with the assumption that cash inflows can be reinvested at a predetermined rate (usually the MARR), a rate that equates the future worth (at the end of a project's life) of cash inflows and outflows.

Next lecture

Categories of projects or investments

1. **Independent**
   - Projects are selected (or excluded) independently of the selection (or exclusion) of other projects.
2. **Mutually exclusive**
   - only one **valid** project (including the status quo option) can be selected amongst competing alternatives.
3. **Contingent (dependent)**
   - the selection of a project depends on the selection of another (one or more) project
   - Excluded from our project analyses

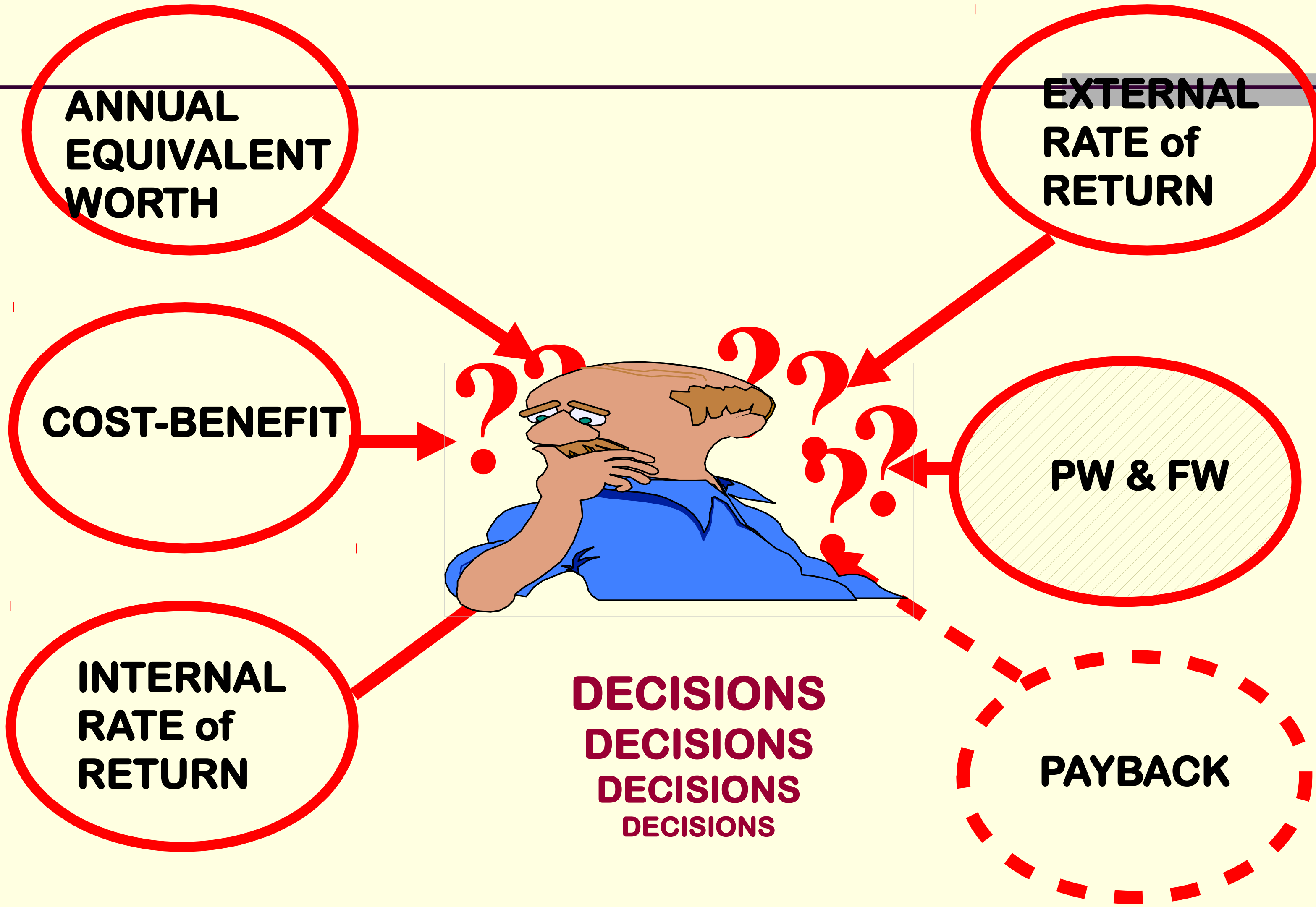
Working assumptions (Repeat)

1. Discrete cash flows & discrete compounding
2. Ownership capital only (no debt capital)
3. No price changes (neither inflation nor deflation during a project's life)
4. Unlimited funds (no capital budgeting): can select all acceptable independent projects
5. Total certainty (no uncertainty or risk)
6. No government (no taxes; no depreciation of capital assets)
7. No imponderables (all project benefits and costs have been converted to their equivalent monetary value)
    - Value of time and lives saved ……………

