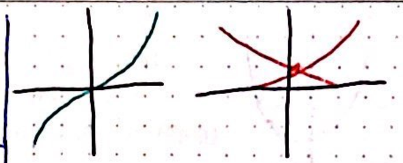
1) Higher power of x dominate large x.
smaller power of x dominate small x.



ODD — $f(x) = -f(-x)$

EVEN — $f(x) = f(-x)$

Ax^n 

↑n = flatter near origin, steeper beyond $x>1$.

↑A stretches vertically, Yaxis.

Hill Functions

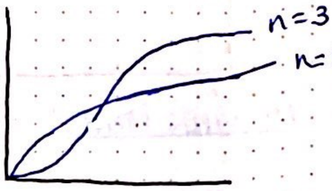
$$y = \frac{Ax^n}{a^n + x^n}$$

$x \geq 0$
$A, a > 0$ (constants)
$n > 0$ (integer)

→ for large x, $f(x) = A$ = horizontal Asymp.

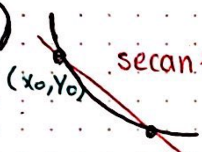
→ for small x, $\frac{A}{a^n} \times x^n$ ∴ power function

a = half max
$f(a) = \frac{1}{2} \times A$



- ↑a = move to right
- goes thru origin.
- ↑n makes middle more steep.
- ↑x = ↑y
- ↑A = ↑ Horizontal Asympt.

2)



secant Line. slope = $\left(\frac{y_1 - y_0}{x_1 - x_0}\right)$
↳ gives Av. rate of change.

POINT SLOPE

$(y_0 - y_1) = m(x_0 - x_1)$

Instantaneous rate of change

Av rate of change = $\frac{f(x+h) - f(x)}{h}$

instantaneous rate = $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Derivative = $\frac{dy}{dx}$ or as $f'(x)$: instantaneous rate of change @ point x_0.
↳ or the slope of tangent line @ point x_0.

Product Rule

$f(x) = g(x) \times h(x)$
$f'(x) = g'(x)h(x) + h'(x)g(x)$

Quotient Rule

$f(x) = \frac{g(x)}{h(x)}$

$f'(x) = \frac{g'(x) \times h(x) - h'(x)g(x)}{(h(x))^2}$

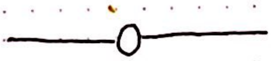
ANTIDERIVATIVES

$f'(x) = x^n$ $(x \neq -1)$
$f(x) = \frac{x^{n+1}}{n+1} + C$

Chain Rule

$f(x) = h(g(x))$
$f'(x) = h'(g(x)) \times g'(x)$

Limit = if $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ exist and are equal, then limit of f at x exists.



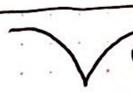
Not continuous, removable continuity.
↳ Limit can exist.

CONTINUITY

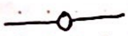
function is continuous at point $x = a$ if

$\lim_{x \to a} f(x) = f(a)$

- check if $f(a)$ exists.
- $\lim_{x \to a} f(x)$ exists
- if they are same



cusp - lim exists continuous



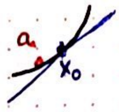
removable discont. lim exists but $f(a) \neq$ lim.

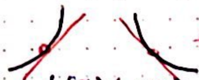
if derivative is continuous, function is continuous.

LINEAR APPROXIMATIONS

$y = L(x) = f(a) + f'(a)(x - a)$

@ approximating point, x and ref point a



Underapprox = 

tangent line is under function.

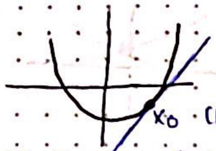
$L(x) < f(x)$ error = $L(x) - f(x)$
→ $f(x)$ is concave up
→ $f''(x)$ is positive (near ref point)

Overapprox = 

tangent line is over function.

$f(x) < L(x)$ error = $f(x) - L(x)$
→ $f(x)$ is concave down
→ $f''(x)$ is negative (near ref point).

Tangent Line



$f(x) = x^2$
$y(x)$ = tangent line
x_0 (point of tangency)

→ @ point of tangency (x_0), $f(x) = y(x)$
→ $f'(x_0) = y'(x_0)$

$$\boxed{X \text{ intercept} = \quad X = X_0 - \frac{f(X_0)}{f'(X_0)}}$$

$$\boxed{y = f(x_0) + f'(x_0)(x - x_0)}$$

↳ general formula for Tangent Line.

POLYNOMIAL

↑x dominates values

Michaelis Menten

$V = \frac{Kx}{K_n + x}$

to make linear:

$\frac{1}{V} = \frac{K_n + x}{Kx}$

$\frac{1}{V} = \frac{K_n}{Kx} + \frac{x}{Kx} = \frac{K_n}{Kx} + \frac{1}{K}$

$\frac{1}{V} = \frac{K_n}{K}\left(\frac{1}{x}\right) + \left(\frac{1}{K}\right)$

new constants $= \frac{K_n}{K} = m$ and $\frac{1}{K} = b$

$\frac{1}{V} = \underbrace{\frac{K_n}{K}}_{\text{slope}}\left(\frac{1}{x}\right) + \underbrace{\frac{1}{K}}_{\text{y int}}$

$\lim_{h \to 0} \frac{\sqrt{3+h} - \sqrt{3}}{h}$ × conjugate $= \frac{\sqrt{3+h} + \sqrt{3}}{\sqrt{3+h} + \sqrt{3}}$

$\frac{3+h-3}{h(\sqrt{3+h}+\sqrt{3})} = \frac{\cancel{h}}{\cancel{h}(\sqrt{3+h}+\sqrt{3})}$

$= \frac{1}{\sqrt{3+h}+\sqrt{3}} \overset{h\to 0}{=} \frac{1}{2\sqrt{3}}$

Limits that you should know:

Function	$x \to$	Limit notation	value
$e^{-ax}, a>0$	∞	$\lim_{x\to\infty} e^{-ax}$	0
$e^{-ax}, a>0$	$-\infty$	$\lim_{x\to-\infty} e^{-ax}$	∞
$e^{ax}, a>0$	∞	$\lim_{x\to\infty} e^{ax}$	∞
e^{kx}	0	$\lim_{x\to 0} e^{kx}$	1
$x^n e^{-ax}, a>0$	∞	$\lim_{x\to\infty} x^n e^{-ax}$	0
$\ln(ax), a>0$	∞	$\lim_{x\to\infty} \ln(ax)$	∞
$\ln(ax), a>0$	1	$\lim_{x\to 1} \ln(ax)$	0
$\ln(ax), a>0$	0	$\lim_{x\to 0} \ln(ax)$	$-\infty$
$x\ln(ax), a>0$	0	$\lim_{x\to 0} x\ln(ax)$	0
$\frac{\ln(ax)}{x}, a>0$	∞	$\lim_{x\to\infty} \frac{\ln(ax)}{x}$	0
$\frac{\sin(x)}{x}$	0	$\lim_{x\to 0} \frac{\sin(x)}{x}$	1