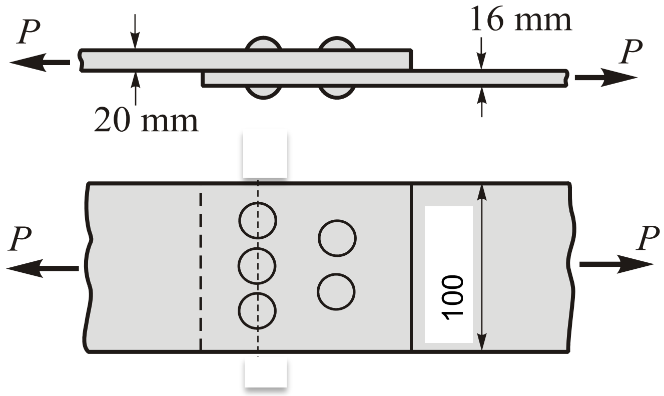
Solutions (there are always different solutions to the same problem)

Question 1: (3 questions)

The lap joint seen in the figure is fastened by five 25 mm diameter rivets. For P = 50 kN, determine (i) the maximum shear stress in the rivets, (ii) the maximum bearing stress in the plate and (iii) the maximum tensile stress. (Assume that the load is divided equally among the rivets)



a) Maximum shear stress:

$$\tau_{max} = \frac{P}{A} = \frac{50 \times 10^3}{5\left(\pi\left(\frac{d}{2}\right)^2\right)} = \frac{50 \times 10^3 N}{5\left(\pi\left(\frac{25}{2}\right)^2 mm^2\right)} = 20.4\ MPa$$

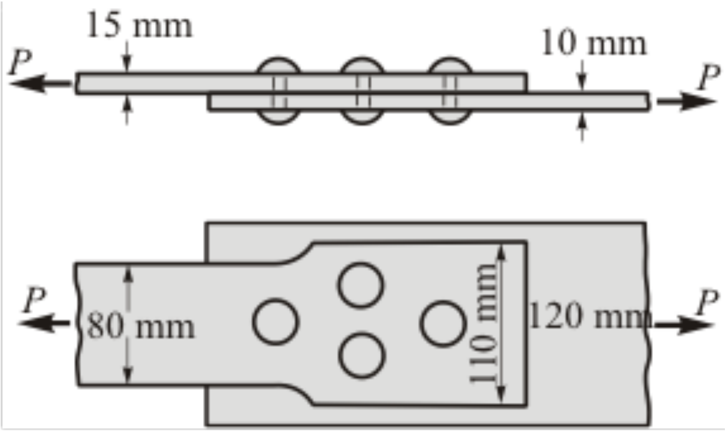
b) Maximum bearing stress:

$$\sigma_{b(max)} = \frac{P}{A_b} = \frac{50 \times 10^3 N}{5dt} = \frac{50 \times 10^3 N}{5(16 \times 25\ mm^2)} = 25\ MPa$$

a) Maximum tensile stress:

$$\sigma_{(max)} = \frac{P}{A} = \frac{50 \times 10^3 N}{(100 - 3 \times 25)(16)} = \frac{50 \times 10^3 N}{400\ mm^2} = 125\ MPa$$

Two plates are joined by four rivets of 20-mm diameter, as shown in the figure. Determine the maximum load P if the shearing, tensile, and bearing stresses are limited to 80, 100, and 140 MPa, respectively. Assume that the load is equally divided among the rivets.



c) Maximum load based on maximum shear stress: (Single shear)

$$\tau_{max} = \frac{P}{A} = \frac{P}{4\left(\pi\left(\frac{d}{2}\right)^2\right)} \rightarrow P = (80)(4)\left(\frac{\pi(20)^2}{4}\right) = 101.48\ kN$$

d) Maximum load based on maximum bearing stress:

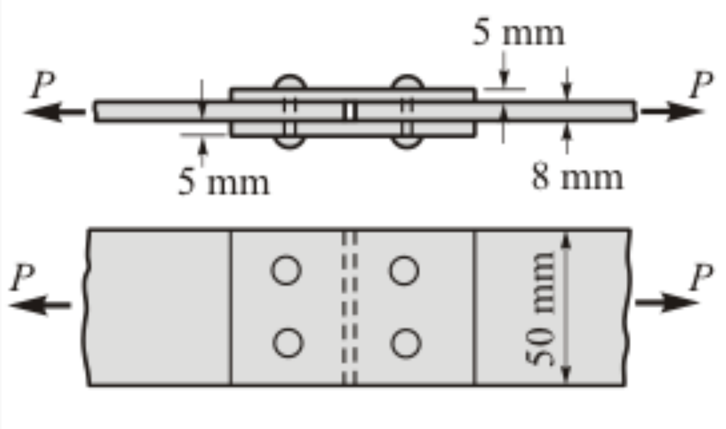
$$\sigma_{b(max)} = \frac{P}{A_b} = \frac{P}{4(dt)} \rightarrow P = (140)(4)(20 \times 15) = 168\ kN$$

b) Maximum load based on maximum tensile stress:

$$\sigma_{(max)} = \frac{P}{A} = \frac{P}{(80-20)(15)} \rightarrow P = (100)(80-20)(15) = 90\ kN$$

