Asgn 2.

Question 1

Equation of motion given:

(a) $\frac{\partial \tau_{yx}}{\partial y} = 0 \rightarrow \tau_{yx} = C_1$

$$\rightarrow -\mu \frac{dv_x}{dy} = C_1 \rightarrow \frac{dv_x}{dy} = -\frac{C_1}{\mu}$$

$$\rightarrow v_x = -\frac{C_1}{\mu} y + C_2$$

BC1) At $y = 0$ $v_x = 0.333$ m/s : $C_2 = 0.333$ (m/s)

BC2) At $y = 0.02$ $v_x = -0.666$ m/s : ($\mu = 0.25$ Pa·s)

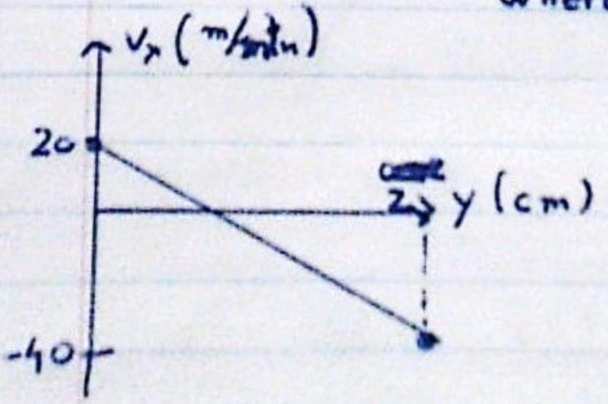
$$-0.666 = -\frac{C_1(0.02)}{0.25} + 0.333 \rightarrow C_1 = 12.5 \text{ Pa}$$

$$\boxed{\tau_{yx} = 12.5 \text{ Pa}}$$

(b) From (a): $v_x = -\frac{12.5}{0.25} y + 0.33\bar{3}$

$\rightarrow v_x = -50y + 0.33\bar{3}$

where v_x in m/s , y in m



$v_x = 0$ at:

$-50y + 0.333 = 0$

$\rightarrow y = 0.0066\bar{6}$ m

$= 0.66\bar{6}$ cm

Question 2.

(a) Navier-Stokes eqn simplifies to:

$$0 = -\frac{dP}{dz} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right)\right]$$

$$\rightarrow \frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) = \frac{1}{\mu}\frac{\partial P}{\partial z} r$$

Take $\frac{1}{\mu}\frac{\partial P}{\partial z} = \alpha$

Then: $\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) = \alpha r$

$$\rightarrow r\frac{\partial V_z}{\partial r} = \frac{\alpha}{2}r^2 + C_1$$

$$\rightarrow \frac{\partial V_z}{\partial r} = \frac{\alpha}{2}r + \frac{C_1}{r}$$

$$\rightarrow V_z = \frac{\alpha r^2}{4} + C_1 \ln r + C_2$$

B.C.1) At $r = R_1$, $V_z = 0$:

$$0 = \frac{\alpha R_1^2}{4} + C_1 \ln R_1 + C_2 \qquad (1)$$

B.C2) At $r = R_2$, $V_z = 0$:

$$0 = \frac{\alpha R_2^2}{4} + C_1 \ln R_2 + C_2 \qquad (2)$$

(1)-(2): $0 = \frac{\alpha}{4}(R_1^2 - R_2^2) + C_1 \ln(R_1/R_2)$

$$\rightarrow C_1 = \frac{\frac{\alpha}{4}(R_1^2 - R_2^2)}{\ln(R_2/R_1)}$$

$$(1) \rightarrow \quad C_2 = -\frac{\alpha}{4} R_1^2 - c_1 \ln R_1$$

$$\rightarrow \quad C_2 = -\frac{\alpha}{4} R_1^2 - \frac{\frac{\alpha}{4}(R_1^2 - R_2^2)}{\ln(R_2/R_1)} \ln R_1$$