Single Sums

1. Present Worth (PW)
2. Future Worth (FW)

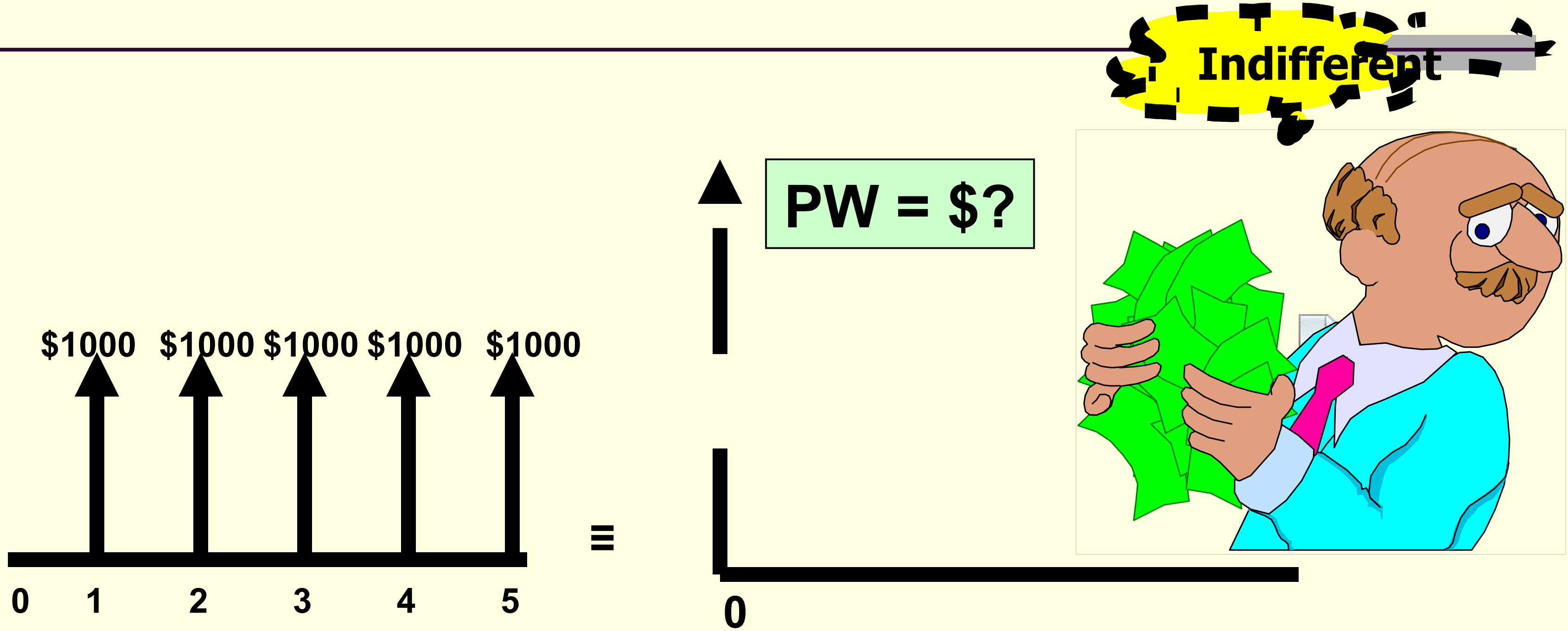
Final decisions



Summary Measure: Single Sums

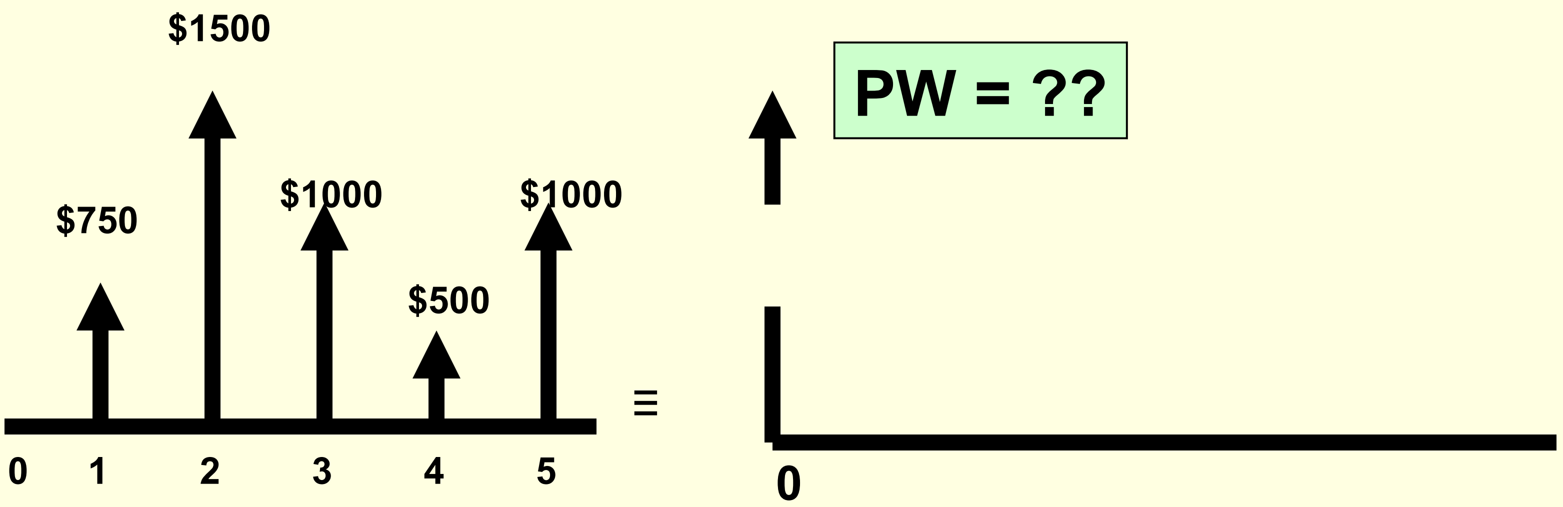
- A project's cash inflows and outflows are summarised in an equivalent single dollar amount at a specific point in time.
- The most common single sums are
  - Present Worth (PW or NPW)
    - all project cash flows are converted to an equivalent value **now** (today)
  - Future Worth (FW or NFW)
    - all project cash flows are converted to an equivalent **future** value (usually at the end of a project's life).

Cash flow diagram: Present Worth (PW)



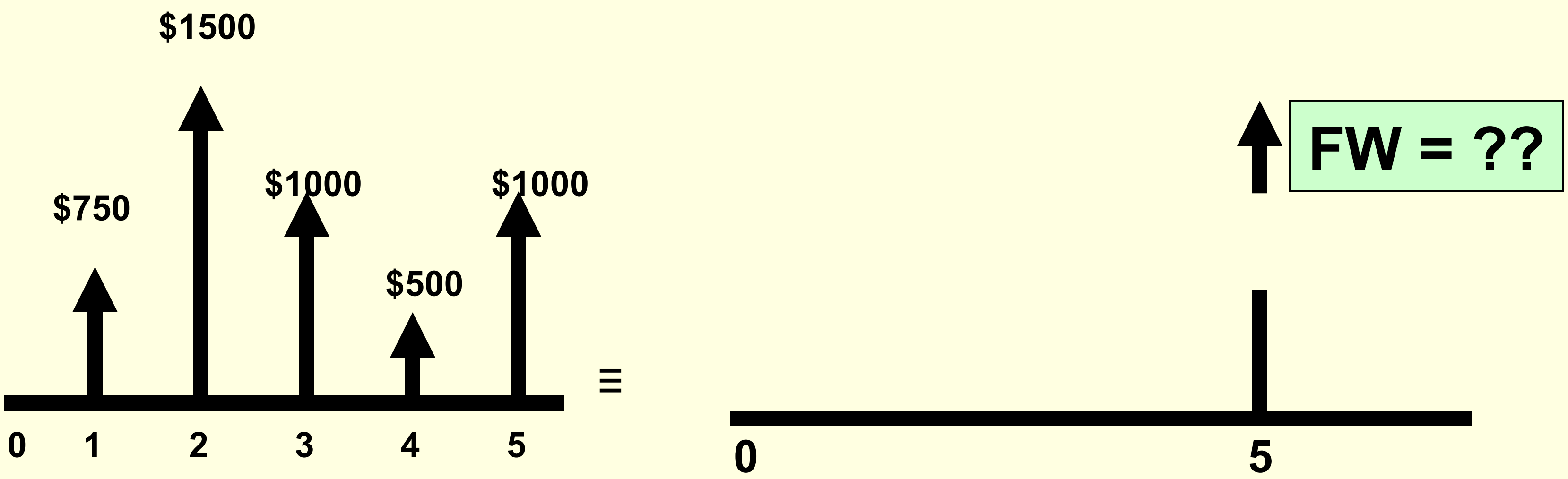
Find the PW of $1,000 annual end-of-year deposits if MARR=10%.

Cash flow diagram: Present Worth (PW)



Find the PW of five irregular end-of-year cash flows if MARR=10%.

Cash Flow Diagram: Future Worth (FW)



Find the FW of five irregular end-of-year cash flows if MARR=10%.

Find PW if MARR=10%

End of Year	Cash Flow ($)
0	0
1	+1,000
2	+2,000
3	+3,000
4	+2,500
5	+2,000
6	+1,500
7	+1,000
8	+500

Sample Financial Table

DISCRETE CASH FLOW AND DISCRETE COMPOUNDING (MARR=10%)							
n	(F/P,i%,n)	(P/F,i%,n)	(A/P,i%,n)	(P/A,i%,n)	(A/F,i%,n)	(F/A,i%,n)	(A/G,i%,n)
1	1.1000	0.9091	1.1000	0.9091	1.0000	1.0000	0.0000
2	1.2100	0.8264	0.5762	1.7355	0.4762	2.1000	0.4762
3	1.3310	0.7513	0.4021	2.4869	0.3021	3.3100	0.9366
4	1.4641	0.6830	0.3155	3.1699	0.2155	4.6410	1.3812
5	1.6105	0.6209	0.2638	3.7908	0.1638	6.1051	1.8101

Problem: PW (MARR =10%)

End of Year	Cash Flow ($)
0	0
1	+1,000
2	+2,000
3	+3,000
4	+2,500
5	+2,000
6	+1,500
7	+1,000
8	+500

One solution:

PW = 1,000(P/F,10%,1)
+2,000(P/F,10%,2)
+3,000(P/F,10%,3)
+2,500(P/F,10%,4)
+ 2,000(P/F,10%,5)
+ 1,500(P/F,10%,6)
+1,000(P/F,10%,7)
+ 500(P/F,10%,8)
= $9,359

Problem: PW (MARR=10%)

End of year	Cash Flow ($)
0	0
1	+1,000
2	+2,000
3	+3,000
4	+2,500
5	+2,000
6	+1,500
7	+1,000
8	+500

A second solution:

PW of the first 3 terms (PW1)

PW1 = A(P/A,i%,N) + G(P/G,i%,N)
= 1,000(P/A,10%,3)
+ 1,000(P/G,10%,3) = $4,816

PW of the last five terms (PW2)
PW2 = A(P/A,i%,N) - G(P/G,i%,N)
= 2,500(P/A,10%,5) - 500(P/G,10%,5)
= $6,046

Move PW2 back from t=3 to t=0 and add to PW1
PW2* = 6,046(P/F,10%,3) = $4,542
PW (series) = $4,816 + $4,542
= $9,358 (rounded)

Excel Calculation

	Column C	
Row		
1	0	
2	1000	
3	2000	
4	3000	
5	2500	
6	2000	
7	1500	
8	1000	
9	500	
	=NPV(10%,C7:C14)+C6	=$9,358

Decision Criteria for PW & FW: Independent Projects

- Select all projects with a non-negative PW or FW
  - if PW or FW > 0, accept project
  - if PW or FW < 0, reject project
  - if PW or FW = 0, accept project
- If all projects have a PW or FW < $0, select the "status quo" (also known as the "Do Nothing" option)
  - Invest available funds at the MARR
- Note that the selection of independent projects is usually constrained by a financial (capital) budget (limited budget or funds).

