FINAL EXAM-MATH 1300
FALL TERM, 2017

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Please circle the name of your professor above.

First Name:________________________ **Last Name:**________________________

I.D. Number____________________________________

Instructions-This final examination consists of 12 multiple choice questions worth 4 points each. Your answers to the multiple choice questions must be clearly marked in the squares below. There are also 4 long answer questions worth a total of 52 points. For the long answer questions, you must show your work **on the exam itself** and clearly display your answers. There are two blank pages at the end of the exam to be used for scrap paper.

NO CALCULATORS. NO BOOKS. NO NOTES.
TURN OFF YOUR CELL PHONES AND PUT THEM AWAY.

Multiple Choice ANSWERS:

E	A	B	C
#1	#2	#3	#4
D	D	A	A
#5	#6	#7	#8
D	E	C	D
#9	#10	#11	#12

PLEASE READ THE FOLLOWING CAREFULLY AND SIGN BELOW

Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to immediately leave the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam or worse.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

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SIGN HERE

Question 1- Find the inverse of the following function:

$$f(x) = \frac{x-3}{x+4}$$

A) $\frac{2x-3}{x+4}$ B) $\frac{4x+3}{x-4}$ C) $\frac{2x+3}{x-1}$ D) $\frac{x+3}{x+4}$ E) $\frac{4x+3}{1-x}$ ✓

$y = \frac{x-3}{x+4}$ $\quad x \neq -4$

$y(x+4) = x - 3$

$yx + 4y = x - 3$

$yx - x = -4y - 3$

$x(y-1) = -4y-3$

$y \neq 1$

$\rightarrow x = \frac{-4y-3}{y-1}$

$y = \frac{-4x-3}{x-1}$

$y = \frac{4x+3}{1-x}$

Question 2-Over what interval is the function $f(x) = xe^{2x}$ concave up?

A) $(-1, \infty)$ ✓ B) $(-\infty, -1)$ C) $(-\infty, 0)$ D) $(0, 1)$ E) This function is always concave up.

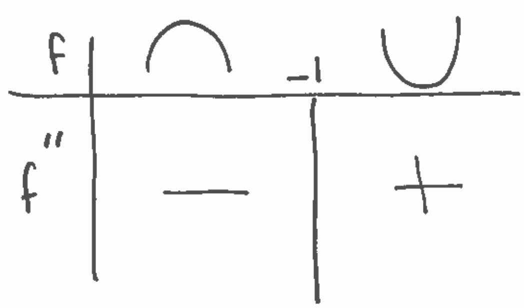
$f'(x) = e^{2x} + x2e^{2x} = e^{2x}(1+2x)$

$f''(x) = 2e^{2x}(1+2x) + 2e^{2x} = 2e^{2x}(1+2x+1)$

$= \underbrace{2e^{2x}}_{\neq 0}(2+2x) = 0$

$2 = -2x$

$x = -1$



Question 3- Suppose the demand function for a product is given by $p(x) = 40 - 2\sqrt{x}$. Find the elasticity of demand when $x = 9$. Is demand elastic?

A) $-\frac{1}{9}$, elastic B) $-\frac{34}{3}$, elastic C) $-\frac{34}{3}$, inelastic D) $-\frac{1}{9}$, inelastic E) $-\frac{3}{50}$, inelastic

$p'(x) = \frac{-2}{2\sqrt{x}} = \frac{-1}{\sqrt{x}}$

$$\eta = \frac{p(x)}{x\,p'(x)} = \frac{40 - 2\sqrt{x}}{\left(\frac{-1}{\sqrt{x}}\right)x}$$

@ $x=9$ $\eta = \frac{40-6}{\left(\frac{-1}{3}\right)9} = \frac{34}{-3}$

$|\eta| \geq 1 \rightarrow$ elastic.