The maximum load (for all stresses) is 90 kN.

The butt joint shown in the figure is fastened by four 18 mm diameter rivets. Determine the maximum load P if the stresses are not to exceed 100 MPa in shear, 140 MPa in tension, and 200 MPa in bearing. Assume that the load is equally divided among the rivets.



e) Maximum load based on maximum shear stress: (Double shear)

$$\tau_{max} = \frac{P}{2A} = \frac{P}{2 \times 4\left(\pi \left(\frac{d}{2}\right)^2\right)} \rightarrow P = (100)(8)\left(\frac{\pi(18)^2}{4}\right) = 203.47\ kN$$

f) Maximum load based on maximum bearing stress:

$$\sigma_{b(max)} = \frac{P}{A_b} = \frac{P}{4(dt)} \rightarrow P = (200)(4)(18 \times 8) = 115.2\ kN$$

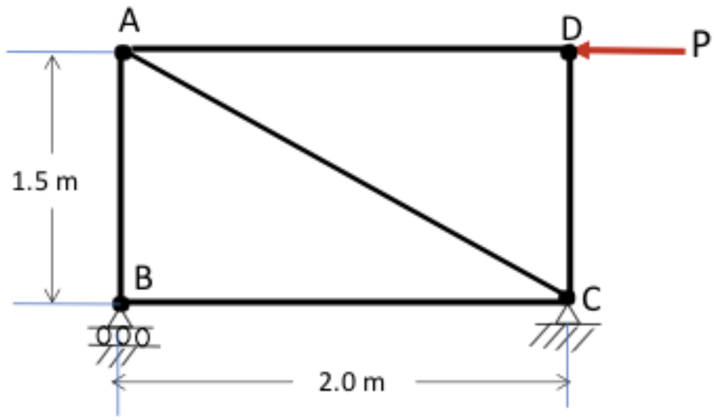
c) Maximum load based on maximum tensile stress:

$$\sigma_{(max)} = \frac{P}{A} = \frac{P}{(50 - 2x18)(8)} \rightarrow P = (140)(50 - 36)(8) = 15.68\ kN$$

The maximum load (for all stresses) is 15.68 kN.

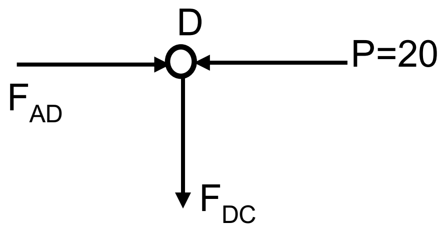
Question 2: (3 questions)

For the truss made of steel bars as shown in the figure, determine the horizontal displacement of joint *D*. Each member has a cross-sectional area of 2000 mm^2. P = 20 kN. E = 200 GPa



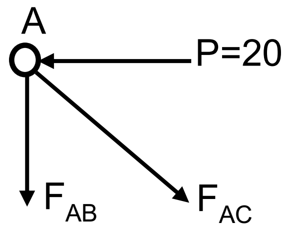
$$Joint\ D: \quad \sum F_x = 0,\ F_{AD} = 20\ kN\ (Compression)$$

$$\sum F_y = 0,\ F_{DC} = 0\ (zero\ force\ member)$$



$$Joint\ A: \quad \sum F_x = 0,\ \frac{4}{5}F_{AC} - 20\ = 0,\ \ F_{AC} = 25\ kN$$

