

Carleton University – Faculty of Engineering  
**ECOR 2050**

Instructor: S. Shams

Assignment #1- Solutions

Due: Friday, May 22, 4:30 pm

SHOW ALL CALCULATIONS & CLEARLY STATE ANY/ALL ASSUMPTIONS!!!

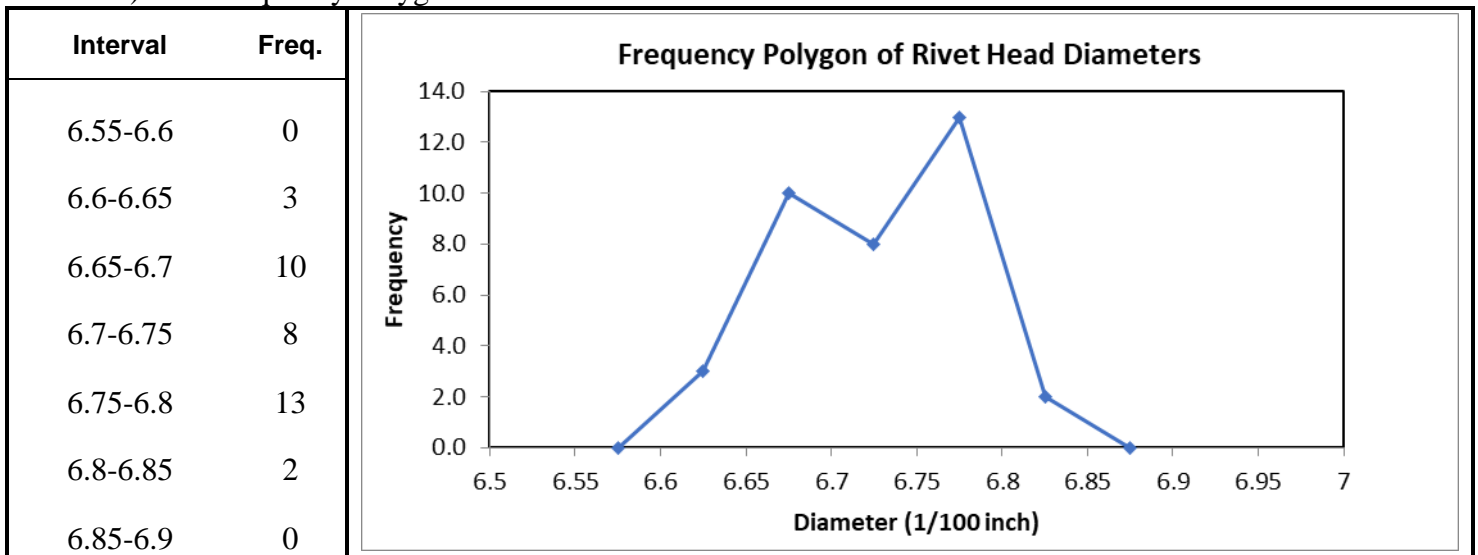
1. (14 marks)

The following data are the measures of the diameters of 36 rivet heads in 1/100 of an inch.

6.72 6.77 6.82 6.70 6.78 6.70 6.62 6.75 6.66 6.66 6.64 6.76  
 6.73 6.80 6.72 6.76 6.76 6.68 6.66 6.62 6.72 6.76 6.70 6.78  
 6.76 6.67 6.70 6.72 6.74 6.81 6.79 6.78 6.66 6.76 6.76 6.72

- a) Provide a **report-quality** frequency polygon of these data along with a table summarizing the grouped data.
- b) Compute the 10<sup>th</sup> percentile of the sample.
- c) Calculate the mean, variance, and coefficient of variation of the diameters.

a) Frequency Polygon



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b)  $n = 36$ ,  $p = 0.10$ ,  $(n+1)p = 3.7$ ,  $k = 3$ ,  $d = 0.7$ ,  $x_k = 6.64$ ,  $x_{k+1} = 6.66$   
 $Q_{0.10} = 6.64 + 0.7 \cdot (6.66 - 6.64) = 6.654$  The 10<sup>th</sup> percentile is 6.654 (1/100 inch).

c)  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \approx 6.726$  The sample mean diameter is 6.726 (1/100 inch).