Question 4- Use implicit differentiation to find $\frac{dy}{dx}$ at $(1, 1)$ when

$$(x^3y - 3xy^2 + 4x - y = 1)'$$

A) $\frac{9}{2}$ B) $-\frac{1}{2}$ C) $\frac{2}{3}$ D) -2 E) 1

$$3x^2y + x^3y' - 3y^2 - 6xyy' + 4 - y' = 0$$

@ $(1,1)$

$$3 + y' - 3 - 6y' + 4 - y' = 0$$

$$-6y' = -4 \qquad y' = 2/3$$

Question 5-Calculate the following limit.

$$\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}$$

A) 1 B) 3 C) 5 ~~D)~~ 2 E) This limit does not exist.

$$\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \lim_{x\to 1} \frac{(x-1)(\sqrt{x}+1)}{(x-1)} = \lim_{x\to 1} \sqrt{x}+1 = 2.$$

Question 6- Evaluate the following improper integral:

$$\int_1^\infty \frac{3}{\sqrt[3]{x}}\,dx$$

A) 0 B) $\frac{1}{2}$ C) $\frac{3}{2}$ D) The integral diverges. E) $\frac{5}{2}$

$$\int_1^\infty 3(x)^{-1/3}\,dx = \lim_{d\to\infty} 3\int_1^d x^{-1/3}\,dx$$

$$= \lim_{d\to\infty} 3\left(\frac{3}{2}x^{2/3}\Big|_1^d\right)$$

$$= \lim_{d\to\infty} \frac{9}{2}\left(d^{2/3}-1\right) = \text{DNE}.$$

Question 7- Calculate the following definite integral

$$\int_0^2 \frac{x}{x^2+1}dx$$

A) $\frac{1}{2}\ln(5)$ ✓ B) $\ln(5)$ C) $\frac{1}{2}$ D) $2\ln(5)$ E) 1

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$x = 0 \rightarrow u = 1$

$x = 2 \rightarrow u = 5$

$$\int_0^2 \frac{x\,dx}{x^2+1} = \int_1^5 \frac{\frac{du}{2}}{u} =$$

$$\frac{1}{2}\int_1^5 \frac{du}{u} = \frac{1}{2}\left(\ln|u|\Big|_1^5\right) =$$

$$\frac{1}{2}(\ln 5 - \underbrace{\cancel{\ln 1}}_{0}) = \frac{1}{2}\ln 5$$

Question 8- Calculate:

$$\int_0^{\ln(2)} xe^x dx$$

A) $2\ln(2) - 1$ ✓ B) $2\ln(2)$ C) $3\ln(2)$ D) $\frac{\ln(2)}{2}$ E) $\frac{1}{2}$

$u = x$

$u' = 1$

$v' = e^x$

$v = e^x$

$$\int_0^{\ln 2} xe^x = x \cdot e^x \Big|_0^{\ln 2} - \int_0^{\ln 2} (1)\, e^x\,dx$$

$$= (\ln 2 \cdot \underbrace{e^{\ln 2}}_{2} - 0) - \left(e^x \Big|_0^{\ln 2}\right)$$

$$= (2\ln 2) - (e^{\ln 2} - 1) =$$

$$2\ln 2 - (2-1) = 2\ln 2 - 1$$

Question 9- Find $f(0)$, when $f'(x) = x^{\frac{1}{3}} - 2$ and $f(8) = 4$.

A) 5 B) 6 C) 7 D) 8 E) 9

$f(x) = \int (x^{1/3} - 2)\,dx = \frac{3}{4} x^{4/3} - 2x + C$

$f(8) = 4$

$\frac{3}{4} \underbrace{(8)^{4/3}}_{\underbrace{(2^3)^{4/3}}_{2^4 = 16}} - 16 + C = 4$

$\underbrace{\frac{3}{4}(16)}_{12} - 16 + C = 4$ $\boxed{C = 8}$

$f(0) = 8$

Question 10- Suppose that for a certain product, the demand function is given by $D(x) = 12 - x$ and the supply function is given by $S(x) = x^2$. Calculate the producer surplus.

