Math 208, Class test, October 23, 2016

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Time: 1 Hour and 30 minutes

Answer all questions. **Only approved calculators are allowed**

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\frac{(1+i)^n - 1}{i}, \quad PV = PMT\frac{1-(1+i)^{-n}}{i}$$

1. **(10 points)** For a quadratic function

$$f(x) = 1.25x^2 - 3.75x + 2.2$$

Find

a)x and y intercepts algebraically.

b)The vertex form of $f(x)$

c) The vertex and the minimum of $f(x)$.

2. **(10 points)** Solve the following equations for x

$$(A) \quad 4^{\sqrt{x+1}} = 64 \cdot 2^{\sqrt{x+1}}$$

$$(B) \quad 2^{x^2-1} - 3^{x^2} = 3^{x^2-1} - 2^{x^2+2}$$

$$(C) \quad \log_{10}(5-x) + 2\log_{10}\sqrt{3-x} = 1$$

$$(D) \quad \frac{\ln(x^2)}{\ln(6x-5)} = 1$$

3. **(10 points)**

(A) In the arithmetic sequence

$$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}$$

$a_3 = 7$ and $a_6 = -2$. Find

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10}$$

(B) In the geometric sequence

$$b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}$$

$\frac{b_3}{b_1} = 4$ and $b_{10} = 64$. Find b_2.

4. **(10 points)**

(A) What is Annual Nominal Rate (compounded continuously) that gives Annual Percentage Yield 9% ?

(B) What is Annual Nominal Rate (compounded bi-monthly) if the Principle Amount doubles in 10 years ?

5. **(10 points)** Beginning in February 2017, you will be depositing 500$ at the end of each two months period into an account earning 8% compounded bi-monthly. Find the interest earned during each year for the first four years.

6. **(10 points)** A family is thinking about buying a new house costing 380 000$. They must pay 110 000$ down and the rest is to be amortized over 25 years in equal monthly payments. If money costs 7% compounded monthly

(A)What will their monthly payment be?

(B)What will be unpaid balance after 20 years?

(C)How much total interest will be paid over the 25 years?

MATH 208 : MIDTERM TEST
FALL 2016 : SOLUTON.

Q1. $f(x) = 1.25x^2 - 3.75x + 2.2$.

$a = 1.25$, $b = -3.75$, $c = 2.2$.

(a) x intercept set $f(x) = 0$

$1.25x^2 - 3.75x + 2.2 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{3.75 \pm \sqrt{(-3.75)^2 - 4(1.25)(2.2)}}{2(1.25)}$

$x = \frac{3.75 \pm 1.75}{2.5} = 2.2, 0.8$

y intercept $= 2.2$

(b) Vertex $V(h, k)$, $h = -\frac{b}{2a} = \frac{3}{2}$

$k = \frac{4ac - b^2}{4a} = -0.6125$

$f(x) = 1.25\left(x - \frac{3}{2}\right)^2 - 0.6125$

(c) Minimum at $\left(\frac{3}{2}, -0.6125\right)$

Min $f\left(\frac{3}{2}\right) = -0.6125$

2(A) $(2^2)^{\sqrt{x+1}} = 2^6 \cdot 2^{\sqrt{x+1}} \Rightarrow 2^{2\sqrt{x+1}} = 2^{6+\sqrt{x+1}}$

$2\sqrt{x+1} = 6 + \sqrt{x+1} \Rightarrow \sqrt{x+1} = 6$

$x + 1 = 36 \Rightarrow x = 35$

(B) $2^{x^2-1} + 2^{x^2-1} \cdot 2^3 = 3^{x^2-1} + 3^{x^2-1} \cdot 3$

$2^{x^2-1}(1+8) = 3^{x^2-1}(1+3)$

$\left(\frac{2}{3}\right)^{x^2-1} = \frac{4}{9} \Rightarrow \left(\frac{2}{3}\right)^{x^2-1} = \left(\frac{2}{3}\right)^2$

$x^2 - 1 = 2 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$.

(c) $\log_{10}(5-x)(3-x) = 1$

$(5-x)(3-x) = 10$

$x^2 - 8x + 15 = 10 \Rightarrow x^2 - 8x + 5 = 0$

$x = \frac{8 \pm \sqrt{64-20}}{2} = 4 \pm \sqrt{11}$

(D) $\ln x^2 = \ln(6x-5)$

$\ln\left(\frac{x^2}{6x-5}\right) = 0 \Rightarrow \frac{x^2}{6x-5} = 1$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 1, 5$

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(3) A. $a_n = a_1 + (n-1)d$

$a_5 = a_1 + 5d$

$a_3 = a_1 + 2d$ $\Rightarrow$ $a_1 + 5d = -2$, $a_1 + 2d = -7$

$3d = -9$

Thus: $d = -3$, $a_1 = 13$

$S_n = \frac{n}{2}[2a_1 + (n-1)d] \Rightarrow S_{10} = 5[26 - 27]$

$S_{10} = -5$

3 (B) $b_n = b_1 r^{n-1} \Rightarrow \frac{b_3}{b_1} = 4 \Rightarrow r^2 = 4 \Rightarrow$

$r = \pm 2$ but $b_{10} = b_1 r^9 = 64$

$b_1 = \frac{2^6}{(\pm 2)^9} = \pm\frac{1}{8} \Rightarrow b_2 = \left(\pm\frac{1}{8}\right)(\pm 2)$

$b_2 = \left(-\frac{1}{8}\right)(-2) = \frac{1}{8}$

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(4) $APY = e^r - 1 \Rightarrow 0.09 = e^r - 1$

(A) $e^r = 1.09 \Rightarrow r = \ln(1.09) =$

(B) $m = 6$, $t = 10$, $r = p$, $A = 2P$,

Formula $A = P\left(1 + \frac{r}{m}\right)^{mt}$

$2P = P\left(1 + \frac{r}{6}\right)^{60} \Rightarrow \left(1 + \frac{r}{6}\right) = 2^{\frac{1}{60}}$

$\frac{r}{6} = 2^{\frac{1}{60}} - 1 \Rightarrow r = 6\left[2^{\frac{1}{60}} - 1\right] = 6.97\%$

5: $m = 6$, $r = 0.08$, $i = 0.01333$, $PMT = \$500$

YR 1: $FV = \frac{PMT[(1+i)^6 - 1]}{i} = \$3101.87$

Interest YR 1 $= 3101.87 - 3000 = \$101.87$

YR 2: $FV = \frac{500[(1.013)^{12} - 1]}{0.013} = 6458.58$

Int YR 2 $= 6458.58 - 3000 - 3101.87$

$= \$356.71$

Int YR 3 $= \$636.25$, Int YR 4 $= \$936.89$

#6 $PV = \$270{,}000$, $i = \frac{0.07}{12} = 0.005833$, $t = 25$ YR

(A) $PMT = \frac{PV \times i}{\left[1 - (1+i)^{-n}\right]}$

$PMT = \frac{270{,}000 \times 0.005833}{1 - (1.005833)^{-300}}$

$= \$1908.30$

(B) Unpaid balance after 60 PMT $= PMT\left[\frac{1 - (1+i)^{-60}}{i}\right]$

$= 1908.30\left[\frac{1 - (1.005833)^{-60}}{0.005833}\right]$

$= \$96,268.92$

(C) Total Int paid $= 300 \times 1908.30 - 270{,}000$

$= \$302,490$.