

- [1] 1. Find the domain of the function  $g(x) = \frac{xe^{\sqrt{1-x}}}{x^2 - 9}$ .

**solution:** We need to make sure that the expression in the square root is non-negative, and we need to make sure that the denominator is not zero. So we must have  $1 - x \geq 0$  which gives  $x \leq 1$  and we also need to have  $x^2 - 9 \neq 0$  which gives  $x \neq \pm 3$ . So the domain is  $(-\infty, -3) \cup (-3, 1]$ .

- [2] 2. Using the definition of the derivative, find  $\frac{d}{dx}\sqrt{x+5}$ .

**solution:**

$$\begin{aligned}
 & \lim_{h \rightarrow 0} \frac{\sqrt{(x+h)+5} - \sqrt{x+5}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{(x+h)+5} - \sqrt{x+5}}{h} \times \frac{\sqrt{(x+h)+5} + \sqrt{x+5}}{\sqrt{(x+h)+5} + \sqrt{x+5}} \\
 &= \lim_{h \rightarrow 0} \frac{(\sqrt{(x+h)+5})^2 - (\sqrt{x+5})^2}{h(\sqrt{(x+h)+5} + \sqrt{x+5})} \\
 &= \lim_{h \rightarrow 0} \frac{((x+h)+5) - (x+5)}{h(\sqrt{(x+h)+5} + \sqrt{x+5})} \\
 &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{(x+h)+5} + \sqrt{x+5})} \\
 &= \lim_{h \rightarrow 0} \frac{1}{(\sqrt{(x+h)+5} + \sqrt{x+5})} \\
 &= \frac{1}{(\sqrt{x+0+5} + \sqrt{x+5})} \\
 &= \frac{1}{(2\sqrt{x+5})}
 \end{aligned}$$

- [4] 3. Evaluate each of the limits. Show your work!

a)  $\lim_{x \rightarrow 1} \frac{x-1}{5x^2 - 5x}$

**solution:**

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{x-1}{5x^2-5x} &= \lim_{x \rightarrow 1} \frac{x-1}{5x(x-1)} \\ &= \lim_{x \rightarrow 1} \frac{1}{5x} \\ &= \frac{1}{5(1)} \\ &= \frac{1}{5}\end{aligned}$$

b)  $\lim_{x \rightarrow \infty} \frac{3x^5 - x^2 - 7x}{4x^5 + \sqrt{x}}$

**solution:**

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{3x^5 - x^2 - 7x}{4x^5 + \sqrt{x}} &= \lim_{x \rightarrow \infty} \frac{3x^5 - x^2 - 7x}{4x^5 + \sqrt{x}} \times \frac{1/x^5}{1/x^5} \\ &= \lim_{x \rightarrow \infty} \frac{3 - 1/x^3 - 7/x^4}{4 + 1/x^{5/2}} \\ &= \frac{3 - 0 - 0}{4 + 0} \\ &= \frac{3}{4}\end{aligned}$$

- [2] 4. Suppose that  $2x^3 + 5xy - e^y = 7$ . Find  $y' = \frac{dy}{dx}$  in terms of  $x$  and  $y$ .

**solution:**

$$\begin{aligned}2x^3 + 5xy - e^y &= 7 \\ 6x^2 + 5y + 5xy' - e^y y' &= 0 \\ 5xy' - e^y y' &= -6x^2 - 5y \\ y' &= \frac{-6x^2 - 5y}{5x - e^y}\end{aligned}$$

- [2] 5. Let  $f(x) = -2x^3 - x$ . Find the equation of the line tangent to  $f$  at the point  $x = 1$ .

**solution:** First we find the derivative:  $f'(x) = -6x^2 - 1$ . The slope is  $f'(1) = -7$ . The value of the function is  $f(1) = -3$ . Then we have the line.

$$\begin{aligned}(y - y_0) &= m(x - x_0) \\ (y - (-3)) &= (-7)(x - 1) \\ y &= -7x + 4\end{aligned}$$

- [6] 6. Differentiate each of the following. Show your work! You do not need to simplify your answer.

a)  $u(t) = (t^3 - 5)^7(e^t + 2)$

**solution:**

$$u'(t) = 7(t^3 - 5)^6(3t^2)(e^t + 2) + (t^3 - 5)^7(e^t)$$

b)  $g(x) = \frac{2^x}{x^2 + \tan(x)}$ .

**solution:**

$$g'(x) = \frac{(2^x \ln(2))(x^2 + \tan(x)) - (2^x)(2x + \sec^2(x))}{(x^2 + \tan(x))^2}$$

c)  $w(r) = \ln(\sqrt{r^2 - 1})$ .

**solution:**

$$w'(r) = \frac{1}{\sqrt{r^2 - 1}} \frac{1}{2} (r^2 - 1)^{-1/2} (2r)$$

- [2] 7. Use logarithmic differentiation to find the derivative of  $f(x) = (x^4)^{\sin(x)}$ . Give your answer in terms of  $x$  (not  $f$ ), but you do not need to simplify.

**solution:** First take  $\ln$  of both sides.

$$\begin{aligned} \ln(f) &= \ln\left((x^4)^{\sin(x)}\right) \\ &= (\sin(x)) \ln(x^4) \end{aligned}$$

Now differentiate implicitly, using product rule and chain rule. Remember to substitute  $f$  at the end.

$$\begin{aligned} \frac{1}{f} f' &= (\cos(x)) \ln(x^4) + (\sin(x)) \frac{1}{x^4} (4x^3) \\ f' &= (x^4)^{\sin(x)} (\cos(x)) \ln(x^4) + (\sin(x)) \frac{1}{x^4} (4x^3) \end{aligned}$$

$$f' = (x^4)^{\sin(x)} \left( 4 \cos(x) + \frac{4 \sin(x)}{x} \right)$$

- [3] 8. A rectangle is changing width and height continuously. When the height is 20cm and the area is  $100\text{cm}^2$ , the width is increasing at a rate of  $1\text{cm/s}$  and the area is increasing at a rate of  $15\text{cm}^2/\text{s}$ . At this moment, what is the rate of change of the height?

**solution:** Consider a rectangle of width  $w$ , height  $h$  and area  $A$ . We know that  $A = wh$ . Differentiating we get  $\frac{dA}{dt} = \frac{dw}{dt}h + w \frac{dh}{dt}$ . When the height is 20 and the area is 100 the width

must be  $100/20 = 5$ . At this moment we know  $\frac{dw}{dt} = 1$  and  $\frac{dA}{dt} = 15$ .

$$\begin{aligned}\frac{dA}{dt} &= \frac{dw}{dt}h + w\frac{dh}{dt} \\ 15 &= (1)(20) + (5)\frac{dh}{dt} \\ \frac{dh}{dt} &= \frac{15 - 20}{5} \\ &= -1\end{aligned}$$

Note that the height is decreasing, since its derivative is negative.