A) 7 B) 12 C) $\frac{2}{3}$ D) $\frac{8}{3}$ E) 18

$D(x) = S(x)$ $12 - x = x^2$

$x^2 + x - 12 = 0$ $(x+4)(x-3) = 0$

$x = 3$ ✓ $x = -4$

$x = 3 \Rightarrow D(3) = 12 - 3 = 9$

$PS = \int_0^3 (9 - x^2)\,dx = 9x - \frac{x^3}{3}\Big|_0^3 = (27 - 9) =$

18

Question 11-How many critical points does the following function of 2 variables have?

$$g(x,y) = -3x^2 - 3y + x^3 + y^3 + 14$$

A) 2 B) 3 C) 4 D) 1 E) 0

$g_x = -6x + 3x^2 = -3x(2-x) = 0$ $x = 0$, $x = 2$

$g_y = -3 + 3y^2 = 0$ $y^2 = 1$ $y = \pm 1$

$(0,1)$ $(2,1)$

$(0,-1)$ $(2,-1)$

Question 12- Suppose $f(x,y) = e^{xy^2+1}$. Find $f_{xx}(1,2)$.

A) $5e^5$ B) $3e^5$ C) $6e^5$ D) $16e^5$ E) $2e^5$

$f_x = y^2 e^{xy^2+1}$

$f_{xx} = y^2(y^2)e^{xy^2+1} = y^4 e^{xy^2+1}$

$f_{xx}(1,2) = (2)^4 e^{(1)(4)+1} = 16e^5$

Long Answer Question 1 (8 points)
For the following 2 parts, do not simplify your answers:

~~8~~ 2 pts **Part 1**: Suppose I invest 30,000 dollars and that interest is compounded continuously. After 4 years, I have 35,000 dollars. What must the interest rate have been?

$P_0 = 30000$ $\qquad A(t) = P_0 e^{rt}$

$A(4) = 35000 = 30000 e^{r(4)}$

$\frac{35}{30} = e^{r4}$ $\qquad \ln(\frac{7}{6}) = 4r$

$r = \ln(7/6)/4$

~~8~~ 6 pts **Part 2**: The number of bacteria in a certain culture is 20,000 at 6am. At 10am, the count is 80,000. Assume the population is growing exponentially. Determine the amount of time it takes for the population to reach 100,000.

$t=0$ 6am $P_0 = 20\,000$

$t=4$ 10am $P(4) = 80000$

$t=?$ $P(?) = 100\,000$

$P(t) = P_0 e^{Kt}$

$8\times10^4 = 2\times10^4 e^{4K}$

$4 = e^{4K}$ $\qquad \ln 4 = 4K$

$K = \frac{\ln 4}{4}.$

3 pts for K

3 pts for t

$P(t) = 10^5 = 2\times10^4 \times e^{t\cdot\frac{\ln 4}{4}}$

$5 = e^{t\frac{\ln 4}{4}}$

$\ln 5 = t\frac{\ln 4}{4}$ $\qquad t = \frac{4\ln 5}{\ln 4}$

Long Answer Question 2 (14 points)

7 pts [a] Evaluate the following indefinite integral:

2 pts

$$\boxed{\int x^5 \ln(x)\,dx = \ln x \cdot \frac{x^6}{6} - \int \frac{1}{x}\cdot\frac{x^6}{6}\,dx}$$

$$\boxed{\begin{aligned} u &= \ln x \\ u' &= \frac{1}{x} \\ v' &= x^5 \\ v &= \frac{x^6}{6} \end{aligned}}$$

~~8~~ 4 pts

$$= \ln x \cdot \frac{x^6}{6} - \frac{1}{6}\int x^5\,dx$$

1 pt.