Decision Criteria: Mutually Exclusive Projects (PW & FW)

- Select the project with the largest non-negative single sum (PW or FW)
  - The selection of the BEST project using the PW or FW methods requires that all projects be analysed using the same period of time.
- If no project has a non-negative single sum, select the “status quo” option
  - invest available funds elsewhere at MARR (e.g., at a financial institution).

Project Selection Example: PW

1. If PW(A) = $500 (valid project)
2. If PW(B) = $5,000 (valid project)
3. If PW(C) = $-500 (invalid project)
4. If PW(D) = $0 (valid project)

- If A, B, C and D are independent projects with no budget constraint
  - select projects A and B; or A, B and C
  - Do not select project D.
- If A, B, C and D are mutually exclusive projects
  - select project B (the best “valid” project).

Caveat: Mutually exclusive projects

- **Single sum methods** (PW & FW)
  - Projects need not be assessed over an identical period of analysis to determine their acceptability (validity)
  - A common period of analysis must be used to determine the "best" mutually exclusive project
- Example 1: One cannot compare a 5-year project to a 10-year project without making appropriate time adjustments
  - known as co-termination where projects have identical start and end times
- Example 2: You cannot select the better of a 1-year and a 2-year magazine subscription strictly on the basis of their respective subscription cost
  - the 1-year subscription cost will always be less than the 2-year subscription cost. Is it always the better deal? No.

Approaches to a common period of analysis (for a single sum approach)

1. **Repeatability**: repeat individual projects to get a common period of analysis.

   Example: Given project A of 5 years, project B of 8 years and project C of 10 years, their least common denominator is 40 years (hence, a 40-year period of analysis)

2. **Study period**: use the shortest project duration as the period of analysis; all projects must have a known salvage value at that point in time.

   Example: Given project A of 5 years, project B of 8 years and project C of 10 years, the study period would match project A's duration (i.e., five years) and the salvage value for projects B and C would have to be estimated after 5 years)

Approaches to a common period of analysis (cont'd)

3. **Best replacement alternative**

Calculate each project's annual equivalent worth and use the project with the highest positive annual worth as the basis for replacement to arrive at a common period of analysis.

Not Required in this course

PW & FW Applications: Finite & infinite periods of analysis

A. Finite periods of analysis (projects of finite duration)

A1. Projects of equal duration

- $N_A = N_B = N_C = 5$ yrs

A2. Projects of unequal duration

- $N_A = 5$ yrs; $N_B = 10$ yrs; $N_C = 20$ yrs

B. Infinite periods of analysis (extremely long projects or periods of analysis)

B1. Projects of finite duration (where project services are required indefinitely)

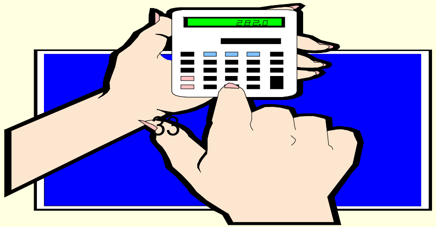
- $N_A = 5$ yrs; $N_B = 10$ yrs; $N_C = 20$ yrs

B2. Projects of infinite duration

- N_A & $N_B \rightarrow$ infinity.

A1. Finite projects of equal duration

Parameters	Project A	Project B	Project C
P	2,500	3,500	5,000
AOC	900	700	1,000 + G (=100)
SV	200	350	-100
N	5	5	5
AOR	1,800	1,900	2,100(15% GROWTH)
MARR (%)	10	10	10



DISCRETE CASH FLOW AND DISCRETE COMPOUNDING (10%)

n	(F/P,i%,n)	(P/F,i%,n)	(A/P,i%,n)	(P/A,i%,n)	(A/F,i%,n)	(F/A,i%,n)	(A/G,i%,n)
1	1.1000	0.9091	1.1000	0.9091	1.0000	1.0000	0.0000
2	1.2100	0.8264	0.5762	1.7355	0.4762	2.1000	0.4762
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5	1.6105	0.6209	0.2638	3.7908	0.1638	6.1051	1.8101

A1. Finite projects of equal duration (cont'd)

- NPW (A) = PW(Project Cash Inflows) – PW (Project Cash Outflows)
- NPW (A) = -2500 - 900(P/A,10%,5) + 1800(P/A,10%,5) + 200(P/F,10%,5) = **$1,036**
- NFW (A) = -2500(F/P,10%,5) - 900(F/A,10%,5) + 1800(F/A,10%,5) + 200 = **$1,668**

The NPW of the "STATUS QUO" option is zero.
= -2500 + 250(P/A,10%,5) + 2500(P/F,10%,5) = $0

A1. Finite projects of equal duration (cont'd)

NPW (B) = -3500 -700(P/A,10%,5)
+ 1900(P/A,10%,5) + 350(P/F,10%,5) = **$1,266**

NFW (B) = -3500(F/P,10%,5)
- 700(F/A,10%,5) + 1900(F/A,10%,5) + 350 = **$2,039**

The NPW of the "STATUS QUO" option is zero.
= -3500 + 350(P/A,10%,5) + 3500(P/F,10%,5) = $0

A1. Finite projects of equal duration (cont'd)

NPW (C) = -5000 - 1000(P/A,10%,5)

- 100(P/G,10%,5) +(2100/(0.1-0.15)){1-(1.15/1.1)^5}

- 100(P/F,10%,5) = **$977**

NFW (C) = -5000(F/P,10%,5)

- 1000(F/A,10%,5) - 100(F/G,10%,5) - 100

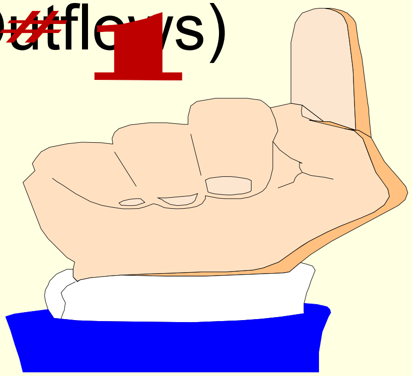
+ (2100/(0.1-0.15)){1-(1.15/1.1)^5}(F/P,10%,5) = **$1,573**

The NPW of the "STATUS QUO" option is zero.

= -5000 + 500(P/A,10%,5) + 5000(P/F,10%,5) = $0

Independent projects

- NPW = PW(Cash Inflows) - PW(Cash Outflows)
- NPW(A) = $1,036
- NPW(B) = $1,266
- NPW(C) = $977
- NPW(MARR) = $0



Decision: Select all projects in the absence of capital rationing.

A1. Finite projects of equal duration: Mutually exclusive projects

- NPW = PW(Cash Inflows) - PW(Cash Outflows)
- NPW(A) = $1,036
- NPW(B) = $1,266
- NPW(C) = $977
- NPW(MARR) = $0
- Projects A, B and C have the same duration (i.e., 5 years)
  - Select the project with the highest non-negative NPW or NFW

Decision: Select Project B (NPW = $1,266)

Finite projects of equal duration

EOY	Row			
0	1	-2500		
1	2	=1800 (		

A2. Finite periods of analysis: finite projects of unequal duration

Parameters	Project A	Project B
P ($)	26,000	36,000
AOC ($)	11,800	9,900
SV ($)	2,000	3,000
N (years)	6	12
AOR ($)	25,000	18,000
MARR (%)	15	15

Single sums: Projects of unequal duration

- If Projects A and B are independent
  - NPW and NFW calculations need not be based on a common period of analysis
  - Use their respective duration (life) to determine if they are valid (acceptable).
- If Projects A and B are mutually exclusive
  - NPW and NFW project calculations MUST be based on a common period of analysis to determine the BETTER project.
    - the period of analysis is usually the least common denominator of the project durations
    - If $N_A = 7$ years and $N_B = 10$ years, the least common denominator is 70 years.

