

DGD 1

MAT1348X

Summer 2020

1. Use the properties of the connectives to show that $(A \rightarrow B) \rightarrow C \equiv (A \wedge C) \vee (\sim B \wedge C)$.

Solution. $(A \rightarrow B) \rightarrow C \equiv (\sim A \vee B) \rightarrow C \equiv \sim(\sim A \vee B) \vee C \equiv (\sim\sim A \wedge \sim B) \vee C \equiv (A \wedge \sim B) \vee C \equiv (A \wedge C) \vee (\sim B \wedge C)$.

2. Use a truth table to show the equivalence relation $\sim(A \equiv B) \equiv (\sim A \wedge B) \vee (A \wedge \sim B)$.

Solution.

A	B	C	$A \equiv B$	$\sim(A \equiv B)$	$\sim A \wedge B$	$A \wedge \sim B$	$(\sim A \wedge B) \vee (A \wedge \sim B)$
T	T	T	T	F	F	F	F
T	T	F	T	F	F	F	F
T	F	T	F	T	F	T	T
T	F	F	F	T	F	T	T
F	T	T	F	T	T	F	T
F	T	F	F	T	T	F	T
F	F	T	T	F	F	F	F
F	F	F	T	F	F	F	F

Since the columns $\sim(A \equiv B)$ and $(\sim A \wedge B) \vee (A \wedge \sim B)$ are identical, this equivalence is true.

3. Write the following formula in CNF: $(A \vee \sim B) \rightarrow C$.

Solution.

$(A \vee \sim B) \rightarrow C \equiv \sim(A \vee \sim B) \vee C \equiv (\sim A \wedge \sim\sim B) \vee C \equiv (\sim A \wedge B) \vee C \equiv (\sim A \vee C) \wedge (B \vee C)$.

4. Find the DNF of a formula X with atomic propositions A , B , and C , such that it is true if and only if A is true, B is true, C is false, or A is true, B is false, C is true, or A is false, B is true, C is true.

Solution. A conjunction clause $A \wedge B \wedge \sim C$ is true if and only if A is true, B is true, C is false.

A conjunction clause $A \wedge \sim B \wedge C$ is true if and only if A is true, B is false, C is true.

A conjunction clause $\sim A \wedge B \wedge C$ is true if and only if A is false, B is true, C is true.

Hence, the formula in DNF: $X \equiv (A \wedge B \wedge \sim C) \vee (A \wedge \sim B \wedge C) \vee (\sim A \wedge B \wedge C)$ is true if and one if one of these three cases occurs.

5. Find the CNF of a formula X with atomic propositions A , B , and C , such that it is true if and only if A is true, B is true, C is false, or A is true, B is false, C is true, or A is false, B is true, C is true.

Solution. To find a CNF, we need to look at the evaluations that make the formula false. In this question, this formula has the truth table like the following:

A	B	C	X
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

There are five evaluations give a false value to X .

For each of the evaluations, we have a disjunction clause that is false:

A	B	C	disjunction clause that is false
T	T	T	$\sim A \vee \sim B \vee \sim C$
T	F	F	$\sim A \vee B \vee C$
F	T	F	$A \vee \sim B \vee C$
F	F	T	$A \vee B \vee \sim C$
F	F	F	$A \vee B \vee C$

Hence, the CNF is false if at least one of them is false. Then

$$X \equiv (\sim A \vee \sim B \vee \sim C) \wedge (\sim A \vee B \vee C) \wedge (A \vee \sim B \vee C) \wedge (A \vee B \vee \sim C) \wedge (A \vee B \vee C).$$

We can simplify this CNF further:

$$\text{Since } (A \vee B \vee \sim C) \wedge (A \vee B \vee C) \equiv (A \vee B) \vee (\sim C \wedge C) \equiv (A \vee B) \vee \mathbf{F} \equiv A \vee B.$$

$$X \equiv (\sim A \vee \sim B \vee \sim C) \wedge (\sim A \vee B \vee C) \wedge (A \vee \sim B \vee C) \wedge (A \vee B).$$

Or, since $(\sim A \vee B \vee C) \wedge (A \vee B \vee C) \equiv (\sim A \wedge A) \vee (B \vee C) \equiv \mathbf{F} \vee (B \vee C) \equiv B \vee C$,

$X \equiv (\sim A \vee \sim B \vee \sim C) \wedge (B \vee C) \wedge (A \vee \sim B \vee C) \wedge (A \vee B \vee \sim C)$.

6. Define the atomic propositions as follows:

G : GG will win the game.

P : Peter will be absent.

E : The game is scheduled on a weekend.

Translate the following sentences into formulas using the defined atomic propositions:

a. GG will win the game only if Peter will not be absent.

b. The game is scheduled on a weekend is not a necessary condition for GG to win the game.

c. Although Peter will be absent and the game is not scheduled on a weekend, GG will win the game anyway.

d. Peter will play every game that is scheduled on weekends.

Solution. a. Since "only if" mean "imply", we have $P \rightarrow \sim G$.

Other way to solve the problem is to rephrase the statement: If Peter is absent, then GG will not win the game. In the way, we have $P \rightarrow \sim G$, which is equivalent to the formula $G \rightarrow \sim P$.

b. Rephrase the statement: It is not true that the game is scheduled on a weekend is a necessary condition for GG to win the game.

The statement "the game is scheduled on a weekend is a necessary condition for GG to win the game" in formula is $G \rightarrow E$.

The answer to this question is $\sim(G \rightarrow E)$.

c. A statement of the type "although X, Y" is translated by disjunction. In this question, X is $P \wedge \sim E$, and Y is G. Hence, the answer is $(P \wedge \sim E) \vee G$.

d. A game scheduled at a weekend is a sufficient condition for Peter to play the game, i.e., not absent. Hence, $E \rightarrow \sim P$.