$$= \ln x \cdot \frac{x^6}{6} - \frac{1}{6}\left(\frac{1}{6}x^6\right) + C$$

$$= \ln x \cdot \frac{x^6}{6} - \frac{1}{36}x^6 + C.$$

But they lose a point if they forget +C

7 pts [b] Evaluate the following indefinite integral:

$$\int \frac{1}{x\ln(x)}\,dx = \int \frac{1}{\ln x}\cdot\frac{1}{x}\,dx$$

$$\boxed{\begin{aligned} u &= \ln x \\ du &= \frac{1}{x}\,dx \end{aligned}}$$

4 pts

$$= \underbrace{\int \frac{1}{u}\,du}_{2\text{ pts}} = \ln|u| + C$$

$$= \underbrace{\ln|\ln x|}_{1\text{ pts.}} + C.$$

only take off 1 point even if they forget +C twice

Long Answer Question 3 (14 points)

Consider the two functions:

$f(x) = 4 - x^2$ and $g(x) = x^2 - 4$

(a) **(2 points)** Find the intersection points of the graphs of the two functions.

(b) **(4 points)** On the next page, graph these functions, and shade the region between the graphs of f and g for x such that $0 \le x \le 3$. 1 pt for each graph 2 pts for shading

(c) **(8 points)** Find the area of the shaded region.

$f(x) = g(x)$

$4 - x^2 = x^2 - 4 \Rightarrow 2x^2 - 8 = 0 \quad x^2 - 4 = 0$

$x^2 = 4 \quad x = \pm 2$

3 pts

$A_1 = \int_0^2 (4 - x^2) - (x^2 - 4)\, dx = \int_0^2 (8 - 2x^2)\, dx$

0 pts if they write $\int_0^4$

$= 8x - \frac{2x^3}{3} \Big|_0^2 = 16 - \frac{16}{3} = \frac{32}{3}$

3 pts

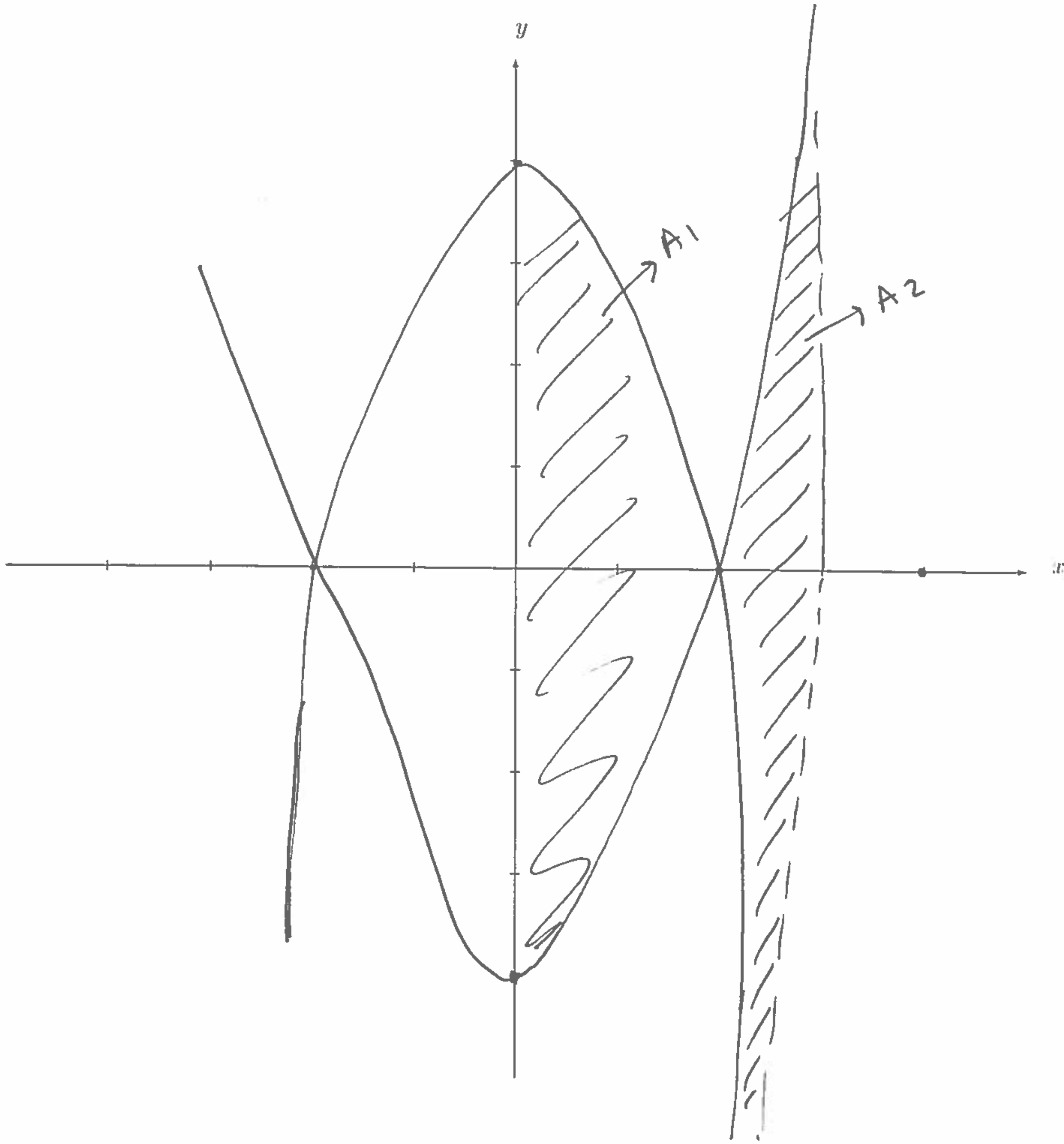
$A2 = \int_2^3 (x^2 - 4) - (4 - x^2) = \int_2^3 (2x^2 - 8)\, dx$

