

1. The exponential function  $f(x) = Ca^x$  whose graph passes through points  $(1, 4)$  and  $(2, 32)$  is
  - A.  $f(x) = 4^x$
  - B.  $f(x) = 5^x$
  - C.  $f(x) = \frac{1}{2} 8^x$
  - D.  $32^{x/2}$
  - E. none of the above
2. How many solutions does the equation  $\sin(2x) = \cos x$  have in the interval  $[0, 2\pi]$  ?
  - A. none
  - B. one
  - C. two
  - D. three
  - E. more than three
3. Which of the following functions has an inverse:
  - A.  $f(x)=3$
  - B.  $g(x) = x^2$
  - C.  $h(x) = x^2 + 1$
  - D.  $k(x) = e^x$
  - E. none of the above
4. If  $f(x) = \ln(x + 5)$  then  $f^{-1}(x) =$ 
  - A.  $e^{x-5}$
  - B.  $e^x - 5$
  - C.  $e^{x-\ln 5}$
  - D.  $\frac{1}{\ln(x-\frac{1}{5})}$
  - E. none of the above
5. (Remember no calculators or electronic aids may be used in the exam)  
 $\log_3 \frac{1}{27} + \log_3 30 - \log_3 10 =$ 
  - A. -3
  - B. 3
  - C. -2
  - D. 2
  - E. none of the above
6.  $\tan(\arctan 8) =$ 
  - A. -8
  - B. 8
  - C.  $\pi - 8$
  - D.  $\frac{\pi}{2} - 8$
  - E. none of the above

7.  $\cos(2 \tan^{-1} x) =$
- A.  $\frac{1+x^2}{1-x^2}$
  - B.  $\frac{1-x^2}{1+x^2}$
  - C.  $\frac{x}{1+x^2}$
  - D.  $\frac{x}{1-x^2}$
  - E. none of the above
8.  $\lim_{x \rightarrow 4^+} \ln(x^2 - 16) =$
- A. 0
  - B. 1
  - C.  $-\infty$
  - D.  $\infty$
  - E. none of the above
9.  $\lim_{h \rightarrow 0} \frac{(3h+1)^2 - 1}{h} =$
- A. 0
  - B. 6
  - C. 9
  - D. 15
  - E. none of the above
10.  $\lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2} =$
- A. 0
  - B.  $-\frac{1}{4}$
  - C.  $-\frac{1}{2}$
  - D. -1
  - E. none of the above
11.  $\lim_{t \rightarrow 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t} =$
- A. -1
  - B. 0
  - C. 1
  - D. 2
  - E. none of the above
12.  $\lim_{x \rightarrow \infty} \arctan(e^{2x}) =$
- A.  $\frac{\pi}{2}$
  - B. 0
  - C.  $\infty$
  - D.  $-\infty$
  - E. none of the above

13.  $\lim_{x \rightarrow -\infty} \frac{x^{1/3} + x^2}{1 + x + 2x^2} =$
- $-\frac{1}{2}$
  - 0
  - $\frac{1}{2}$
  - 1
  - none of the above
14. For the graph of the function  $y = \frac{2x^2 + x - 1}{x^2 + x - 2}$  which of the following statements is true ?
- This graph has two vertical asymptotes and one horizontal asymptote
  - $x=3$  is a vertical asymptote
  - $y=3$  and  $y=0$  are horizontal asymptotes
  - there are no vertical asymptotes
  - statements A,B,C,D are false
15. Equality  $\lim_{h \rightarrow 0} \frac{\sin(\frac{\pi}{2} + h) - 1}{h} = f'(a)$  holds for
- $f(x) = \sin x, a = 0$
  - $f(x) = \sin x, a = \frac{\pi}{2}$
  - $f(x) = \sin(\frac{\pi}{2} + x), a = \frac{\pi}{2}$
  - $f(x) = \sin(\frac{\pi}{2} + x) - 1, a = 0$
  - none of the above
16.  $\frac{d}{dx} e^5 =$
- 0
  - $e^5$
  - $5e^4$
  - $\ln 5$
  - none of the above
17. The slope of the tangent line to the graph of  $y = \sin x - \cos x$  at  $(\pi, 1)$  is
- 2
  - 1
  - 0
  - 1
  - none of the above
18. If  $xe^y = x - y$  then the value of  $\frac{dy}{dx}$  when  $x = 1$  and  $y = 0$  is
- 0
  - $\frac{1}{2}$
  - 1
  - 2
  - none of the above
19. If  $f(x) = \ln(\ln(x^2 + 1))$  then  $f'(1) =$
- $\ln(\ln 2)$
  - $\frac{2}{\ln 2}$
  - $\frac{1}{\ln 2}$
  - $\frac{1}{2 \ln 2}$
  - none of the above

20.  $\frac{d}{dx} \cos(e^x + x) =$
- A.  $\sin(e^x + x)$
  - B.  $-\sin(e^x + x)$
  - C.  $(e^x + 1) \sin(e^x + x)$
  - D.  $-(e^x + 1) \sin(e^x + x)$
  - E. none of the above
21.  $\lim_{x \rightarrow -3} \frac{x+3}{x^2-9} =$
- A.  $\frac{1}{6}$
  - B.  $-\frac{1}{6}$
  - C.  $\infty$
  - D. 0
  - E. none of the above
22. An equation of the tangent line to  $y = 3e^{x-1}$  at  $(1, 3)$  is
- A.  $y = 3x$
  - B.  $y = \frac{3}{e}x$
  - C.  $y = 3x - 1$
  - D.  $y = 3x - e$
  - E. none of the above
23. If  $g(x) = \sin(f(x) + \pi)$ ,  $f(\pi) = 0$ , and  $f'(\pi) = 2$ , then  $g'(\pi) =$
- A.  $\pi$
  - B. 2
  - C. 0
  - D. -2
  - E. none of the above

In the questions that follow you are required to show your work. Questions without sufficient justification will not receive credit.

28. Let

$$f(x) = \begin{cases} \frac{x^2+x-6}{|x-2|} & \text{if } x \neq 2 \\ 5 & \text{if } x = 2 \end{cases}$$

(i) Find  $\lim_{x \rightarrow 2^-} f(x)$

(ii) Find  $\lim_{x \rightarrow 2^+} f(x)$

(iii) Find  $\lim_{x \rightarrow 2} f(x)$

(iv) Is  $f(x)$  continuous at  $x = 2$  ? Why ?

(v) Is  $f(x)$  continuous at  $x = 0$  ? Why ?

29. Use the Intermediate Value Theorem to show that the equation

$$4^x = 5 - 3x$$

has a solution in the interval  $(0, 1)$ .

30. For

$$f(x) = \cos(x^2 + \arctan(e^x \sin x))$$

find  $f'(x)$ .

31. If  $F(x) = f(g(x))$  where  $f(\pi) = 8$ ,  $f'(\pi) = 4$ ,  $f'(6) = 3$ ,  $g(6) = \pi$ , and  $g'(6) = 7$ , find  $F'(6)$ .

32. (a) If  $x^2 - xy + y^2 = 3$ , use implicit differentiation to find  $\frac{dy}{dx}$  when  $x = -1$ ,  $y = 1$ .

(b) Use your answer in part (a) to find an equation of the tangent line to the curve  $x^2 - xy + y^2 = 3$  at  $(-1, 1)$ .