CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	203	All
Examination	**Date**	**Pages**
Final	December 2015	3
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Special Instructions:	Only approved calculators are allowed **Show all your work for full marks.**	

MARKS

[11] **1.** **(a)** Solve for x (find the EXACT solution,*do not give a decimal approximation*):
$2^{2x+2} = 3^{x-7}$

(b) Let $f(x) = \ln(x^2 - 1)$ and $g(x) = 1 - x$. Find $f \circ g$ and determine its domain.

(c) Find the inverse function f^{-1} of $f(x) = \ln(1 - 2x)$, and determine the range of $f^{-1}(x)$.

[7] **2.** Find the limit if it exists, otherwise explain why it does not exist:

(a) $\lim\limits_{x\to 2} \dfrac{|\,x-2|\,(x+3)}{x^2+x-6}$ **(b)** $\lim\limits_{x\to 1} \dfrac{x-1}{3-\sqrt{x^2+8}}$

[6] **3.** Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{\sqrt{9x^2+1}}{x^2-25} \cdot \frac{x^2+1}{x+5}$$

.

[15] **4.** Find the derivatives of the following functions (you don't need to simplify your final answer, but you must show how you calculate it):

(**a**) $f(x) = x^e\, e^x + e^2$

(**b**) $f(x) = \dfrac{1+\ln(x^2)}{1+x^2}$

(**c**) $f(x) = \arctan[\sin(e^{x^2\cos x})]$

(**d**) $f(x) = \sqrt{x}\,(x^{3/2} - x^{-1/2})\,(x+1)$

(**e**) $f(x) = (1+x^2)^{\tan(x)}$ (use logarithmic differentiation)

[12] **5.** Consider the function $y = \sqrt{25 + x}$.

(a) Use the **definition of derivative** to find the formula for dy/dx.

(b) Find the linearization $L(x)$ of the function $y(x)$ at $a = 0$

(c) Use this linearization to approximate $\sqrt{30}$.

[7] **6.** Let $f(x) = x^3 - 2x + 3$.

(a) Find the slope m of the secant line joining the points $(-2, f(-2))$ and $(0, f(0))$.

(b) Find all points $x = c$ (if any) on the interval [-2,0] such that $f'(c) = m$.

[17] **7.** **(a)** Verify that the point (2,1) belongs to the curve defined by the equation $x^2 + 2y^2 + 2 = x^3\, y^3$, and find an equation of the tangent line to the curve at this point.

(b) A spherical snowball is melting in such a way that its diameter D is decreasing at the rate of $dD/dt = -0.1\,\text{cm/min}$. At what rate the volume V of the snowball is decreasing when the diameter is $9\,\text{cm}$?
(NOTE: the volume of a sphere with rtadiu r is $V = 4\pi r^3/3$)

(c) Use l'Hôpital's rule to evaluate the $\lim\limits_{x\to 0} \dfrac{\sin^2(3x)}{1-\cos(2x)}$.

[11] **8.** **(a)** Find the point (x_0, y_0) on the line $y = x + 6$ that is closest to the origin.

(b) A rectangle is inscribed with its base on the x-axis and its upper corners on the parabola $y = 12 - x^2$. What is the largest possible area of such rectangle?

[14] **9.** Given the function $f(x) = 2x^3 - 21x^2 + 36x - 9$.

(a) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and all critical numbers on the x-axis where $f(x)$ has local maximum or local minimum.

(b) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).

(c) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question.** Is it possible to have a function $f(x)$ such that $f(0) = 0$, $f(2) = 4$, **and** $f'(x) < 2$ for all x on the interval $[0, 2]$? Give an example of such function or prove that it is impossible.