DGD 2

MAT1348X
May 19, 2020

1. Determine whether the following set of formulas is consistent:

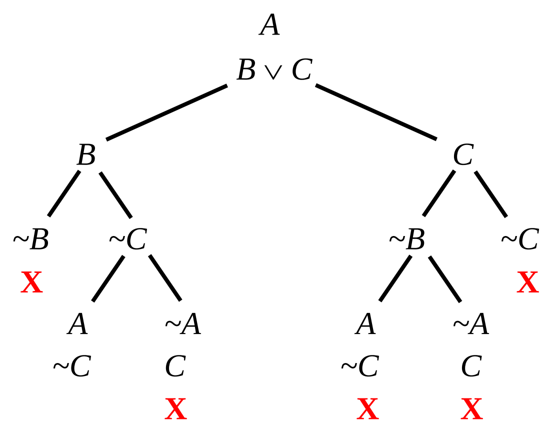
$\{(A \wedge (B \vee C), \sim(B \wedge C), \sim(A \equiv C)\}$.

If it is consistent, find an evaluation so that all formulas in this set are true.

Solution. Construct a truth tree.

$A \wedge (B \vee C)$	(1)
$\sim(B \wedge C)$	(2)
$\sim(A \equiv C)$	(3)

From (1):
From (2):
From (3):

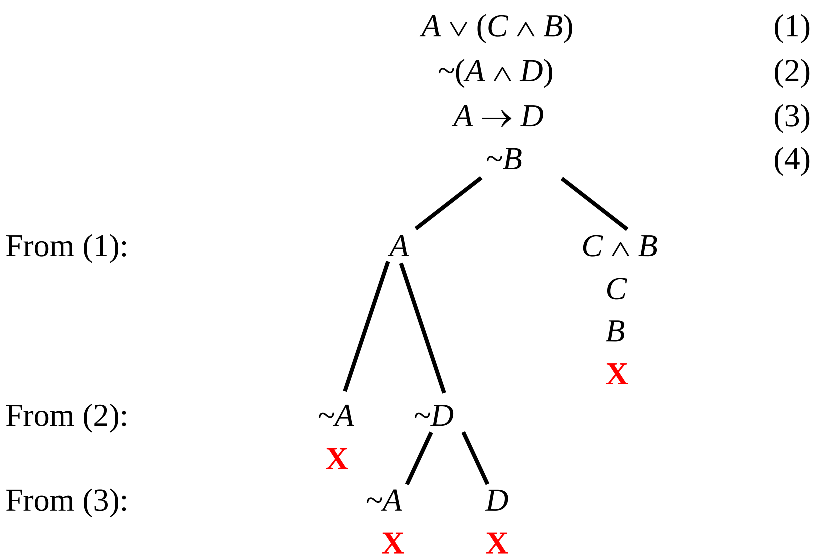


This tree has an open branch. This set is consistent. An evaluation to make all formulas are true is *A*: true, *B*: true, and *C*: false.

Explain: How the evaluation is found from the open branch.

2. Determine whether the set of formulas $\{A \vee (C \wedge B), \sim(A \wedge D), A \rightarrow D, \sim B\}$ is consistent.

Solution. Construct a truth tree as follows:



This set of formulas is inconsistent.

3. Determine whether the following argument is valid;

Premises:
Mary went to school unless it was raining and her bicycle was broken.
If Mary went to school, then it was raining or John was with her.
It was not raining and John was with her.

Conclusion:
Her bicycle was broken.

Solution.

Define propositions:

S: Mary went to school.
R: It was raining.
B: Her bicycle was broken.
J: John was with her.