2 pts for math nethe

$= \left(\frac{2}{3}x^3 - 8x\right)\Big|_2^3 = \left(\frac{2}{3}(27) - 24\right) - \left(\frac{16}{3} - 16\right)$

$= 18 - 24 + 16 - \frac{16}{3} = \frac{14}{3}$

$\text{Area} = \frac{32}{3} + \frac{14}{3} = \frac{46}{3} =$



y
x
A1
A2

Long Answer Question 4 (16 points)
Consider the function of two variables

$$f(x, y) = \frac{1}{3}y^3 + x^2 - 2xy - 15y + 12$$

(a) **(2 points)** Calculate the first-order partial derivatives.

(b) **(6 points)** Find all critical points.

(c) **(8 points)** Identify what type of critical points they are (local max, local min or saddle point).

a) $\begin{cases} f_x = 2x - 2y & 1pt \\ f_y = y^2 - 2x - 15 & 1pt \end{cases}$

b) $\begin{cases} ① f_x = 0 \\ ② f_y = 0 \end{cases}$

2 pts for knowing this

$2x = 2y \Rightarrow x = y$ ③

sub ③ in ②

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = 5 \quad x = -3$

from ③ $x = y \Rightarrow (5,5)$
$(-3,-3)$
critical points

2 pts for each CP

c) 2 pts
$\boxed{\begin{matrix} f_{xx} = 2 & f_{yy} = 2y \\ f_{xy} = -2 & f_{yx} = -2 \end{matrix}}$

$\boxed{D(x,y) = 4y - (-2)^2 = 4y - 4}$

$D(5,5) = 20 - 4 > 0 \rightarrow f_{xx} > 0$ 2 pts $\boxed{(5,5) \text{ local Min}}$

$D(-3,-3) = -12 - 4 = -16 < 0$ 2 pts $\boxed{(-3,-3) \text{ saddle point}}$

2 pts

Extra page for additional work

Extra page for additional work