University of Guelph
Department of Mathematics and Statistics

STAT*2040
Statistics I

Test 2 (White version)

Solutions

March 17 2017

Examiner: Jeremy Balka

This exam is 70 minutes in duration

Please clearly write your name, and write your signature, but DO NOT include your student ID on this booklet. (Make sure you include both your name and student ID on the scantron sheet.)

Name:

Signature:
Please read the instructions:

1. Fill out your name and ID number above.
2. When the examination starts, make sure your question paper is complete. You should have 19 multiple choice questions, along with formula sheets and Z and t tables in a separate handout. The first question is just a bookkeeping question, and does not count for marks, but please fill it in to ensure your exam is properly graded.
3. Do all rough work on this paper.
4. You are allowed to bring in a calculator, and pens and pencils.
5. There is only **one** correct answer for each question. Fill in only one bubble for each question.
6. Fill out the computer answer sheet in pencil as you go. *There will be no extra time given at the end of the exam to fill in the sheet.*
7. The answers given in the exam are often rounded versions of the correct answer. Choose the closest value.

1. The colour of the first page of this examination booklet (the cover sheet) is:

   (a) White
   (b) Yellow

2. According to the Canadian Health Measures Survey, the waist-to-hip ratio of twenty-year-old Canadian females is approximately normally distributed with a mean of 0.83 and a standard deviation of 0.06. The 90th percentile of the waist-to-hip ratio of twenty-year-old Canadian females is closest to which one of the following?

   (a) 0.90
   (b) 0.91**
   (c) 0.92
   (d) 0.93
   (e) 0.94

   The 90th percentile of the standard normal distribution is approximately 1.28 (if you don't know how to find this, go back and refresh your memory on how to use the standard normal table). Since $z = \frac{x-\mu}{\sigma}$, this implies $x = \mu + \sigma z = 0.83 + 1.28 \cdot 0.06 = 0.9068$.

3. Which one of the following statements is FALSE? (If A-D are all true, answer option E.)

   (a) If $P(A) = 1$, and B is any event, then A and B are independent. True. If $P(A) = 1$, then $P(A \cap B) = P(B)$ for any B, and thus $P(A \cap B) = P(A)P(B)$ for any B. In a hand-waving argument: If A is certain to occur, knowledge that it has occurred can't possibly change the probability of event B.
   (b) If $P(A) = 0.5$, $P(B) = 0.5$, and $P(A \cap B) = 0.25$, then A and B are independent. True, since $P(A \cap B) = P(A)P(B)$.
   (c) If $P(A) > 0$, $P(B) > 0$, and A and B are mutually exclusive, then A and B are dependent. True. If A and B are mutually exclusive, then $P(A \cap B) = 0$, and it cannot equal $P(A)P(B)$.
   (d) If $P(A|B) = P(A)$, then $P(A \cap B) = P(A)P(B)$. True.
   (e) None of the above. **

4. Suppose 10 values are selected randomly and independently from a Uniform distribution that has a minimum of 100 and a maximum of 200. What is the probability that all 10 values are less than 195? (Choose the closest value.)

   (a) 0.50
   (b) 0.55
   (c) 0.60 **
   (d) 0.65
   (e) 0.70

   The probability any individual value is less than 195 is $\frac{195-100}{200-100} = 0.95$. If 10 values are selected randomly and independently, the probability they are all less than 195 is $0.95^{10} = 0.5987$.

5. Suppose we are about to sample 9 values randomly and independently from a normally distributed population where $\mu = 10$ and $\sigma^2 = 36$.

   Consider the following statements:

I. The sampling distribution of the sample mean has a mean of 10. True. The mean of the sampling distribution of the sample mean is the mean of the population from which we are sampling. That is, $\mu_{\bar{X}} = \mu$.

II. The sampling distribution of the sample mean has a standard deviation of 2. True. $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{9}} = 2$.

III. The sampling distribution of the sample mean is normal. True. If we are sampling from a normally distributed population, the sample mean is normally distributed (regardless of the sample size).