$$\sum F_y = 0,\ F_{AB} - \frac{3}{5}(25) = 0,\ F_{AB} = 15\ kN$$



$$Joint\ B: \sum\ F_x = 0, F_{BC}\ = 0, (zero\ force\ member)$$

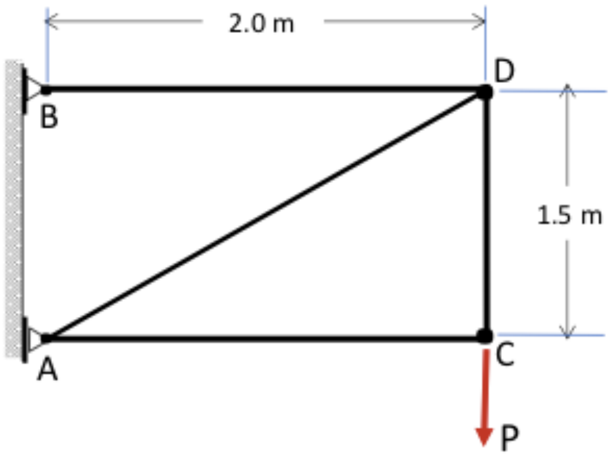
$$L_{AC} = \sqrt{1.5^2 + 2^2} = 2.5\ m$$

Member	P (kN)	L (m)	P^2L (kN^2.m)
AD **AC** **AB**	20 25 15	2 2.5 1.5	800 1562.5 337.5
			$\sum P^2L = 2700$

$$U = \sum \frac{P^2L}{2EA} = 1.5625\, J$$

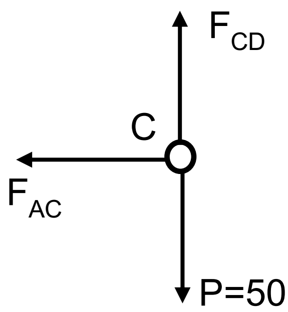
$$\frac{1}{2}P\delta_D = 1.5625 \rightarrow \delta_D = 0.1562\, mm$$

For the truss made of steel bars as shown in the figure, determine the vertical displacement of joint *C*. Each member has a cross-sectional area of 2400 mm^2. P = 50 kN. E = 200 GPa



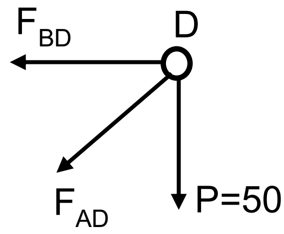
$$Joint\ C: \quad \sum F_x = 0,\ \ F_{AC} = 0$$

$$\sum F_y = 0,\ \ F_{CD} - P = 0\ , F_{CD} = 50\ kN$$



$$Joint\ D: \quad \sum F_y = 0,\ \frac{3}{5}F_{AD} - P\ = 0,\quad F_{AD} = 83.33\ kN$$

$$\sum F_x = 0,\ \frac{4}{5}F_{AD} - F_{BD}\ = 0,\quad F_{BD} = 66.66\ kN$$



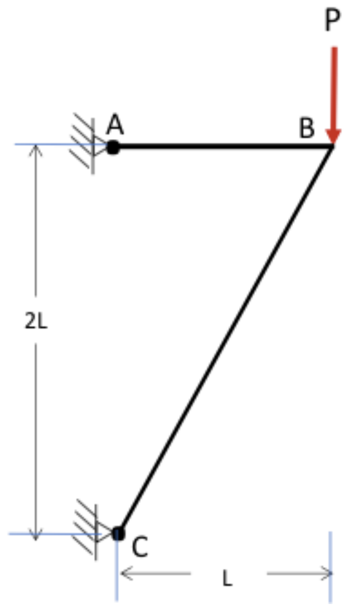
$$L_{AD} = \sqrt{1.5^2 + 2^2} = 2.5\ m$$

Member	P (kN)	L (m)	P^2L ($kN^2.m$)
AD **BD** **CD**	83.33 66.66 50	2.5 2 1.5	17359.72 8887.11 3750
			$\sum P^2L = 29996.83$

$$U = \sum \frac{P^2L}{2EA} = 31.2466\ J$$

$$\frac{1}{2}P\delta_C = 31.2466\ \rightarrow \delta_C = 1.25\ mm$$

Determine the vertical displacement of joint B. Member AB is made of steel of cross-sectional area, A_{st}, and modulus of elasticity, E_{st}. Member BC is made of aluminum of cross-sectional area, A_{al} and modulus of elasticity, E_{al}. The applied load is P.



