Purpose:

The purpose of this experiment was to use the frictional torque of the apparatus to determine the acceleration due to gravity of a specific location. The frictional torque would be measured by Atwood's Machine.

Theory:

There are two weights in the system, m_1 and m_2, and it is assumed that $m_1 < m_2$. These weights are hung onto a pulley with a radius *r*. The frictional force is in the form of torque which slows the rotation of the pulley. Since of the heavier weight is hung on the right the pulley turns counter-clock wise.

$$m_2 g - T_2 = m_2 a \quad \text{Eq. 1}$$

$$T_1 - m_1 g = m_1 a \quad \text{Eq. 2}$$

T_1 and T_2 are the tensions on the string. g is the gravity applied to each mass. m_1 and m_2 are the masses. a is the acceleration in the system.

$$m_2 g - m_1 g - (T_2 - T_1) = a(m_2 + m_1) \quad \text{Eq. 3}$$

When the torque is accounted for the equation changes to:

$$I\alpha = T_2 r - T_1 r - \Gamma \quad \text{Eq. 4}$$

I is the rotational inertia. Γ is the torque. α is the angular acceleration. Since there is no slipping of the string, one can assume that $\alpha = \frac{a}{r}$. Plugging that into the equation, the equation becomes:

$$\frac{Ia}{r^2} + \frac{\Gamma}{r} = T_2 - T_1 \quad \text{Eq. 5}$$

If the above two equations are combined into one, the equation becomes:

$$a = \frac{(m_2 - m_1)g - {}^{\Gamma}/_{r}}{M + {}^{I}/_{r^2}} \quad \text{Eq. 6}$$

M is equal to the combination of m_2 and m_1.

The height of the mass in the system's equation is as follows

$$h = \frac{at^2}{2} \quad \text{Eq. 7}$$

To graph the data and find g we first have to combine equations 6 and 7, which gives the following equation:

$$\frac{1}{t^2} = \frac{g}{2h(M + {}^{I}/_{r^2})}\Delta m - \frac{\Gamma}{2h(M + {}^{I}/_{r^2})}$$ Eq. 8

Once this equation is graphed with the data found the slope will be found by the following equation:

$$m = \frac{g}{2hA}$$ Eq. 9

Which g can be found by rearranging the equation into:

$$g = 2mhA$$ Eq. 10

The error on this equation can be found from the following equation:

$$\sigma_g = g\sqrt{\frac{\sigma_m^2}{m^2} + \frac{\sigma_h^2}{h^2} + \frac{\sigma_A^2}{A^2}}$$ Eq. 11

For M, the total mass of the system, it and its error can be found by the following equations:

$$M = m_1 + m_2 + 10m_w$$ Eq. 12

$$\sigma_M = \sqrt{\sigma_{m_1}^2 + \sigma_{m_2}^2 + 100\sigma_{m_w}^2}$$ Eq. 13

When gathering data, the inefficient statistic and the standard deviation will have to be calculated. These are their equations:

$$\sigma^{IS} = \frac{x_{max} - x_{min}}{\sqrt{N}}$$ Eq. 14

$$\sigma_{mean} = \frac{\sigma_{IS}}{\sqrt{N}}$$ Eq. 15

Finding the difference in the masses, the following equations are used:

$$\Delta m_{10-0} = (m_2 + 10m_w) - m_1$$

$$\Delta m_{9-1} = (m_2 + 9m_w) - (m_1 + m_w)$$ Eq. 16

...

In this system the I/r^2 is defined as $(80 \pm 1)\ g$. From equation 8 we can use the y-intercept to find the torque. If the change in mass is at zero the y-intercept will be found and the equation would be:

$$b = -\frac{\Gamma}{2hrA}$$ Eq. 17

Which is rewritten to find torque as:

$$\Gamma = -2bhrA$$ Eq. 18

The error of torque is found by the following equation:

$$\sigma_\Gamma = \Gamma\sqrt{\frac{\sigma_b^2}{b^2} + \frac{\sigma_h^2}{h^2} + \frac{\sigma_r^2}{r^2} + \frac{\sigma_A^2}{A^2}}$$ Eq. 19

