Practice Problems (Part I)

MAT1348, Summer 2020

1. Use the formula $A \to B \equiv \sim A \lor B$ to show that $A \to B \equiv \sim B \to \sim A$.

2. Construct the truth table of the formula $(A \lor C \equiv B) \to A \land \sim B$.

3. Use a truth table to prove the formula $(A \to B) \land (B \to A) \equiv (A \equiv B)$ is a tautology.

4. Find a formula X in CNF, which involves atomic propositions A, B and C with the following truth table (Do not simplify):

A	*B*	*C*	*X*
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

5. Find a DNF that is equivalent to formula $(A \lor C \equiv B) \to (A \land \sim B)$. Simplify if possible.

6. Define atomic propositions as follows:

A: *A* is a knight.
B: *B* is a knight.
C: *C* is a knight.

Then

~*A*: *A* is a knave.
~*B*: *B* is a knave.
~*C*: *C* is a knave.

Translate each of the following statements into a propositional logic formula using the atomic propositions defined above:

a. *A* is a knight if and only if *B* and *C* are knaves.

b. A is a knight only if B or C is a knave.

c. "A is a knight" is a necessary condition of that B is a knave.

d. Among A, B, and C, exactly two of them is a knight.

e. Among A, B, and C, at most two of them are knaves.

f. A is a knight whenever exactly one of B and C is a knight.

g. Even though A is a knight, B or C is a knave.

h. Even if both A and B are knaves, C does not have to be a knave.

7. Find a formula P that involves only B and C such that the set of formulas
$\{ B \to (A \vee C), C \to (\sim A \wedge B), P\}$ is inconsistent.

8. On the island of knights and knaves, is the following argument valid? If it is not, give a counter-example, i.e., an evaluation such that the premises are true, but the conclusion is false.

Premises:

A is a knight if and only if B is a knave and C are knights.
If C and D are both knights, then E is a knight.
E and B are knaves.

Conclusion:

A is a knave.

9. Which one of the following formulas can be used to replace the conclusion (i.e., $\sim A$) in the previous argument so that the argument is valid?

(1) $A \vee C$; (2) $A \vee \sim C$; (3) $A \vee D$; (4) $A \vee \sim D$; (5) $C \vee D$; (6) $\sim C \vee D$.

10. Define the following predicates:

Px: x is a person in this party.
Sy: y is a spicy dish.
Exy: x eats y.

Translate each of the following sentences into a predicate logic formula using the predicates defined above:

a. Everybody in this party eats a certain spicy dish.

b. Someone in this party eats all spicy dishes.

c. Someone in this party does not eat any spicy dish.

d. One spicy dish is eaten by everybody in this party.

e. Every one in this party eats every spicy dish.

11. Write the negation of the predicate logic formula (a) and (b) that you created in the previous question such that the negation is only before a predicate, and state the sentence that this formula represents.