Université d'Ottawa · University of Ottawa
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and Statistics

Discrete Mathematics for Computing
MAT1348B Winter 2020
University of Ottawa
Midterm 2

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Directives :
- This is an open book exam.
- You can use your class notes, the material posted on Brightspace and any useful books.
- This document has 8 pages (including this page).
- This midterm is taking place on March 23rd from 13 :00 to 16 :00 only.
- This midterm has 10 questions for a total of 35 points.
- **You must show your work when appropriate**
- **The last page of the document contains a table of set identities.**

IMPORTANT :

You must write all your answers on a piece of paper of your choice or an electronic tablet. It is not necessary to write directly on a printed copy of the exam (but those who can and wish to do so can)

Instructions to submit your midterm online

- You must send a scan or a picture of your work. It must clear enough so that we can read what you wrote. We are only marking what we receive so make sure to send complete files.
- .pdf, .jpg, jpeg, .docx files are accepted.
- Make sure to submit your work at the appropriate link of the exam on Brightspace.
- Only the last version of a file or document that you submit is markek.

Question 0. This exam is open book. However, it is not an exam to do as a team. I would kindly ask you to do this exam without communicating with anyone. The only person that you can communicate with is me. You can contact me by email during the midterm. Of course, I cannot verify directly that you do not communicate with others but I would kindly as you to promise (code of honour) that you accept these conditions.

Signature : ____

(For students who do not write on a printed version of the exam, I would ask you to sign your name after writing Question 0.)

1. Let $A = \{1, 3, 5, 7, 9\}$ and $B = \{1, 2, 4, 8, 9\}$ be subsets of the universel set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Determine the following subsets or number :
a) $A - B$ b) $\overline{A \cup B}$ c) $A \oplus B$ d) $(A \cup B) - (A \cap B)$ e) $|A \cap B|$ f) |P(A)|
(3 points)

2. Prove the following results (without using the table of identities)

a) If A, B, C are sets then $(A \cup B) \subseteq (A \cup B \cup C)$. (1 point)
b) If A, B, C are sets then $(A \cap B \cap C) \subseteq (A \cap B)$. (1 point)
c) If A and B are sets and $A \subseteq B$, then $A \cup B = B$. (3 points)

3. Use the table of identities of sets (available on the last page) to show that $\overline{(\overline{A} - B)} \cap C = (C \cap A) \cup (C \cap B)$. Write the number of the rule that you use at each step. Do not skip any steps and apply only one property per step. (3 points)

4. a) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = x^3 + 1$. Show that f is a bijective function. (4 points)

b) Determine the inverse function of f. You must show your work. (2 points)

5. Consider f, g, h functions from $\{1, 2, 3, 4\} \to \{1, 2, 3, 4\}$ such that :
$f(1) = 2, f(2) = 1, f(3) = 4, f(4) = 3$; $g(1) = 1, g(2) = 4, g(3) = 2, g(4) = 3$; $h(1) = 4, h(2) = 3, h(3) = 2, h(4) = 1$. Determine the following functions :

a) $f \circ g$ b) $h \circ f$ (3 points)

6. Let $A = \mathbb{N} \times \mathbb{N}$ et $R = \{((a_1, a_2), (b_1, b_2)) \in A \times A \mid a_1 + b_2 = a_2 + b_1\}$. Show that R is an equivalence relation (4 points)

7. Determine if the binary relation $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid a \geq b^2\}$ on the set $\mathbb{Z}$ is reflective, symmetric, antisymmetric and transitive. For each case, you must show why if you think that R has this given property. If not, you must give a counterexample. (4 points)

8. Let $A =$ the set of binary codes of length 5 and
$R = \{(a, b) \in A \times A \mid \text{the first and last bit of a and b are the same}\}$. R is an equivalence relation (you do not need to show that).
a) Determine the equivalence class of 10101. Here, we want a complete list of the elements in the equivalence class and not just the definition of an equivalence class. (1 point)
b) a) Determine the equivalence class of 01011. Here, we want a complete list of the elements in the equivalence class and not just the definition of an equivalence class. (1 point)

9. Let $A = \mathbb{R} \times \mathbb{R}$ and the equivalence relation
$R = \{((x_1, y_1), (x_2, y_2)) \in \mathbb{A} \times \mathbb{A} \mid x_1^2 + y_1^2 = x_2^2 + y_2^2\}$ (you do not need to show that it is an equivalence relation). Give a **geometric description** of the equivalence class of the point (3,4) in A. Your final answer should be as simple and elegant as possible. (2 points)

10. Let $A = \mathbb{Z} \times \mathbb{Z}$ and the equivalence relation
$R = \{((a, b), (c, d)) \mid a \equiv c \ (mod\ 2), b \equiv d \ (mod\ 5)\}$ (you do not need to show that it is an equivalence relation). Determine the partition of A induced by the relation R. Give a complete description of the elements forming that partition. (2 points)

Table of identities on sets

Number	Identities
1.	$A - B = A \cap \overline{B}$
2.	$A \cup \overline{A} = U$
3.	$A \cap \overline{A} = \emptyset$
4.	$A \cup \emptyset = A$
5.	$A \cap U = A$
6.	$A \cup U = U$
7.	$A \cap \emptyset = \emptyset$
8.	$A \cup A = A$
9.	$A \cap A = A$
10.	$\overline{\overline{A}} = A$
11.	$A \cup B = B \cup A$
12.	$A \cap B = B \cap A$
13.	$A \cup (B \cup C) = (A \cup B) \cup C$
14.	$A \cap (B \cap C) = (A \cap B) \cap C$
15.	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
16.	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
17.	$\overline{A \cup B} = \overline{A} \cap \overline{B}$
18.	$\overline{A \cap B} = \overline{A} \cup \overline{B}$