The consistency of the results is found by the following equation:

$$t = \frac{|x_1 - x_2|}{\sqrt{\sigma_1^2 + \sigma_2^2}}$$ Eq. 20

Apparatus:

Atwood Machine

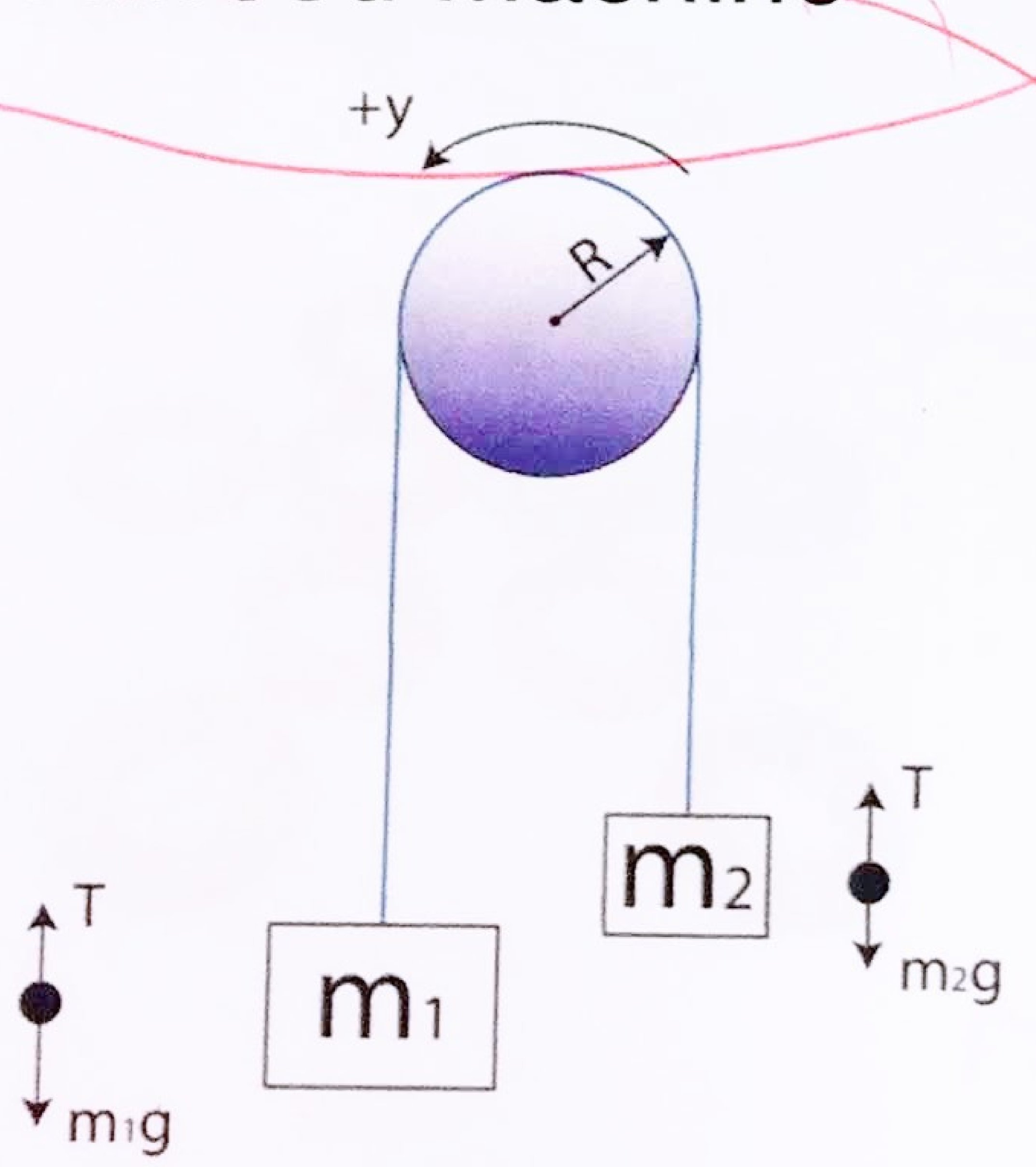

Figure #1