Thus:

$$v_z = \frac{\alpha}{4} r^2 + c_1 \ln r + C_2$$

$$v_z = \frac{\alpha}{4} r^2 + \frac{\frac{\alpha}{4}(R_1^2 - R_2^2)}{\ln(R_2/R_1)} \ln r$$

$$- \frac{\alpha}{4} R_1^2 - \frac{\frac{\alpha}{4}(R_1^2 - R_2^2)}{\ln(R_2/R_1)} \ln R_1$$

$$\boxed{v_z = \frac{\alpha}{4}\left[r^2 - R_1^2 + \frac{R_1^2 - R_2^2}{\ln(R_2/R_1)} \ln(r/R_1) \right]}$$

$$\left(\alpha = \frac{1}{\mu}\frac{dP}{dz}\right)$$

$$(b) \quad \tau_{rz} = -\mu \frac{\partial v_z}{\partial r}$$

$$\tau_{rz} = -\mu\left[\frac{\alpha}{4}(2r) + \frac{\frac{\alpha}{4}(R_1^2 - R_2^2)}{\ln(R_2/R_1)} \frac{1}{r} \right]$$

$$\tau_{rz} = -\mu \frac{\alpha}{2}\left[r + \frac{(R_1^2 - R_2^2)}{2\ln(R_2/R_1)} \left(\frac{1}{r}\right) \right]$$

$$\tau_{rz} = -\underbrace{\frac{\partial P}{\partial z}}_{\text{constant}} \frac{1}{2}\left[r + \frac{(R_1^2 - R_2^2)}{2\ln(R_2/R_1)} \left(\frac{1}{r}\right) \right]$$

(c) $v_z = v_{max}$ where $\frac{\partial v_z}{\partial r} = 0$

$$\rightarrow r + \frac{(R_1^2 - R_2^2)}{2\ln(R_2/R_1)} \cdot \frac{1}{r} = 0$$

$$\rightarrow r^2 = \frac{R_2^2 - R_1^2}{2\ln(R_2/R_1)}$$

$$\rightarrow \boxed{r = \sqrt{\frac{(R_2^2 - R_1^2)}{2\ln(R_2/R_1)}}}$$ (ans to (e))

and therefore:

$$v_{max} = \frac{a}{4}\left[\frac{(R_2^2 - R_1^2)}{2\ln(R_2/R_1)} - R_1^2 + \frac{(R_1^2 - R_2^2)}{\ln(R_2/R_1)} \ln\left(\frac{\sqrt{\frac{R_2^2 - R_1^2}{2\ln(R_2/R_1)}}}{R_1}\right)\right]$$

$v_z = 0$ (minimum) at ~~the~~ $r = R_1$ and $r = R_2$

(d) $Q = 2\pi \int_{R_1}^{R_2} v_z \, r \, dr$

$$= 2\pi \int_{R_1}^{R_2} \frac{\alpha}{4}\left[r^2 - R_1^2 + \frac{(R_1^2 - R_2^2)}{\ln(R_2/R_1)} \ln(r/R_1)\right] r\,dr$$

$$= \frac{\pi\alpha}{2} \int_{R_1}^{R_2} \left(r^3 - R_1^2 r + \frac{R_1^2 - R_2^2}{\ln(R_2/R_1)} \, r \ln(r/R_1) \right) dr$$

$$\cancel{= \frac{\pi\alpha}{2}\left[\frac{r^4}{4} - \frac{R_1^2}{2} r^2 + \left(\frac{R_1^2 - R_2^2}{\ln(R_2/R_1)}\right)\right.}$$

$$\left\{ \text{Note: } \int_{R_1}^{R_2} r \ln(r/R_1)\, dr = \int_{R_1}^{R_2} (r \ln r - r \ln R_1)\, dr \right.$$

$$= \left[\frac{1}{2} r^2 \ln r - \frac{1}{4} r^2 - \frac{\ln R_1}{2} r^2 \right]_{R_1}^{R_2}$$