$$Joint\ B: \quad \sum F_x = 0, \quad \frac{P_{CB}}{\sqrt{5}} - P_{AB} = 0, \ P_{AB} = \frac{P_{CB}}{\sqrt{5}}$$

$$\sum F_y = 0, \quad \frac{2}{\sqrt{5}} P_{CB} - P = 0, \ P_{CB} = P\frac{\sqrt{5}}{2}$$

$$P_{AB} = \frac{P}{2}$$

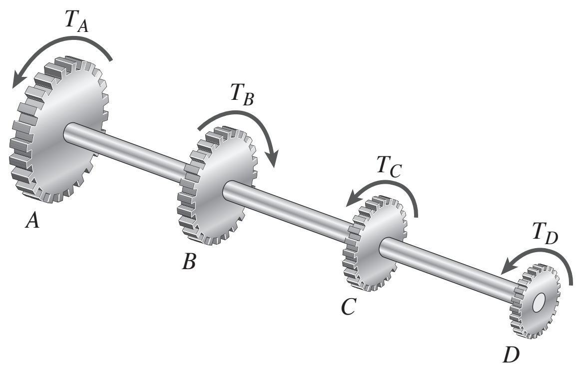
$$\frac{1}{2}P\delta = \sum \frac{P^2L}{2AE} = \frac{P_{AB}^2 L_{AB}}{2A_{AB}E_{AB}} + \frac{P_{CB}^2 L_{CB}}{2A_{CB}E_{CB}}$$

$$\frac{1}{2}P\delta = \frac{P^2L}{8A_{AB}E_{AB}} + \frac{5P^2\sqrt{5}L}{8A_{CB}E_{CB}}$$

$$\delta = P\left[\frac{L}{4A_{AB}E_{AB}} + \frac{5\sqrt{5}L}{4A_{CB}E_{CB}}\right]$$

Question 3: (4 questions)

A gear shaft transmits torques T_A = 1000 N.m, T_B = 1600 N.m, T_C = 750 N.m and T_D = 875 N.m. Determine the required shaft diameter if the allowable shear stress is 50 MPa.



T_{AB} = 1000 N.m

T_{BC} = -600 N.m

T_{CD} = 875 N.m

$$\tau = \frac{T\frac{d}{2}}{\frac{\pi}{32}d^4} = \frac{16d}{\pi d^3}$$

Segment AB: $d = \left(\frac{16T_{AB}}{\pi\tau}\right)^{\frac{1}{3}} = \left(\frac{16(1000\ N.m)}{\pi(50\ MPa)}\right)^{\frac{1}{3}} = 46.7\ mm$ (required)

Segment BC: $d = \left(\frac{16T_{AB}}{\pi\tau}\right)^{\frac{1}{3}} = \left(\frac{16(600\ N.m)}{\pi(50\ MPa)}\right)^{\frac{1}{3}} = 39.4\ mm$

Segment CD: $d = \left(\frac{16T_{AB}}{\pi\tau}\right)^{\frac{1}{3}} = \left(\frac{16(875\ N.m)}{\pi(50\ MPa)}\right)^{\frac{1}{3}} = 44.7\ mm$

A solid circular shaft of 40 mm diameter is to be replaced by a hollow circular tube. If the outside diameter of the tube is limited to 60 mm, what must be the thickness of the tube for the same linearly elastic material working at the same maximum stress.

$$\tau_{max} = \frac{Tc_1}{\frac{\pi}{2}c_1^4} = \frac{Tc_o}{\frac{\pi}{2}(c_o^4 - c_i^4)}, \qquad T(c_o^4 - c_i^4) = T\frac{c_o}{c_1}(c_1)^4$$

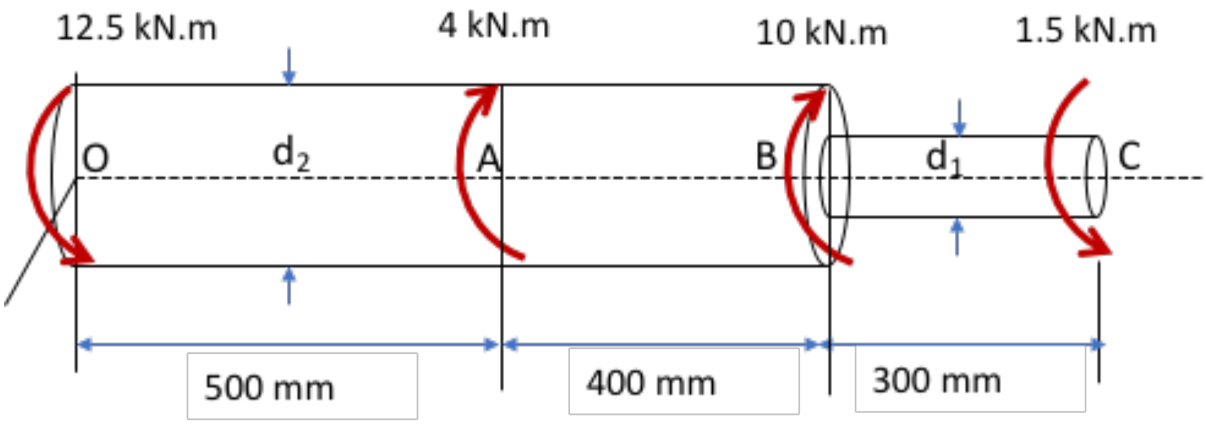
$$T(30^4 - c_i^4) = T\frac{30}{20}(20)^4, \quad c_i = 27.475\ mm$$

$$t = c_o - c_i = 30 - 27.475 = 2.525mm$$

The torsion member shown in the Figure is made of steel (G = 77.5 GPa) and is subjected to torsional loads as shown. Neglect the effect of stress concentrations and assume that the material remains elastic. Diameters d_1 = 25 mm and d_2 = 50 mm

a) Determine the maximum shear stress in the member.

