Oct 20
2020

Lecture 12

Linear Momentum

- The integral version of Newton's Second law
- Useful in problems with forces applied during given time intervals
- Problems with impacts and impulse forces

IMPULSE – MOMENTUM

Linear momentum (Particle)

$\underline{G} = m\underline{V}$ vector quantifying the amount of motion

UNITS:

SI: $kg \cdot \frac{m}{s}$ OR $N \cdot s$

US: Slug-ft/sec OR lb-sec

Newton's Second Law

$$\Sigma \underline{F} = m\underline{a} = m\underline{\dot{V}} = \frac{d\underline{G}}{dt} = \boxed{\underline{\dot{G}} = \Sigma \underline{F}}$$

Resultant of external forces equals the rate of change of linear momentum

$\underline{G} = m\underline{V}$ is parrallel to $\underline{V}$

vector equation: $\Sigma F_x = \dot{G}_x$

$\Sigma F_y = \dot{G}_y$

$\Sigma F_z = \dot{G}_z$

$\boxed{\text{Linear Impulse - Momentum Principle}}$: effect of $\Sigma \underline{F}$ during a time interval $t_2 - t_1$ on the change of linear momentum of the particle

$$\frac{d\underline{G}}{dt} = \Sigma \underline{F} \quad \text{multiply by } dt$$

$$d\underline{G} = \underbrace{(\Sigma \underline{F})dt}_{\text{linear impulse of } (\Sigma \underline{F})}$$

$$\underbrace{\int_{t_1}^{t_2} (\Sigma \underline{F})dt}_{\text{total linear impulse}} = \underline{G}_2 - \underline{G}_1 \rightarrow$$ The total linear impulse during the time interval $t_2 - t_1$ equals the change in linear momentum

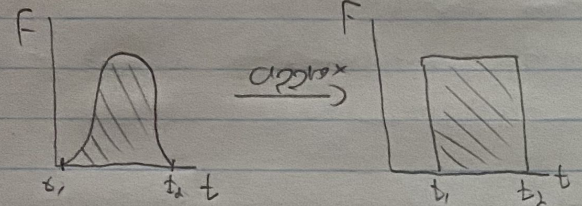
$$\underline{G}_1 + \int_{t_1}^{t_2} (\Sigma \underline{F})dt = \underline{G}_2 \rightarrow mV_{x_1} + \int_{t_1}^{t_2} (\Sigma F_x)dt = mV_{x_2}$$

$\rightarrow mV_{y_1} + \ldots$

Impulsive Forces: Relatively large forces with short duration

ex: Baseball hit by bat → During impact the contact forces are considerably larger than weights of ball and bat
↳ Approximation by considering only impulsive forces during the time interval of impact

Impulsive forces are considered constant during their application



- Non-impulsive forces → considered negligible during the action of impulsive forces
- If forces given by experimental data, calculation may require numerical integration

Conservation of linear Momentum

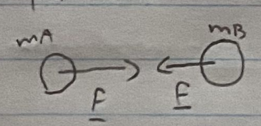
$$\underline{G}_1 + \int_{t_1}^{t_2} (\Sigma \underline{F})\, dt = \underline{G}_2$$

If impulse of external forces is zero during the time interval $[t_1, t_2]$

↓

$$\underline{G}_1 = \underline{G}_2 \rightarrow \Delta \underline{G} = 0$$ conservation of linear momentum

- One particle: $\Sigma \underline{F} = \underline{0}$
- Two particles

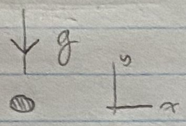


$$(\underline{G}_A)_1 + (\underline{G}_B)_1 + \underbrace{\int_{t_1}^{t_2} (\underline{F}\, dt) + \int_{t_1}^{t_2} (-\underline{F}\, dt)}_{0} = (\underline{G}_A)_2 + (\underline{G}_B)_2$$

$$\Delta \underline{G}_A + \Delta \underline{G}_B = 0$$

$$\Delta \underline{G} = 0$$

• The linear momentum is a vector, and therefore it can be conserved in one direction but not in all directions ⟶ choose carefully the coordinates



$$(\underline{G}_x)_1 = (\underline{G}_x)_2 \rightarrow \Delta \underline{G}_x = 0$$

$$(\underline{G}_y)_1 + \int_{t_1}^{t_2} -mg\,dt = (\underline{G}_y)_2$$