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \approx 0.002870$$
 The sample variance is 0.002870 (1/100 inch)<sup>2</sup>.

$$COV = \frac{s}{\bar{x}} \times 100\% \approx 0.7965\%$$
 The sample coefficient of variation is 0.7965%.

2. (6 marks)

A collection of 7 artworks contains 2 non-original items. An art collector would like to buy 3 artworks (by random) from this collection.  $X$  is the number of non-original items purchased by the art collector, and the probability distribution of  $X$  in tabular form is as follows:

$x$	0	1	2
$f(x)$	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{1}{7}$

a) What is the probability  $X$  is greater than 0?

b) Find mean and variance of  $X$ .

**Solution:**

a)

$$F(X > 0) = 1 - f(X \leq 0) = 1 - \frac{2}{7} = \frac{5}{7}$$

OR:

$$F(X > 0) = F(X=1) + F(X=2) = \frac{4}{7} + \frac{1}{7} = \frac{5}{7}$$

b)

$$\text{Mean: } E[X] = \sum xf(x) = (0)\left(\frac{2}{7}\right) + (1)\left(\frac{4}{7}\right) + (2)\left(\frac{1}{7}\right) = \frac{6}{7}$$

$$\text{Variance: } \text{VAR}[X] = E[X^2] - (E[X])^2 = (0)\left(\frac{2}{7}\right) + (1)\left(\frac{4}{7}\right) + (4)\left(\frac{1}{7}\right) - \left(\frac{6}{7}\right)^2 = \frac{20}{49}$$

3. (12 marks)

Given a normal distribution with  $\mu = 30$  and  $\sigma = 6$ , find

(a) the normal curve area between  $x = 32$  and  $x = 41$ ;

(b) the value of  $x$  that has 80% of the normal curve area to the left;

(c) the two values of  $x$  that contain the middle 75% of the normal curve area.

**Solution:**

NORMAL,  $\mu = 30$ ,  $\sigma = 6$

$$\begin{aligned} \text{a) } P(32 < X < 41) &= P(Z < (41-30)/6) - P(Z < (32-30)/6) \\ &= P(Z < 1.83) - P(Z < 0.33) \\ &= 0.9664 - 0.6293 \\ &= 0.3371 \end{aligned} \quad \therefore P(32 < X < 41) = 0.3371$$

$$\begin{aligned} \text{b) } P(X < x) = 0.8 &\rightarrow P(Z < (x-30)/6) = P(Z < 0.84) \\ &\rightarrow x = 6(0.84) + 30 = 35.04 \end{aligned} \quad \therefore 80^{\text{th}} \text{ percentile is } 35.04$$

$$\begin{aligned} \text{c) } P(X < x) = 0.125 &\rightarrow P(Z < (x-30)/6) \\ \text{From z-table: } z &= -1.15 \\ &\rightarrow x = 6(-1.15) + 30 = 23.10 \end{aligned}$$

$$\begin{aligned} P(X < x) = 0.875 &\rightarrow P(Z < (x-30)/6) \\ \text{From z-table: } z &= 1.15 \\ &\rightarrow x = 6(1.15) + 30 = 36.90 \end{aligned}$$

$\therefore$  the middle 75% of the normal curve lies between  $x = 23.10$  and  $x = 36.90$

4. (8 marks)

The average grade for an exam is 74, and the standard deviation is 7.

Grades are curved to follow a normal distribution and 12% of the class are given **A** s.

a) What is the lowest possible **A** and the highest possible **B**?

b) What is the 60<sup>th</sup> percentile grade?

Solution

a)  $P(X > x) = 0.12$

$$P(X < x) = 1 - P(X > x) = 1 - 0.12 = 0.88$$

$$\rightarrow P(Z < (x-74)/7) = 0.88$$

From z-table:  $z = 1.18$

$$\rightarrow x = 7(1.18) + 74 = 82.26$$

Therefore, the lowest A is 83 and the highest B is 82.

b)  $P(X < x) = 0.6$

$$\rightarrow P(Z < (x-74)/7) = 0.6$$

From z-table:  $z = 0.25$

$$\rightarrow x = 7(0.25) + 74 = 75.75$$

Therefore, 60% of the grades are 75 or less.