Projects A & B are independent

Project A

- NPW(A) = -26,000
  + 2,000(P/F,15%,6)
  + (25,000 – 11,800)(P/A,15%,6)
  = $24,820 (Acceptable)

Project B

- NPW (B)
  = -36,000 + 3,000(P/F,15%,12)
  + (18,000-9,900)(P/A,15%,12)
  = $8,468 (Acceptable)

PW(6) = 24,820
PW(12) = 35,550
FW (6) = 57,410
FW (12) = 190,202
AEW = 6.558

PARAMETERS	PROJECT A	PROJECT B
P ($)	26000	36000
AOC ($)	11800	9900
SV ($)	2000	3000
N (years)	6	12
AOR ($)	25000	18000
MARR (%)	15	15

Projects A & B are mutually exclusive

Use a 12-year period of analysis

Project A

- NPW(A) = -26,000 -26,000(P/F,15%,6)
  + 2,000(P/F,15%,6) +2,000(P/F,15%,12)
  + (25,000 – 11,800)(P/A,15%,12)
  = $35,550 (Acceptable)

Project B

- NPW (B)
  = -36,000 + 3,000(P/F,15%,12)
  + (18,000-9,900)(P/A,15%,12)
  = $8,468 (Acceptable)

Decision: Select Project A over project B.

PW(6) =	24,820
PW(12) =	35,550
FW (6) =	57,410
FW (12) =	190,202
AEW =	6,558

PARAMETERS	PROJECT A	PROJECT B
P ($)	26000	36000
AOC ($)	11800	9900
SV ($)	2000	3000
N (years)	6	12
AOR ($)	25000	18000
MARR (%)	15	15

B. Infinite periods of analysis: projects of finite or infinite duration

Parameters	Pipeline (Finite duration)	Tunnel (Infinite duration)
P ($)	**5,000,000**	**5,500,000**
AOC ($)	**10,000**	**15,000**
SV ($)	**0**	**n.a.**
N (years)	**50 years**	**∞**
MARR (%)	**6**	**6**

Note: Revenues are excluded from the analysis because they are the same for the pipeline and the tunnel.

(P/A,i%,∞)

(P/A,i,N) where 'N' is finite

$$= \{(1+i)^N - 1\} \div \{i(1+i)^N\} \quad \ldots\ldots\ldots \quad 1)$$

Divide 1) by $(1+i)^N$

$$= \frac{\{(1+i)^N \div (1+i)^N\} - 1 \div \{(1+i)^N\}}{i(1+i)^N \div (1+i)^N} \quad \ldots\ldots\ldots 2)$$

$$= \left\{ \frac{\{1 - \{1 \div (1+i)^N\}}{i} \right\} \quad = 1 \div i \ldots\ldots\ldots\ 3)$$

B. Infinite period of analysis: projects of finite or infinite duration

- **The tunnel and the pipeline have different durations.**
- **A common period of analysis must be determined.**
- **The repeatability assumption will be used for the finite project i.e., the pipeline. It will be repeated every fifty years forever.**

PW(PIPELINE) = A(P/A,6%,infinity) = A/i
327,000/0.06 = $5,450,000

PW(TUNNEL) = A(P/A,6%,infinity)= A/i
345,000/0.06 = $5,750,000

Decision: Select the pipeline over the tunnel (less costly).

The PW of a project's cash outflows (where the project's life tends to infinity) is known as its "capital cost".

Your Turn: What's your **best offer** on the following income-generating property?

- Your friendly real estate agent provides the following information on an investment property:
  - Annual income = \$15,000
  - Annual expenses = \$5,000
  - Selling price after 10 years = \$160,000 (guaranteed)
  - MARR = 10%

Your Turn: What's your best offer on the following income-generating property?

Details of the investment property:

- **Annual income = $15,000**
- **Annual expenses = $5,000**
- **Selling price after 10 years = $160,000 (guaranteed)**
- **MARR = 10%**

Present Worth (PW)

= (15,000-5,000)(P/A,10%,10)

+ 160,000(P/F,10%,10)

= 10,000(6.145) + 160,000(0.3855)

= 61,450 + 61,680

= $123,130

Note: Paying more than $123,130 for this property would decrease the return on your investment below MARR (=10%)

Your Turn: Maximum offer on a used car

- You plan to use your entire monthly income of $500 (after taxes) to make monthly payments on a car loan.
- The terms of your loan are:
  - $500 monthly; $0 down-payment
  - 18% compounded monthly
  - 36 month repayment period
  - $0 salvage value after 36 months.

What is your maximum purchase price?

Your Turn: Best offer on a car?

- Purchase details
  - No down payment
  - 36 monthly loan repayments of $500.
- PW = 500(P/A,[18%/12],36)

  =500(P/A,1.5%,36) = $13,830.50

✂Your best offer is $13,830.50

  - Assumption: Your car has no resale value after 36 months.

Annual Equivalent Worth (AEW)

AEW Applications: finite & infinite periods of analysis

A. Finite periods of analysis (projects of finite duration)

A1. Projects of equal duration

- $N_A = N_B = N_C = 5$ yrs

A2. Projects of unequal duration

- $N_A = 5$ yrs; $N_B = 10$ yrs; $N_C = 20$ yrs

B. Infinite periods of analysis

B1. Projects of finite duration

- $N_A = 5$ yrs; $N_B = 10$ yrs; $N_C = 20$ yrs

B2. Projects of infinite duration

- N_A & $N_B \rightarrow$ infinity.

Annual equivalent worth summary measure (AEW)

- Transforms all project cash flows over a period of time to an equivalent annuity (a series of equivalent dollar cash flows)
- Method does NOT require projects to be co-terminated to determine the best mutually exclusive project.
  - Why?
    - Repeating a project one or several times when its parameters are the same each time will have no impact on its AEW.

Annual equivalent worth (AEW)

1. AEW = P(A/P,i%,N) - SV(A/F,i%,N)
   (most common)

from (A/P,i%,N) = (A/F,i%,N) + i%

2. AEW = (P - SV)(A/P,i%,N) + i%SV
3. AEW = (P - SV)(A/F,i%,N) + i%P

AEW Decision Criterion: Independent Projects

- Select all projects with non-negative annual equivalents (up to the budgetary constraint if any)
  - if no project has a non-negative equivalent, select the status quo (that is, invest available money at the going interest rate i.e. MARR)
  - If AEW > 0, accept project
  - If AEW < 0, reject project
  - If AEW = 0, indifferent (accept or reject project)
- Decision: select all projects with non-negative AEWs (if no budget constraint i.e., no capital rationing)

AEW Decision Criterion: Mutually Exclusive Projects

- Select THE project (only one project MUST be selected) with the largest non-negative annual equivalent value
- If no project has a non-negative annual equivalent value, select the “status quo” option
  - invest available funds at the MARR

Cash flows with costs only (minimisation criterion)

- Without cash inflows (i.e., revenues), it is impossible to ensure the selection of “acceptable" projects.
- If you must select a project for which revenues are not provided (for whatever reason), select the project with the smallest negative annual equivalent worth (i.e., minimise costs).

