Practice Problems (Part II)

MAT1348, Summer 2020

1. Prove that $\sqrt[3]{3}$ is an irrational number.

2. Prove the following result by mathematical induction: When n is an odd positive integer, then $n^2 - 1$ is a multiple of 8.

3. Explain why $\varnothing \subseteq \{\{\varnothing\}\}$, but $\varnothing \notin \{\{\varnothing\}\}$.

4. Let A, B, and C, be sets. Prove the identity $(B - A) \cup (C - A) = (B \cup C) - A$ by the following two methods:

(i) By the definition of the equality of sets.

(ii) By a membership table.

5. Let $S = \{(x, y) \mid x \text{ and } y \text{ are positive integers}\}$. A relation R from S to S is defined as $(x_1, y_1)\ R\ (x_2, y_2)$ if and only if $x_1 + y_2 = x_2 + y_1$. Show that R is an equivalence relation.

6. Let $A = \{1, 2, 3, \dots\}$ be the set of all positive integers, and let $B = \{2, 4, 6, \dots\}$ be the set of all positive even integers.

(i) Define a function from A to B that is injective but not surjective.

(ii) Define a function from B to A that is surjective but no injective.

(iii) Define a function from A to B that is neither surjective, nor injective.

Justify that the function that you define has the required property.

7. Let f_1 be a function from A to B, and let f_2 be a function from B to C. Prove, by contradiction that, if the composite function $f_2 \circ f_1$ is an injection and f_1 is a surjection, then f_2 is an injection.

8. Write $(1101101)_2$ in decimal, octal, and hexadecimal, forms.

9. Find the greatest common divisor and the least common multiplier of 1869 and 2331.

10. Find the smallest positive integer that satisfies the system of congruences

$x \equiv 7 \pmod{11}$, (1)
$x \equiv 2 \pmod{17}$, (2)
$x \equiv 3 \pmod{23}$. (3)