Midterm Exam MATH 205
Differential and Integral Calculus II

Winter 2020 **March 8, 2020** **Time allowed: 1h 15min**

Problem 1.

(a) Derive the sigma notation formula for the right Riemann sum R_n of the function $f(x) = x^2 + 2x$ on the interval $[-2, 0]$ using n subintervals of equal length. Then calculate the integral $\int_{-2}^{0} f(x)dx$ as the limit of R_n at $n \to \infty$.

(b) Define $F(x) = \int_{x}^{x^2} \frac{dt}{\ln(t)}$ for $x \geq 2$. Calculate $F'(x)$ for $x \geq 2$. Is F increasing or decreasing at $x = 2020$?

Problem 2.

If $f'(x) = \frac{x+\arctan(x)}{x^2+1}$ and $f(0) = 1$,what is exact value of $f(1)$?

Problem 3.

Calculate the following indefinite integrals

(a) $\int \frac{3x^2+x+2}{x(x^2+1)} dx$

(b) $\int (1 + x^2)e^{-2x} dx$

Problem 4.

Find the average value of the function $f(x) = \sin^3(x)\cos^7(x)$ on the interval $[0, \pi/2]$.

Problem 5.

Evaluate the following definite integrals

(a) $\int_0^3 \frac{3x+4}{\sqrt{x+1}} dx$

(b) $\int_0^{\pi/3} \tan^3(x)\sec(x)dx$

Problem 6.

Find the volume of a solid obtained by rotating the region enclosed by the curve $y = 2 - \sqrt{x}$

and the lines $y = 2$ and $x = 4$ about the x-axis.

Bonus.

Let $F(x) = \left(1 + \int_1^x e^{-t^2}\,dt\right)^2$. Calculate $F''(x)$ to determine whether the graph of F is concave upward or concave downward at $x = 1$.