A1. Finite projects of equal duration

Parameters	Project A	Project B	Project C
P	2,500	3,500	5,000
AOC	900	700	1,000+(G=100)
SV	200	350	-100
N	5	5	5
AOR	1,800	1,900	2,100(15% GROWTH)
MARR (%)	10	10	10

DISCRETE CASH FLOW AND DISCRETE COMPOUNDING (10%)

n	(F/P,i%,n)	(P/F,i%,n)	(A/P,i%,n)	(P/A,i%,n)	(A/F,i%,n)	(F/A,i%,n)	(A/G,i%,n)
1	1.1000	0.9091	1.1000	0.9091	1.0000	1.0000	0.0000
2	1.2100	0.8264	0.5762	1.7355	0.4762	2.1000	0.4762
3	1.3310	0.7513	0.4021	2.4869	0.3021	3.3100	0.9366
4	1.4641	0.6830	0.3155	3.1699	0.2155	4.6410	1.3812
5	1.6105	0.6209	0.2638	3.7908	0.1638	6.1051	1.8101

A1. Finite Projects of Equal Duration

PARAMETERS	PROJECT A	PROJECT B	PROJECT C
P	2500	3500	5000
AOC	900	700	1000+(G=100)
SV	200	350	-100
N	5	5	5
AOR	1800	1900	2100(15% GROWTH)
I%	10	10	10

AEW (A) = -2,500(A/P,10%,5)
- 900 + 1,800 + 200(A/F,10%,5)
= $273

AEW (B) = -3,500(A/P,10%,5)
- 700 + 1,900 + 350(A/F,10%,5)
= $334

AEW (C) = -5,000(A/P,10%,5)
1,000 - 100(A/G,10%,5)
- 100(A/F,10%,5)
+ **(2,100/(0.1-0.15)){1-(1.15/1.1)^5}(A/P,10%,5)**
= $258

A1. Finite Projects of Equal Duration

EOY	Row			
0	1	-2		
1	2	='		

Finite Projects of Equal Duration: AEW Decision

- If projects A, B and C are independent (without capital rationing)
  - select projects A, B, C
- If projects A, B and C are mutually exclusive
  - select project B (largest AEW > 0)
- Project selection must be the same under single sum and annual equivalent methods.

A2. Finite periods of analysis: AEW and finite projects of unequal duration

Parameters	Project A	Project B
P ($)	26,000	36,000
AOC ($)	11,800	9,900
SV ($)	2,000	3,000
N (years)	6	10
AOR ($)	25,000	18,000
MARR (%)	15	15

Co-termination

- A co-terminated period of analysis is NOT required when the annual equivalent method is applied to mutually exclusive projects.
- The underlying assumption is that projects can always be repeated without changes to their original parameters i.e., P; SV; AOR …) OR that an identical project (with the same parameters) can be found.

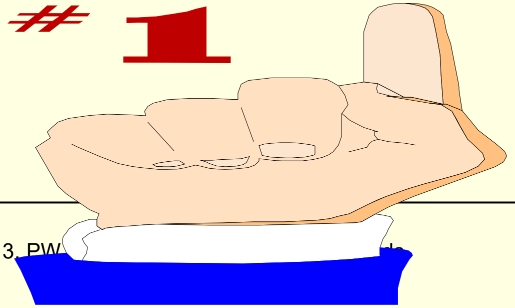
A2. Finite periods of analysis: AEW and finite projects of unequal duration

- AEW (A) = -26,000(A/P,15%,6) – 11,800 + 25,000 + 2,000(A/F,15%,6)
  = $<u>6,558</u>
- AEW (B) = -36,000(A/P,15%,10) – 9,900 + 18,000 + 3,000(A/F,15%,10)
  = **$<u>1,075</u>**

PARAMETERS	PROJECT A	PROJECT B
P ($)	26000	36000
AOC ($)	11800	9900
SV ($)	2000	3000
N (years)	6	10
AOR ($)	25000	18000
I%	15	15

Decision

- If projects A and B are mutually exclusive, select project A (largest positive AEW)
- I projects A and B are independent, select both projects (in the absence of capital rationing)
- Note: Same decisions as with NPW & NFW



B. Infinite periods of analysis: projects of finite or infinite duration

Parameters	Pipeline (Finite duration)	Tunnel (Infinite duration)
P ($)	5,000,000	5,500,000
AOC ($)	10,000	15,000
SV ($)	0	n.a.
N (years)	50 years	∞
MARR (%)	6	6

Note: Revenues are excluded from the analysis because they are the same for the pipeline and the tunnel.

Remember (PW discussion)

- $(P/A, i\%, \infty) = 1 \div i$
- $(A/P, i\%, \infty) = 1 \div (P/A, i\%, \infty) = i$

B. Infinite period of analysis: projects of finite (B1) & infinite duration (B2)

Assume that the pipeline and tunnel will generate the same water taxes (i.e., government revenues)

- AEW (Pipeline)

  = 5,000,000(A/P,6%,50) + 10,000 = $327,000/year
- AEW (Tunnel)

  = 5,500,000(0.06) + 15,000 = $345,000/year

Decision

- **Select the pipeline (less costly).**

PARAMETERS	PIPELINE	TUNNEL
P ($)	5,000,000	5,500,000
AOC ($)	10,000	15,000
SV ($)	0	n.a.
N (years)	50	infinity
AOR ($)	not given	not given
i% (=MARR)	6	6

Review Problem 1: Effective Interest Rate

Two options were advertised for the purchase of a car:
1. Cash: $3,575 or
2. Financing: $375 down with 45 monthly payments = $93.41

What is the effective rate of interest charged by the car dealer?

From $A = P(A/P,i\%,N)$,

$93.41 = (3{,}575 - 375)(A/P,i\%,N)$ or

$(A/P,i\%,45) = 93.41/3200 = 0.0291906$

(continued on next slide)

Review Problem 1: Effective Interest Rate

From financial tables:

- (A/P,1.5%,45) = 0.03072 and
- (A/P,i%,45) = 93.41/3200 = 0.0291906
- (A/P,1%,45) = 0.02771
- By interpolation: (A/P,i%,45)

= 1% + {[0.0291906-0.02771]/[0.03072-0.02771]}0.5

= 1% + 0.245947 = 1.245947%

- Therefore, the effective interest rate is $[1+0.01245947]^{12} - 1$ = 16.02% compounded annually.

Review Problem 2: Effective Interest Rate

- An investor purchased today real property worth $75,000.
- Financing
  - $20,000 cash down-payment
  - Loan of $55,000 at 12% compounded monthly
    - Monthly payments = $650.

a) What is the effective annual interest rate?

Effective rate = $[1 + (r/m)]^m - 1$

$= [1+0.01]^{12} - 1 = 0.127$ or 12.7%

Review Problem 2: Effective Interest Rate (cont'd)

b) How long will it take to pay off the loan?

- From P = A(P/A,i%,N), 55,000 = 650(P/A,1%,N)

  [55,000/650] = (P/A,1%,N) = 84.615
- Solve for N; N = 190.4 months

Review Problem 2: Effective Interest Rate (cont'd)

c) What would be the total interest expense ($) if the borrower opted for the full 190.4 months to repay the loan?

Total interest expense

= 190.4($650) - $55,000 = $68,760

Review Problem 2: Effective Interest Rate (cont'd)

d) How much would the borrower have to pay to clear the outstanding mortgage immediately after making the 48th payment of $650?

There would be (190.4 - 48) = 142.4 monthly payments of $650 outstanding after the 48th payment.

The PW of the remaining payments (immediately after the 48th payment) would be

650(P/A, 1%,142.4) = 650(75.7542) = $49,240.

A payment of $49,240 is required to pay off the outstanding (i.e., leftover or unpaid) loan.

Subscription options to a scientific journal

Given options:

- One year: $100
- Two years: $180
- Four years: $340

Which option is best if ….

- Money will always earns 10%.
- The subscriber is expected to live another 100 years (at least).

Revisiting the subscription to an engineering periodical

PW approach → four-year period of analysis

Option 1

PW(4@one-year subscriptions)

= 100 (now) + 100(P/A,10%,3) = $349

Option 2

PW(2@two-year subscriptions)

= 200+200(P/F,10%,2) = $365.29

Option 3

PW(1@four-year subscription) = $340

Decision: Option 3 is best (least costly).

Review Problem: Which power line option is better?

Power Line Details	Around the lake	Under the lake
Length (km)	15	5
First cost per km	$5,000	25,000
Maintenance per km	$200	400
Use life (yrs)	15	15
Salvage value per km	$3,000	5,000
Yearly power loss per km	$500	500
Annual property taxes	2% of first cost	2% of first cost

Review Problem: Which power line option is better?

Power line around the lake

- AEW = 15(5000)(A/P,10%,15) + 15(200) +15(3000)(A/F,10%,15) + 15(500) + 2%(15)(5000)

OR

- AEW = 5000(A/P,10%,15) + 200 + 3000(A/F,10%,15) + 500 + 2%(5000)

Review Problem: Which power line option is better?

Power line under the lake

- AEW = 5(25000)(A/P,10%,15) + 5(400) + 5(5000)(A/F,10%,15) + 5(500) + 2%(5)(25000)

OR

- AEW = 25000(A/P,10%,15) + 400 + 5000(A/F,10%,15) + 500 + 2%(25000)

Decision: Select the alternative with the lower annual cost.

