CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	205	All
Examination	**Date**	**Pages**
Final	April 2019	2
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Special Instructions:	Only approved calculators are allowed **Show all your work for full marks**	

MARKS

[10] **1.** **(a)** Sketch the graph of $f(x) = 2x + x^2$ on the interval $[-1, 2]$, and write in sigma notation the formula for the right Riemann sum R_n for $f(x)$ with partitioning of the interval $[-1, 2]$ into n subintervals of equal length. Then calculate $\int_{-1}^{2} f(x)\,dx$ as the limit of R_n at $n \to \infty$

NOTE: you may need the formulas $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

(b) Use the Fundamental Theorem of Calculus to calculate the derivative of $F(x) = \int_{-x^4}^{0} \sin(t^2)\,\mathrm{d}t$. Is the function F increasing or decreasing at $x = -1$?

[15] **2.** Calculate the following indefinite integrals:

(**a**) $\displaystyle\int \frac{x^2+3}{x^2-9}\,dx$ (**b**) $\displaystyle\int \sqrt{x}\ \ln x\ dx$ (**c**) $\displaystyle\int \frac{x^3}{\sqrt{x^2+1}}\,dx$

[6] **3.** Find the antiderivative $F(t)$ of the function $f(t) = \dfrac{e^t}{4+e^{2t}}$ such that $F(\ln 2) = \dfrac{\pi}{8}$

[12] **4.** Evaluate the following definite integrals (give the **exact answers**):

(**a**) $\displaystyle\int_0^{\pi} \cos^2(x)\,\sin^3(x)\,\mathrm{d}x$ (**b**) $\displaystyle\int_0^4 x^2\sqrt{1+2\,x}\ dx$

[8] **5.** Evaluate the given improper integral or show that it diverges:

(**a**) $\displaystyle\int_1^{\infty} \frac{1}{x\,(1+\ln x)^3}\,dx$ (**b**) $\displaystyle\int_0^1 \frac{1}{(1-x)^{2/3}}\,dx$

[17] **6.** **(a)** Sketch the curves $x = y^2 - 4y$ and $x = 2y - y^2$, find their points of intersection, and find the area enclosed by the curves.

(b) Sketch the region between $y = \sin(x)$ and the x-axis on the interval $x \in [0, \pi]$, and find the volume of the solid generated by rotating this region about the line $y = -1$.

(c) Suppose $g\,(x) = \int_0^x f\,(t)\,dt$ where $f'\,(t) > 0$ for all t and $f\,(1) = 0$. Investigate whether $g(x)$ has **(i)** a local maximum or local minimum, **(ii)** an inflection point, or **(iii)** none of the above at $x = 1$. **Explain**.

[6] **7.** Find the limit of the sequence $\{a_n\}$ or prove that the limit does not exist:

(**a**) $a_n = \dfrac{2n^2\cos(\pi n)}{\sqrt{100 + 4n^4}}$ (**b**) $a_n = \sqrt{n+1} - \sqrt{n}$

[12] **8.** Determine whether the series is divergent or convergent, and if convergent, whether absolutely or conditionally:

(**a**) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, n}{1 + n^2}$ (**b**) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{10n}}{n!}$ (**c**) $\displaystyle\sum_{n=2}^{\infty} \frac{\sin(n)}{n^2}$

[6] **9.** Find **(a)** the interval of convergence, and **(b)** the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(6x+3)^n}{n+1}$$

[8] **10.** **(a)** Derive the Maclaurin series of $f(x) = x^2\,\ln(1 + 2x)$ (HINT: start with the series for $\ln(1 + z)$ where $z = 2x$).

(b) Use the differentiability of power series to find the sum $S(x) = \sum_{n=1}^{\infty} \dfrac{x^{2n}}{n}$ in the form of an elementary function within the radius of convergence of $S(x)$. (HINT: find first the sum for the derivative of $S(x)$, then antidifferentiate).

[5] **Bonus question.** If f is continuous, prove that

$$\int_0^{\pi/2} f(\cos x)dx = \int_0^{\pi/2} f(\sin x)dx$$