

- [1] 1. Find the domain of the function $g(x) = \frac{xe^{\sqrt{2-x}}}{x^2 - 9}$.

solution: We need to make sure that the expression in the square root is non-negative, and we need to make sure that the denominator is not zero. So we must have $2 - x \geq 0$ which gives $x \leq 2$ and we also need to have $x^2 - 9 \neq 0$ which gives $x \neq \pm 3$. So the domain is $(-\infty, -3) \cup (-3, 2]$.

- [2] 2. Using the definition of the derivative, find $\frac{d}{dx}\sqrt{2x+1}$.

solution:

$$\begin{aligned}
 & \lim_{h \rightarrow 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \times \frac{\sqrt{2(x+h)+1} + \sqrt{2x+1}}{\sqrt{2(x+h)+1} + \sqrt{2x+1}} \\
 &= \lim_{h \rightarrow 0} \frac{(\sqrt{2(x+h)+1})^2 - (\sqrt{2x+1})^2}{h(\sqrt{2(x+h)+1} + \sqrt{2x+1})} \\
 &= \lim_{h \rightarrow 0} \frac{(2(x+h)+1) - (2x+1)}{h(\sqrt{2(x+h)+1} + \sqrt{2x+1})} \\
 &= \lim_{h \rightarrow 0} \frac{2h}{h(\sqrt{2(x+h)+1} + \sqrt{2x+1})} \\
 &= \lim_{h \rightarrow 0} \frac{2}{(\sqrt{2(x+h)+1} + \sqrt{2x+1})} \\
 &= \frac{2}{(\sqrt{2(x+0)+1} + \sqrt{2x+1})} \\
 &= \frac{2}{(2\sqrt{2x+1})} \\
 &= \frac{1}{1(\sqrt{2x+1})}
 \end{aligned}$$

- [4] 3. Evaluate each of the limits. Show your work!

a) $\lim_{x \rightarrow -3} \frac{x+3}{4x^2+12x}$

solution:

$$\begin{aligned}\lim_{x \rightarrow -3} \frac{x+3}{4x^2+12x} &= \lim_{x \rightarrow -3} \frac{x+3}{4x(x+3)} \\ &= \lim_{x \rightarrow -3} \frac{1}{4x} \\ &= \frac{1}{4(-3)} \\ &= -\frac{1}{12}\end{aligned}$$

b) $\lim_{x \rightarrow \infty} \frac{x^2 - \sqrt{x} - 7}{x^2 + x}$

solution:

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{x^2 - \sqrt{x} - 7}{x^2 + x} &= \lim_{x \rightarrow \infty} \frac{x^2 - \sqrt{x} - 7}{x^2 + x} \times \frac{1/x^2}{1/x^2} \\ &= \lim_{x \rightarrow \infty} \frac{1 - 1/x^{3/2} - 7/x^2}{1 + 1/x} \\ &= \frac{1 - 0 - 0}{1 + 0} \\ &= 1\end{aligned}$$

- [2] 4. Suppose that $x^4 + xy - e^y = -1$. Find $y' = \frac{dy}{dx}$ in terms of x and y .

solution:

$$\begin{aligned}x^4 + xy - e^y &= -1 \\ 4x^3 + y + xy' - e^y y' &= 0 \\ xy' - e^y y' &= -4x^3 - y \\ y' &= \frac{-4x^3 - y}{x - e^y}\end{aligned}$$

- [2] 5. Let $f(x) = -3x^4 - x$. Find the equation of the line tangent to f at the point $x = 1$.

solution: First we find the derivative: $f'(x) = -12x^3 - 1$. The slope is $f'(1) = -13$. The value of the function is $f(1) = -4$. Then we have the line.

$$\begin{aligned}(y - y_0) &= m(x - x_0) \\ (y - (-4)) &= (-13)(x - 1) \\ y &= -13x + 9\end{aligned}$$

- [6] 6. Differentiate each of the following. Show your work! You do not need to simplify your answer.

a) $g(x) = \frac{x^2 + 2^x}{\tan(x)}$

solution:

$$g'(x) = \frac{(2x + 2^x \ln(2)) \tan(x) - (x^2 + 2^x)(\sec^2(x))}{(\tan(x))^2}$$

b) $u(t) = (t + e^t)^7(e^t + 2)$.

solution:

$$u'(t) = 7(t + e^t)^6(1 + e^t)(e^t + 2) + (t + e^t)^7(e^t)$$

c) $w(r) = \ln(\sqrt{r - r^2})$.

solution:

$$w'(r) = \frac{1}{\sqrt{r - r^2}} \frac{1}{2} (r - r^2)^{-1/2} (1 - 2r)$$

- [2] 7. Use logarithmic differentiation to find the derivative of $f(x) = (x^2 + 2)^{\cos(x)}$. Give your answer in terms of x (not f), but you do not need to simplify.

solution: First take \ln of both sides.

$$\begin{aligned} \ln(f) &= \ln\left((x^2 + 2)^{\cos(x)}\right) \\ &= (\cos(x)) \ln(x^2 + 2) \end{aligned}$$

Now differentiate implicitly, using product rule and chain rule. Remember to substitute f at the end.

$$\begin{aligned} \frac{1}{f} f' &= (-\sin(x)) \ln(x^2 + 2) + (\cos(x)) \frac{1}{x^2 + 2} (2x) \\ f' &= (x^2 + 2)^{\cos(x)} (-\sin(x)) \ln(x^2 + 2) + (\cos(x)) \frac{1}{x^2 + 2} (2x) \end{aligned}$$

- [3] 8. A rectangle is changing width and height continuously. When the height is 20cm and the area is 100cm², the width is increasing at a rate of 1cm/s and the area is increasing at a rate of 5cm²/s. At this moment, what is the rate of change of the height?

solution: Consider a rectangle of width w , height h and area A . We know that $A = wh$. Differentiating we get $\frac{dA}{dt} = \frac{dw}{dt}h + w\frac{dh}{dt}$. When the height is 20 and the area is 100 the width

must be $100/20 = 5$. At this moment we know $\frac{dw}{dt} = 1$ and $\frac{dA}{dt} = 5$.

$$\begin{aligned}\frac{dA}{dt} &= \frac{dw}{dt}h + w\frac{dh}{dt} \\ 5 &= (1)(20) + (5)\frac{dh}{dt} \\ \frac{dh}{dt} &= \frac{5 - 20}{5} \\ &= -3\end{aligned}$$

Note that the height is decreasing, since its derivative is negative.