Payback Methods

(Simple and Discounted)

Lecture Objectives

1. Payback Method
   - Simple
   - Discounted
2. Comparison with other summary measures (e.g., PW, FW, AEW..)
3. Applications

Simple Payback: definition

- The time required (e.g. years) to recover the initial investment (P) when the un-recovered investment (i.e. Project Balance) has **no opportunity cost** (i.e., MARR = 0%)
  - The time value of money is zero **($1 tomorrow** has the same value as **$1 today**)
- Note that a project's **salvage value** is not considered neither in the simple nor in the discounted payback methods.

Two Approaches

Approach 1

- Find N* (time to recover initial investment P) where N* ≤ N (project's life)
- **$\sum(OR_i - OC_i$ – opportunity cost of a negative project balance) = P where i = 1 to N***

Approach 2

Not applicable to this course

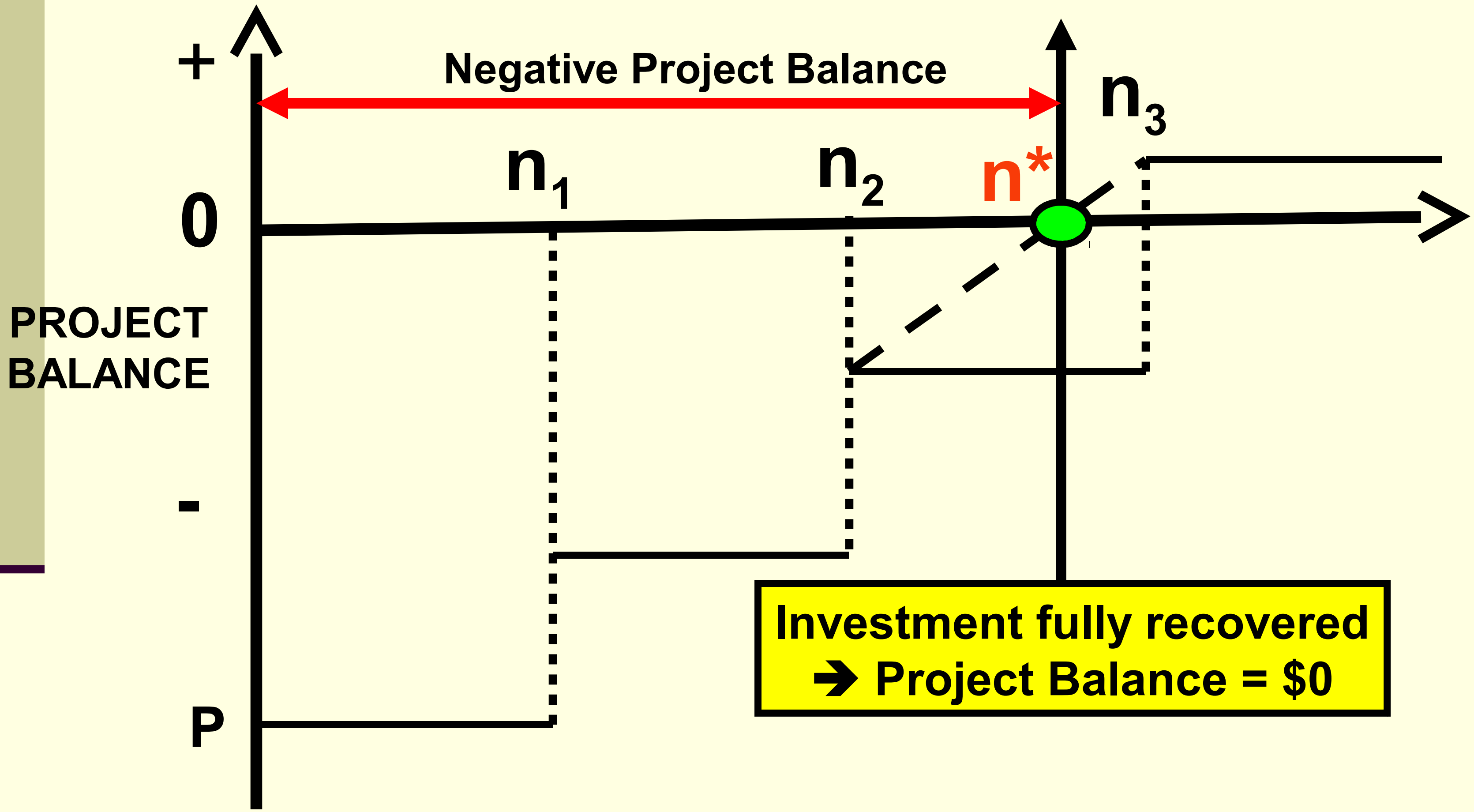
- **$\sum(OR_i - OC_i$ – opportunity cost of a negative project balance +/- Salvage value) = (P – SV), where i = 1 to N***

OR = Operating Revenue; OC = Operating Cost

Project Balance

- A project's balance at any point during the life of a project is
  - Simple payback
    - Its un-recovered investment (P)
  - Discounted payback
    - Its un-recovered investment (P) + the annual opportunity cost (i.e., interest charges) of the un-recovered investment
- Positive net annual revenues (revenues minus costs) should decrease a project's balance over time
  - Simple method: Definitely.
  - Discounted method: depends on the relationship between the opportunity cost of the project's balance at the beginning of a year and its net revenues for that year.
- A project balance is negative until the initial investment has been fully recovered.
- Our interest is limited to negative project balances ➔ opportunity (i.e., economic) costs

Project Balance (cont'd)



Simple Payback: Characteristics

- Payback methods (both simple and discounted) are concerned exclusively with **liquidity**
  - Other decision criteria (e.g., PW, AEW) are concerned exclusively with profitability)
- Liquidity: how quickly can the investment (P) be recovered from the project's cash flows.
- Usually based on before-tax calculations
- Simple payback: MARR = 0%.
- The salvage value is excluded from recovery calculations.
  - A project balance has NO opportunity cost.
  - **Project Balance = P - $\sum(OR_i – OC_i)$, i = 0, 1 ….N**

Project Balance: Simple Payback

- EOY n_0: Project Balance (PB_0) = P (negative value)
- EOY n_1: $PB_1 = PB_0 + (OR_1 – OC_1)$
- EOY n_2: $PB_2 = PB_1 + (OR_2 – OC_2)$
- EOY n_3: $PB_3 = PB_2 + (OR_3 – OC_3)$
- ………..

Where OR ≡ operating revenue; OC ≡ operating cost; EOY ≡ End-of-year.

Simple Payback: MARR = 0%

Decision Rules: Simple Payback

- $N^{**} \equiv$ the industry standard or threshold
- $N^{*} \equiv$ recovery period for a specific project
- $N \equiv$ life or duration of a project

Independent Projects (no budget constraint)

- **Select ALL projects for which $N^{*} \leq N^{**}$**

Mutually Exclusive Projects

- **Select the project with the shortest N^{*} among the projects for which ($N^{*} \leq N^{**}$)**

Decision Rule: Example

Project		Payback Period
A	**Accept**	N_A= 4.5 years
B	**Reject**	N_B = 7
C	**Accept**	N_C = 3
D	**Accept**	N_D = 4
E	**Reject**	N_E = 6
F	**Accept**	N_F = 4.7
If Industry Standard (N) = 5 years**		

Example 1: Simple Payback (Focus on liquidity not profitability)

EOY	Cash Flow: Project A	Cash Flow: Project B
0	-15,000	-15,000
1	1000	3000
2	2000	3000
3	3000	3000
4	4000	3000
5	5000	3000
6	6000	3000
7	7000	3000
8	8000	3000

Projects have a 5-year payback period

Example 1: Simple Payback (Focus on liquidity)

- Projects A and B have the same five-year recovery period (i.e., project balance is zero after five years)
- However, they have very different profitability profiles (with Project A being more profitable than Project B)
- Since this method focuses ONLY on liquidity (how FAST can I recover my initial investment), you would be indifferent between projects "A" and "B" even if they have very different profitability.