b) Determine the angle of twist of sections A, B, and C relative to the left end O of the member



T_{OA} = -12.5 kN.m

T_{AB} = -8.5 kN.m

T_{BC} = 1.5 kN.m

Since $\boldsymbol{T_{OA}}$ is larger than $\boldsymbol{T_{AB}}$, the maximum shear stress in the segment OAB occurs in segment OA.

$$\tau_{OA} = \frac{T_{OA}\left(d_2/_2\right)}{\frac{\pi}{32}d_2^4} = \frac{(-12.5)(25)}{\frac{\pi}{32}(25^4)} = 509.55\ MPa\ \text{(max)}$$

$$\tau_{BC} = \frac{T_{BC}\left(d_1/_2\right)}{\frac{\pi}{32}d_1^4} = \frac{(1.5)(12.5)}{\frac{\pi}{32}(12.5^4)} = 489\ MPa$$

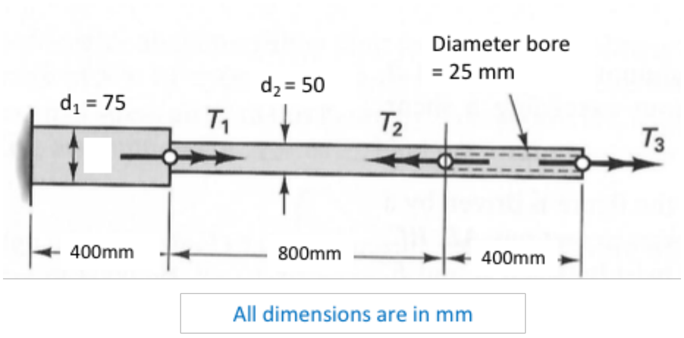
$$\phi_{OA} = \frac{T_{OA}L_{OA}}{I_{OA}G} = \frac{(-12.5\times 10^6\ N.mm)(500mm)}{\frac{\pi}{32}(50\ mm)^4(77.5\times 10^3 MPa)} = -0.131\ rad$$

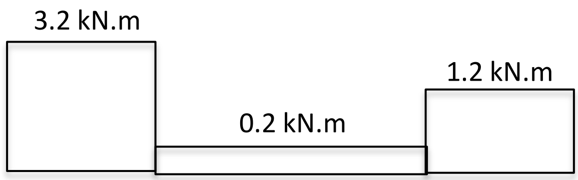
$$\phi_{AB} = \frac{T_{AB}L_{AB}}{I_{AB}G} = \frac{(-8.5\times10^6\ N.mm)(400mm)}{\frac{\pi}{32}(50\ mm)^4(77.5\times10^3MPa)} = -0.071\ rad$$

$$\phi_{BC} = \frac{T_{BC}L_{BC}}{I_{BC}G} = \frac{(1.5\times10^6\ N.mm)(300mm)}{\frac{\pi}{32}(50\ mm)^4(77.5\times10^3MPa)} = 0.151\ rad$$

$$\phi = \phi_{OA} + \phi_{AB} + \phi_{BC} = -0.131 - 0.071 + 0.151 = -0.051\ rad$$

A circular steel shaft of the dimensions shown in the figure is subjected to three torques: T_1 = 3 kN.m, T_2 = 1 kN.m and T_3= 1.2 kN.m. Determine the angle of twist of the right end due to the applied torques. G = 83 GPa





$$\phi = \phi_1 + \phi_2 + \phi_3$$

$$\phi = \frac{(3.2\times10^6 N.mm)(400mm)}{\frac{\pi}{32}(75)^4(83\times10^3MPa)} + \frac{(0.2\times10^6 N.mm)(800mm)}{\frac{\pi}{32}(50)^4(83\times10^3MPa)} + \frac{(1.2\times10^6 N.mm)(400mm)}{\frac{\pi}{32}(50^4-25^4)(83\times10^3MPa)}$$

$$\phi = 0.01816\ rad$$