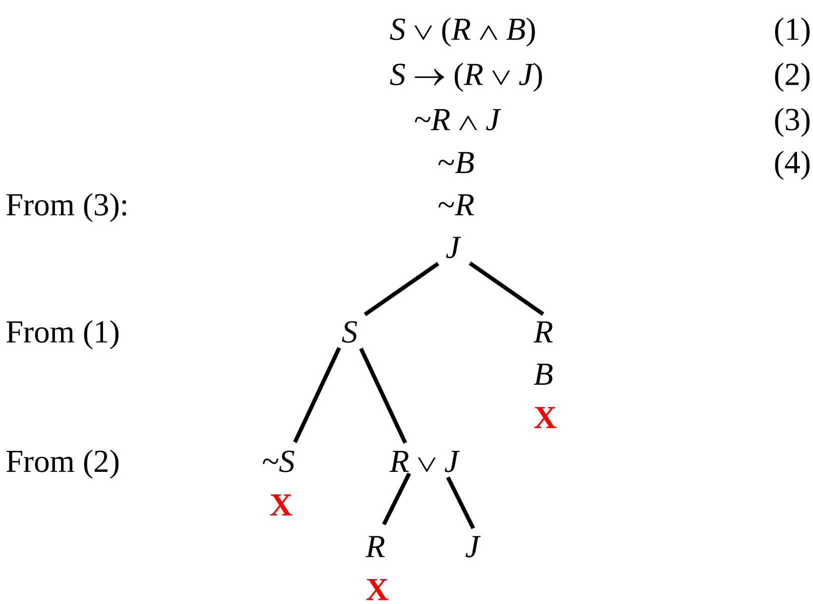
Then the argument is:

$\{S \vee (R \wedge B), S \rightarrow (R \vee J), \sim R \wedge J \mid B\}$

To check the validity of this argument, we check the consistency of the set

$\{S \vee (R \wedge B), S \rightarrow (R \vee J), \sim R \wedge J, \sim B\}$

Construct a truth tree as follows:



This tree has an open branch. The argument is invalid. The open branch gives a counter-example: When Mary went to school, it was not raining, and John was with her, we can still have the case that the bicycle was not broken.

Explain:

a. Why "unless" is translated as "or": "*Y* unless *X*" means "If *X* is not true, then *Y* must be true". In propositional logic formula, this gives $\sim X \rightarrow Y$, which is equivalent to $X \vee Y$.

b. The relation between the validity of an argument and the consistency of a set.

c. Why do we use (3) first in constructing the truth tree?

4. Define the predicates:

Sx: *x* is a student in this class.

Ex: *x* is an exam.
Wxy: *x* writes *y*.

(1) All students write all exams.

$\forall x(Sx \to \forall y(Ey \to Wxy)$.

(2) If someone writes an exam, then he/she must be a student in this class.

$\forall x(\exists y(Ey \wedge Wxy) \to Sx)$.

(3) Someone who writes an exam does not have to be a student in this class.

This is the negation of the statement in (2).

$\sim(\forall x(\exists y(Ey \wedge Wxy) \to Sx)) \equiv \exists x(\sim(\exists y(Ey \wedge Wxy) \to Sx)) \equiv \exists x(\sim(\sim\exists y(Ey \wedge Wxy) \vee Sx))$
$\equiv \exists x(\exists y(Ey \wedge Wxy) \wedge \sim Sx) \equiv \exists x(\exists y(Ey \wedge Wxy) \wedge \sim Sx)$

Say it in non-negation form:

There is someone who writes an exam and this person is not a student in this class.

Explain: Let *Px* be a predicate to specify that *x* has a property *P*. Let *Qx* be a predicate to specify that *x* has a property *Q*.

To say that there is an *x* with property *P* also has property *Q*, use
$\exists x(Px \wedge Qx)$.

To say that all *x* with property *P* have property *Q*, use

$\forall x(Px \to Qx)$.

The negation of $\exists x(Px \wedge Qx)$ is $\forall x(\sim(Px \wedge Qx)) \equiv \forall x(\sim Px \vee \sim Qx) \equiv \forall x(Px \to \sim Qx)$.

The negation of $\forall x(Px \to Qx)$ is $\exists x(\sim(Px \to Qx)) \equiv \forall x(\sim(\sim Px \vee Qx)) \equiv \forall x(Px \wedge \sim Qx)$.