Example 2: Simple Payback (Uniform Cash Flow)

Parameters	Machine X	Machine Y
Annual Cost	20	30
P	200	400
Annual Revenue	95	200
SV	50	150
N (years)	6	12
MARR (%)	10	10

Assume the industry standard = 3 years

Example 2: Simple Payback (Uniform Cash Flow) cont'd

Machine X

200 ÷ (95-20) = 2.7 years

(less than industry standard)

Machine Y

400 ÷ (200-30) = 2.4 years

(both recovery periods are less the industry standard)

Parameters	Machine X	Machine Y
Annual Cost	20	30
P	200	400
Annual Revenue	95	200
SV	50	150
N (years)	6	12
MARR (%)	10	10

Example 2: Simple Payback (Uniform Cash Flow) cont'd

Decision

- If machines X and Y are independent
  - select both machines (since their respective recovery periods are less than 3 years).
- If machines X and Y are mutually exclusive
  - select machine Y (shorter valid recovery period)

Example 3: Simple Payback (Irregular Cash Flow)

Year	Cash Flow	Project Balance
0	-1,000	-1,000
1	-500	-1,500
2	+500	-1,000
3	+700	-300
4	+1,000	+700

= +1 000

Recovery period = 3 years + 300÷1,000 = 3.3 years

Example 4: Simple Payback (Uniform Cash Flows)

Year	Cash Flow	Opportunity Cost (0% of BP)	Project Balance (PB)
0	-15,000		-15,000
1	3,000	$0	-12,000
2	3,000	$0	-9,000
3	3,000	$0	-6,000
4	3,000	$0	-3,000
5	3,000	$0	0
6	3,000	$0	+3,000
7	3,000	$0	+6,000
8	3,000	$0	+9,000
9	3,000	$0	+12,000
10	3,000	$0	+15,000

Discounted Payback

- Time required to recover the initial investment (P) when the un-recovered investment (i.e. Project Balance) has an opportunity cost (MARR > 0%, usually = MARR)
- Economic definition of cost
  - $ tied up in the project **CANNOT** be invested elsewhere to generate investment income

Discounted Method: Characteristics

- The annual opportunity cost of a negative project balance is determined by the beginning of year project balance and the MARR.
- A project's end-of-year balance is equal
  - project balance at the beginning of the year
    - \- the opportunity cost for that year
    - \+ net annual revenues (i.e., OR – OC)
- Calculations are usually performed before taxes.
- Focus on liquidity exclusively; no concern for profitability

Project Balance: Discounted Payback

- At EOY0: Project Balance (PB_0) = P (negative value)
- At EOY1: $PB_1 = PB_0 +$ **PB_0@MARR** $+(OR_1 – OC_1)$
- At EOY2: $PB_2 = PB_1 +$ **PB_1@MARR** $+ (OR_2 – OC_2)$
- At EOY3: $PB_3 = PB_2 +$ **PB_2@MARR** $+ (OR_3 – OC_3)$
- ………..

where PB=project balance; OR=operating revenues; OC=operating costs; EOYN =End-of-year N

DISCOUNTED PAYBACK: MARR > 0% (e.g., 10%)

Decision Rules: Summary

N** = industry standard or threshold

N* = recovery period for a specific project

N = life or duration of a project

Independent Projects (no budget constraint)

Select ALL projects for which $N^* \leq N^{**}$

Mutually Exclusive Projects

Among valid (acceptable, $N^* \leq N^{**}$) projects, select the project with the **shortest** N*

Decision Rule: Example

Project	Payback Period
A	N_A= 4.5 years
B	N_B = 7
C	**BEST** N_C = 3
D	N_D = 4
E	N_E = 6
F	N_F = 4.5
Assume industry standard (N) = 5 years**	

Example 4A: Discounted Payback
(Opportunity Cost Method)

Year	Cash Flow	Opportunity Cost (15% of BP)	Project Balance
0	-15,000		-15,000
1	3,000	-15,000(0.15) = 2,250	-14,250
2	3,000	2,137.5	-13,387.5
3	3,000	2,008.1	-12,395.6
4	3,000	1,859.3	-11,254.9
5	3,000	1,688.2	-9,943.2
6	3,000	1,491.5	-8,434.7
7	3,000	1,265.2	-6,699.9
8	3,000	1,005.	-4,704.9
9	3,000	705.7	-2,410.6
10	3,000	361.6	+227.8

Example 4B: Discounted Payback
(Cumulative Discounted Cash Flows)

Year	Cash Flow	Cumulative Discounted Cash Flow (MARR = 15%)
0	-15,000	-15,000
1	3,000	-12,391.3
2	3,000	-10,122.9
3	3,000	-8,150.3
4	3,000	-6,435.1
5	3,000	-4,943.5
6	3,000	-3,646.6
7	3,000	-2,518.7
8	3,000	-1,538.0
9	3,000	-685.3
10	3,000	56.3

Examples 4A & 4B: Discounted Payback

Example 4A

Recovery period = 9 + 2,410.6 ÷ (2,410.6+227.8)

= 9.91 years

Example 4B

Recovery period = 9 + 685.3 ÷ (685.3+56.3)

= 9.92 years

Decision: If the industry threshold (N**) for this type of project is 8 years, the project is invalid (N* > N**).

Example 5: Discounted Payback (Irregular Cash Flow)

Year	Cash Flow	Opportunity Cost (15%)	Project Balance
0	-15,000		-15,000
1	1,000	-15,000(0.15)=2,250	-16,250
2	2,000	2,437.5	-16,687.5
3	3,000	2,503.13	-16,190.63
4	4,000	2,428.6	-14,619.2
5	5,000	2192.9	-11,812.1
6	6,000	1,771.8	-7,583.9
7	7,000	1,137.6	-1,721.5
8	8,000	258.2	+6,020.2
9	9,000		
10	10,000		

Example 5: Discounted Payback (Irregular Cash Flow) cont'd

Recovery period

= 7 years + 1,721.5 ÷ (1,721.5+6,020.2)

= 7.22 years

Decision

If it takes 5 years, on average, to recover the initial investment for this type of project, this project is unacceptable.

Summary Problem

Parameters	Project A	Project B	Project C
First Cost ($)	10,000	18,000	30,000
Salvage Value ($)	0	-1,000	3,000
Life (Years)	10	10	20
Net Annual Profits (Annual operating revenues minus annual operating costs)($)	2,000	3,850	$2,800 at the end of the first year followed by annual increases of $400 (i.e., $3,200 at the end of the 2nd year; $3,600 at the end of the 3rd year, etc.)
MARR (%)	10	10	10

Net Present Worth (NPW)

NPW (A) = -P + A(P/A,i%,N) + SV(P/F,i%,N)
= -10,000 + 2,000(P/A,10%,10) + 0(P/F,10%,10)
= <u>$2,289</u>

NPW (B)= -P + A(P/A,i%,N) + SV(P/F,i%,N)
= -18,000 + 3,850(P/A,10%,10) - 1,000(P/F,10%,10)
= <u>$5,271</u>

NPW (C) = -P + A(P/A,i%,N) + G(P/G,i%,N)
+ SV(P/F,i%,N)
= -30,000+2,800(P/A,10%,20)+400(P/G,10%,20)
+3,000(P/F,10%,20)
= <u>$16,447</u>

Net Future Worth (NFW)

$$FW(A) = -P(F/P,i\%,N) + A(F/A,i\%,N) + SV$$
$$= -10{,}000(F/P,10\%,10)+2{,}000(F/A,10\%,10) + 0$$
$$= \underline{\$5{,}937}$$

$$FW(B) = -P(F/P,i\%,N)+ A(F/A,i\%,N) + SV$$
$$= -18{,}000(F/P,10\%,10)+3{,}850(F/A,10\%,10)-1{,}000$$
$$= \underline{\$13{,}672}$$

$$FW(C) = -P(F/P,i\%,N) + A(F/A,i\%,N) + G(F/G,i\%,N) + SV$$
$$= -30{,}000 + 2{,}800(F/A,10\%,20) + 400(F/G,10\%,20) + 3{,}000$$
$$= \underline{\$110{,}642}$$

Independent Projects

A common period of analysis is NOT required to determine the validity (acceptability) of independent projects.

If A, B, and C are Independent Projects

- In the absence of capital rationing, select the three projects since
  - NPW(A), NPW(B) and NPW(C) ≥ 0;
  - NFW(A), NFW(B) and NFW(C) ≥ 0.

