Practice Problems (Part III)

MAT1348, Summer 2020

1. On a wedding ceremony, in how many ways can we take a picture of three guests with the new couple such that the bride and the bridegroom are not side by side?

2. Find the number of ways to rearrange the letters in word MISSISSIPPI .

3. Find the number of ways to re-arrange the letters in word ENGINEERING such that all vowels are side by side, and all consonants are side by side.

4. Find the coefficient of the term x^{-1} in expansion of $(x^2 + x^{-3})^{12}$.

5. Find the number of ways to distribute 12 identical balls into 4 distinct boxes such that box 1 has at most 2 balls, box 2 has at most 3 balls and box 3 has at most 4 balls.

6. Find the number of ways to arrange three couples in a row such that no couple is side by side.

7. How many bit string are there with five 1's and seven 0's such that at most three 1's may be side by side.

8. Find the number of bit strings of length 10 such that it has exactly four or five 1's or the first and the last bits are both 0.

9. There are five restaurants near my home, and I am going to a restaurant to have dinner every weekday. In how many ways can I going to these restaurants in two weeks (10 weekdays) such that every restaurant is visited at least once?

10. Prove that, if a set S of eight integers are selected randomly, then there must be x and y in S such that $x \equiv y \pmod 7$.

11. How many integers do we have to select so that there must exist at least seven selected integers that are congruent modulo 7?