$$= \frac{1}{2} R_2^2 \ln R_2 - \frac{1}{2} R_1^2 \ln R_1 - \frac{1}{4}(R_2^2 - R_1^2) - \frac{\ln R_1}{2}(R_2^2 - R_1^2)$$

$$\left. = \frac{1}{2} R_2^2 \ln R_2 - \frac{1}{2} R_1^2 \ln R_1 - \frac{(2\ln R_1 + 1)}{4}(R_2^2 - R_1^2) \right\}$$

$$Q = \frac{\pi\alpha}{2}\left[\frac{R_2^4 - R_1^4}{4} + \frac{R_1^2}{2}(R_2^2 - R_1^2) \right.$$

$$\left. + \left(\frac{R_1^2 - R_2^2}{\ln(R_2/R_1)}\right)\left\{ \frac{1}{2} R_2^2 \ln R_2 - \frac{1}{2} R_1^2 \ln R_1 - \frac{(1 + 2\ln R_1)}{4}(R_2^2 - R_1^2) \right\} \right.$$

(can simplify more)

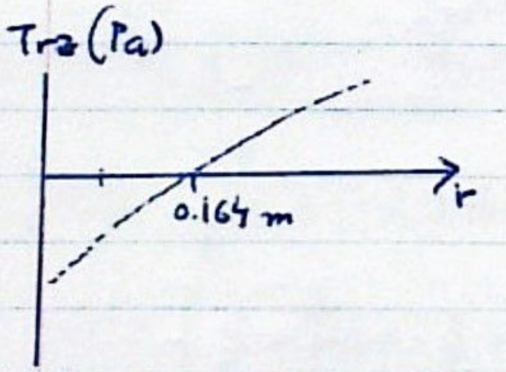
For part (f) use excel with found equations for