If A, B and C are Mutually Exclusive Projects

- Select the BEST project (as long as its NPW and NFW ≥ 0)
- Remember!!!!!!
  - the selection of the BEST mutually exclusive project MUST be based on a *common period of analysis* when a single sum method (NPW and NFW) is used.

If A, B and C are Mutually Exclusive Projects

- Since projects A, B and C have different lives, a common period of analysis MUST be used to determine the BEST PROJECT when a single sum method (PW and FW) is applied.
- Using the "repeatability" assumption, the least common denominator is 20 years
  - hence, projects A and B MUST be repeated a second time to be compatible with the 20-year life of project C

If A, B and C are Mutually Exclusive Projects

Since projects A and B have a common duration (life), we can determine the better project based on a 10-year period of analysis.

NPW(A) = -P + A(P/A,i%,N) + SV(P/F,i%,N)
= -10,000 + 2,000(P/A,10%,10) + 0(P/F,10%,10)
= $2,289

NPW(B)= -P + A(P/A,i%,N) + SV(P/F,i%,N)
= -18,000 + 3,850(P/A,10%,10) - 1,000(P/F,10%,10)
= $5,271

Conclusion: Project B is better than project A

If A, B and C are Mutually Exclusive Projects

- Projects A and B have a common life (= 10 years).
- Therefore, the better project can be determined on the basis of a 10-year period of analysis.

$$FW(A) = -P(F/P,i\%,N) + A(F/A,i\%,N) + SV$$
$$= -10{,}000(F/P,10\%,10)+2{,}000(F/A,10\%,10) + 0$$
= **<u>$5,937</u>**

$$FW(B) = -P(F/P,i\%,N)+ A(F/A,i\%,N) + SV$$
$$= -18{,}000(F/P,10\%,10)+3{,}850(F/A,10\%,10)-1{,}000$$
= **<u>$13,672</u>**

If A, B and C are Mutually Exclusive Projects

Compare projects B and C over 20 years

PW(B) = -P[1 + P(P/F,i%,10)] + A(P/A,i%,20)
+ SV[(P/F,i%,10) + (P/F,i%,20)]
= -18,000[1+(P/F,10%,10)]+ 3,850(P/A,10%,20)
- 1,000[(P/F,10%,10)+(P/F,10%,20)]
= $7,303

PW(C) = -P + A(P/A,i%,N) + G(P/G,i%,N) + SV(P/F,i%,N)
= -30,000 + 2,800(P/A,10%,20) + 400(F/G,10%,20)
+ 3,000(P/F,10%,20)
= $16,447

Conclusion: Project C is better than project B

If A, B and C are Mutually Exclusive Projects

<u>Compare projects B and C over 20 years</u>

FW(B) = -P[(F/P,i%,20)+(F/P,i%,10)] +A(F/A,i%,20)+SV[1+(F/P,i%,N)]
= -18,000[(F/P,10%,20)+(F/P,10%,10)]
+ 3,850(F/A,10%,20) - 1,000[1+(F/P,10%,10)]
= <u>$49,131</u>

FW(C) = -P(F/P,i%,N) + A(F/A,i%,N) + G(F/G,i%,N) + SV
= -30,000(F/P,10%,20) + 2,800(F/A,10%,20)
+ 400(P/G,10%,20) + 3,000
= <u>$110,642</u>

Conclusion: Project C is better than project B

Annual Equivalent Worth (AEW)

AEW(A) = -P(A/P,i%,N) + A + SV(A/F,i%,N)
= -10,000(A/P,10%,10) + 2,000 + 0(A/F,10%,10)
= <u>$373</u>

AEW(B) = -P(A/P,i%,N) + A + SV(A/F,i%,N)
= -18,000(A/P,10%,10) + 3,850 - 1,000(A/F,10%,10)
= <u>$858</u>

AEW(C) = -P(A/P,i%,N) + A + G(A/G,i%,N) + SV(A/F,i%,N)
= -30,000(A/P,10%,20) + 2,800 + 400(A/G,10%,20)
+3,000(A/F,10%,20)
= <u>$1,932</u>

Annual Equivalent Worth (AEW)

Note:

- The AEW method DOES NOT require that a **common period of analysis** to determine neither the acceptability nor the better (best) mutually exclusive project.
- The underlying assumption is that projects can be repeated, as required, without any change to their respective parameters (P, SV ….)

Annual Equivalent Worth (AEW)

Decision:

- If Projects A, B and C are independent, select the three projects because their AEWs > 0 (without capital rationing).
- If projects A, B and C are mutually exclusive, select Project C (the largest project) because it has the largest non-negative AEW.

Project A: Simple Payback

Year	Cash Flow	Project Balance
0	($10,000)	($10,000)
1	2,000	(8,000)
2	2,000	(6,000)
3	2,000	(4,000)
4	2,000	(2,000)
5	2,000	0
6	2,000	+2,000

RECOVERY PERIOD = 10000 ÷ 2000 = 5 years

Project B: Simple Payback

Year	Cash Flow	Project Balance
0	($10,000)	($18,000)
1	3,850	(14,150)
2	3,850	(10,300)
3	3,850	(6,450)
4	3,850	**(2,600)**
5	3,850	**+1,250**
6	3,850	+5,100

RECOVERY PERIOD = 4+(2600 ÷3850) = 4.68 years

Project C: Simple Payback

Year	**Cash Flow**	Project Balance
0	($30,000)	(30,000)
1	2,800	(27,200)
2	3,200	(24,000)
3	3,600	(20,400)
4	4,000	(16,400)
5	4,400	(12,000)
6	4,800	(7,200)
7	5,200	**(2,000)**
8	5,600	**+3,600**

Recovery period = 7 + (2000÷5600) = 7.36 years

Project A: Discounted Payback

Year	Cash Flow	Opportunity Cost MARR=10%	Project Balance
0	($10,000)		($10,000)
1	2,000	($1,000)	(9,000)
2	2,000	(900)	(7,900)
3	2,000	(669)	(6,690)
4	2,000	(536)	(5,359)
5	2,000	(389)	(3,895)
6	2,000	(228)	(2,284)
7	2,000	(51)	**(513)**
8	2,000	+144	**+1,436**

Recovery Period = 7 + 513 ÷ 1949 = 7.3 years

Project B: Discounted Payback

Year	Cash Flow	Opportunity Cost of -ve Project Balance	Project Balance
0	($10,000)		($18,000)
2	3,850	($1,595)	(15,950)
3	3,850	(1,370)	(13,695)
4	3,850	(1,121)	(11,215)
5	3,850	(849)	(8,486)
6	3,850	(548)	(5,485)
7	3,850	(218)	**(2,183)**
8	3,850	(145)	**+1,449**

Recovery Period = 7+(2183÷3632) = 7.6 years

Project C: Discounted Payback

Year	Cash Flow	Opportunity Cost of -ve Project Balance	Project Balance
0	($30,000)		($30,000)
1	2,800	($3,000)	(30,200)
2	3,200	(3,020)	(30,020)
3	3,600	(3,002)	(29,422)
4	4,000	(2,942)	(28,364)
5	4,400	(2,836)	(26,801)
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11	6,800	(944)	**(3,582)**
12	7,200	(358)	**+3,260**

Recovery Period = 11+(3582÷6842) = 11.52 years

Summary Measures: Decisions

Summary Measures	**Acceptable Independent Projects**	**Best Mutually Exclusive Project**
Present Worth	A, B and C	C
Future Worth	A, B and C	C
Annual Equivalent	A, B and C	C
IRR	A, B and C	C
ERR	A, B and C	C
Simple Payback (Standard = 7 yrs)	**A & B**	**B**
Discounted Payback (Standard = 8 yrs)	**A & B**	**A**

Consistent Decision-making

- Single Sum (PW and FW) and EAW methods <u>ALWAYS</u> lead to the same decision as to
  - Acceptable **independent** projects
  - Better (best) **mutually exclusive** project
- Project decisions with the payback methods (simple and discounted) need not lead to the same conclusions because
  - their focus is on liquidity not profitability
  - they exclude some project information (e.g., salvage value) in the decision-making process.

Next Week: Lecture 4

Project decision-making based on:

1. Internal Rates of Return (IRR)
2. External Rates of Return (ERR)

Engineering Economics
ECO 1192

Topic 3: Single sums, Annuities and Payback Methods

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