UNIVERSITÉ Concordia UNIVERSITY

COMP 233/2

Probability and Statistics for Computer Science

Week 5

Midterm #1 "Review" Abbreviated Solutions

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INSTRUCTIONS FOR MIDTERM #1: PLEASE READ CAREFULLY

- This midterm is based on material covered during Weeks 1 to 3.
- It contains 10 multiple choice questions (1 points each; no negative marking).
- The midterm will be available (strictly) on Friday, Oct. 16, 1:30pm-2:30pm. You will have 30 minutes to complete the midterm. Your question attempts will be automatically submitted at 2:30pm (no attempts after this time will be marked).
- You will only have one attempt to answer the midterm.
- Online search of materials is not permitted. You may reference the Midterm 1 Formulae Sheet (see Week 5).

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Topics Covered

Week	Topics Covered	Text Sections
Week 1	Introduction, Axioms of probability, Sample spaces having equally likely outcomes	§3.1-3.5
Week 2	Conditional probability, Bayes formula, Independent events	§3.6-3.8
Week 3	Random variables, Joint probability mass function	§4.1-4.3

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Elements of Probability

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Combinatorics

- Each of the 11 letters in `examination` is written on a sheet of paper. The sheets of paper are then shuffled so that each sequence of 11 sheets of paper are equally likely. What is the probability that after the shuffling, the order of the sheets spells the word "`examination`" correctly?

A) 11! B) 1/11! C) 8/11! D) 11!/2!2!2!

$C)$

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$$P(E) = \frac{N(E)}{N(S)}$$

Multiplication Rule

- Ruby picks 5 cards from a deck of 52 cards. We say that there are two distinct pairs (the hand "two pairs") if the 5 cards contain 2 cards of a first denomination (i.e., one of "A", "2", ..., or "K"), 2 cards of a different denomination, and a single card of third different denomination.
- What is the probability of Ruby getting two distinct pairs? Assume all 5-card hands are equally likely.

$$\frac{\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{44}{1}}{\binom{52}{5}}$$

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Conditional Probability

- Three cards are randomly chosen without replacement from an ordinary deck of 52 playing cards.
- Given that the ace of spades is chosen, what is the probability that all three cards are aces?

$$\frac{\binom{3}{2}}{\binom{51}{2}}$$

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Question

- Forty percent of the people in a town watch both basketball games and hockey games regularly. 55 percent watch basketball games and 63 percent watch hockey games regularly. If one person is chosen randomly, what is the probability that this person watches neither basketball games nor hockey games regularly?

.22

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Bayes' Rule

- Cell phones in a store's inventory are produced at three different plants, 50 percent from plant A, 30 percent from plant B and the rest from plant C. The quality of manufacturing varies between plants: A produces 5 percent defectives, B produces 7 percent defectives, and C yields 8 percent defectives. A customer purchases a cell phone from the store and it turns out to be defective. What is the probability that it is from plant A?

0.403

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Independence

- The probability that a certain critical part of a spaceship working during a flight is determined to be 0.99. Since the engineers are reluctant to risk the probability 0.01 of failure, they insert two more of the same part in parallel. This means that failure occurs if and only if all three parts fail. Assume that all three parts operate independently. What is the probability of at least one of the parts functioning successfully?

0.999999

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Random Variables and Expectation, etc.

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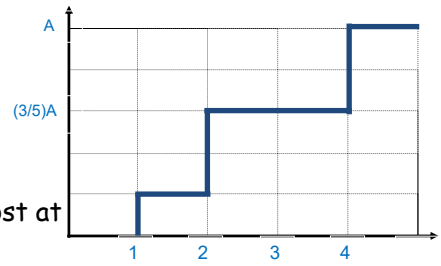
Probability Mass Functions



Here there are only masses at most at k=1,2,3,4. Shown is the CDF.

1) What is the value of A? Why?

1

2) What is the Probability Mass Function.

$$p_1 = \frac{1}{5}, \quad p_2 = \frac{2}{5}, \quad p_3 = 0, \quad p_4 = \frac{2}{5}$$

3) What is the probability that X < 3?

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PDF

Decide whether the following function f(x) can be a PDF for a continuous RV.

$$f(x) = \begin{cases} 2x & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Where the answer is positive, find the respective CDF, F(x).

Yes

$$F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

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PDF

- Decide whether the following function f(x) can be a PDF for a continuous RV.

$$f(x) = \begin{cases} 1 - x^2 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

- Where the answer is positive, find the respective CDF, F(x), the expectation and the variance.

NO

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Joint Probability Mass Function

- Consider the following joint PMF:

i	j=0	1	2	3	P{X=i}
i=0	0.1	0.1	0.05	0.05	0.3
1	0.1	0.1	0.05	0.05	0.3
2	0.1	0.05	0.025	0.025	0.2
3	0.1	0.05	0.025	0.025	0.2
P{Y=j}	0.4	0.3	0.15	0.15	1

- Determine $F(2,1)$ and $F_Y(2)$. Are RV X and Y independent?

$F(2,1) = 0.55$
$F_Y(2) = 0.7$
Not independent

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Joint Probability Mass Function

- The function f is the joint probability density function of continuous RVs X and Y where:

$$f(x,y) = \begin{cases} e^{-x} & 0 \le x < \infty, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

- Are X and Y independent?

Yes

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