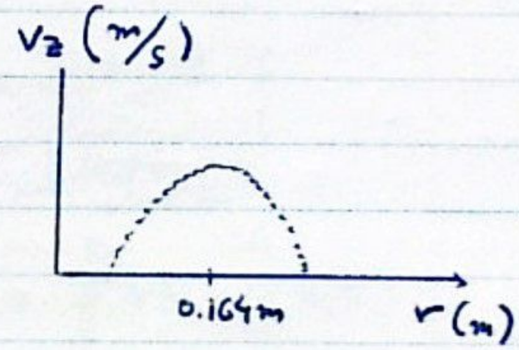
$R_1 = 0.13\ m$

$R_2 = 0.2\ m$

$\mu = 0.001\ Pa \cdot S$

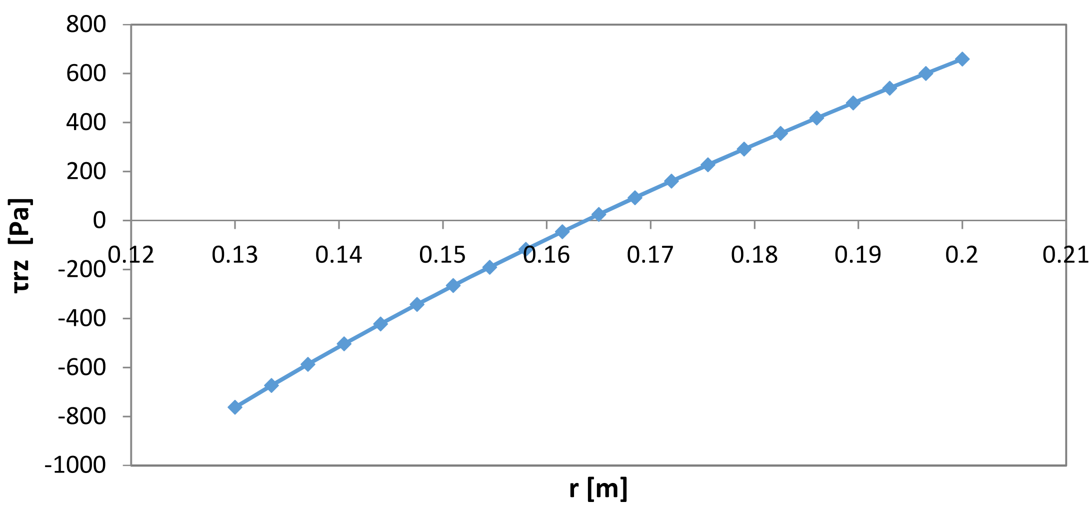
$dP/dz = -2000\ Pa/m$

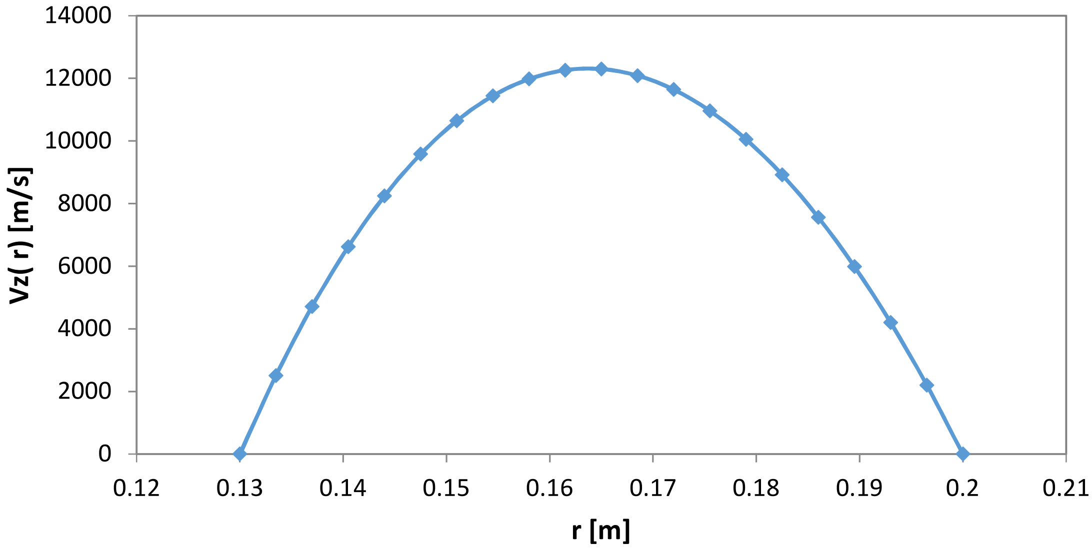




R1 (cm)	13
R1 (m)	0.13
R2 (cm)	20
R2 (m)	0.2
μ (cP)	1
μ (Pa.s)	0.001
dP/dz (kPa/m)	-20
dP/dz (Pa/m)	-20000
r max (m)	0.1637426
Vmax (m/s)	12312.695
α (1/m/s)	-2.0E+07

r (m)	Vz (m/s)	dVz/dr (1/s)	τrz (Pa)
0.13	0	762434.5	-762.434
0.1335	2511.808	673363.2	-673.363
0.137	4716.764	587054.6	-587.055
0.1405	6624.172	503302.4	-503.302
0.144	8242.645	421920	-421.92
0.1475	9580.179	342738.9	-342.739
0.151	10644.21	265605.8	-265.606
0.1545	11441.65	190381.8	-190.382
0.158	11978.96	116939.8	-116.94
0.1615	12262.17	45163.98	-45.164
0.165	12296.92	-25051.6	25.05162
0.1685	12088.51	-93804.3	93.80426
0.172	11641.9	-161183	161.1832
0.1755	10961.74	-227271	227.2708
0.179	10052.42	-292143	292.1426
0.1825	8918.076	-355869	355.8686
0.186	7562.601	-418514	418.5135
0.1895	5989.673	-480137	480.1373
0.193	4202.766	-540795	540.7954
0.1965	2205.171	-600540	600.5395
0.2	0	-659418	659.4176





14000
12000
10000
8000
6000
4000
2000
0
Vz(r) [m/s]
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.2
0.21
r [m]