Which of these statements are true?

(a) None of them.
(b) Just I and II
(c) Just I and III
(d) Just II and III
(e) All of them. **

6. A researcher wishes to investigate a possible placebo effect involving patients with high blood pressure. They intend to give each of 20 patients a placebo pill, which contains only inert substances with no pharmacological effect. They will then measure the change in systolic blood pressure after 24 hours. The researcher wants to test the null hypothesis that that placebo will have no effect on systolic blood pressure, against the alternative hypothesis that it will have an effect. Which one of the following is the most appropriate symbolic representation of those hypotheses?

(a) H_0: $\sigma^2 = 0$, H_a: $\sigma^2 \neq 0$ **
(b) H_0: $\mu = 0$, H_a: $\mu \neq 0$ **
(c) H_0: $\mu \neq 0$, H_a: $\mu = 0$
(d) H_0: $\bar{X} = 0$, H_a: $\bar{X} \neq 0$
(e) H_0: $\bar{X} \neq 0$, H_a: $\bar{X} = 0$

I'll accept either A or B. B is the answer I intended, but one could make an argument for A. (One could argue that since σ^2 is not clearly defined here, that it could represent the added variability of the placebo, which could be considered an effect. I think it's a bit of a stretch here, but I'll accept it.)

7. Suppose we sample 12 values from a normally distributed population, and find:

$$\bar{X} = 12.2, \sqrt{\frac{\sum(X_i - \bar{X})^2}{11}} = 6.1$$

Which one of the following is a 95% confidence interval for μ?

(a) (7.9,16.5)
(b) (8.3,16.1) **
(c) (8.7,15.7)
(d) (9.1,15.3)
(e) (9.5,14.9)

The standard deviation is based on sample data, and thus the interval has endpoints:

$$\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$
$$12.2 \pm 2.201 \frac{6.1}{\sqrt{12}}$$

8. Which one of the following statements best describes the important implications of the central limit theorem, as discussed in class, the text, exercises, and videos.

   (a) If we sample a large number of values from any distribution, a histogram of those values will be approximately normal. False. A histogram of a large number of values sampled from a distribution will look like the distribution.
   (b) The population mean will be approximately normally distributed, provided we have a large sample size. False. The population mean is a fixed value, and does not have a probability distribution.
   (c) When sampling from non-normal distributions, if the sample size is large then the sampling distribution of the sample mean will be approximately normal. ** True, and what the central limit theorem tells us.
   (d) When sampling from a normal distribution, the sample mean is normally distributed, regardless of the sample size. This is a true statement, but it has nothing to do with the central limit theorem.
   (e) All random variables have a normal distribution, provided we are sampling only a single value. False, and just a silly statement.

9. An American roulette wheel consists of 18 red slots, 18 black slots, and 2 green slots. In the game of roulette, the wheel is spun and a ball lands randomly in one of the slots. Each of the slots can be considered equally likely, and the results of successive spins of the wheel can be considered independent.

   In the next ten spins of such a roulette wheel, what is the probability that the first spin results in black, the second spin results in black, and black comes up exactly 6 times in the ten spins? (Choose the closest value.)

   (a) 0.03
   (b) 0.07 **
   (c) 0.13
   (d) 0.21
   (e) 0.25

   Similar to a question from Test 1. The probability the first two spins are black is $\frac{18}{38} \cdot \frac{18}{38}$. The probability the next 8 spins have 4 black is $\binom{8}{4} \frac{18}{38}^4 (1 - \frac{18}{38})^4$. Thus the answer is $\frac{18}{38} \cdot \frac{18}{38} \cdot \binom{8}{4} \frac{18}{38}^4 (1 - \frac{18}{38})^4 = 0.061$. This is closest to 0.07. (The fact that I listed 0.07 instead of 0.06 was an typo.)

10. Suppose we sample 12 observations from a normally distributed population where $\sigma = 8$, and we wish to test $H_0 : \mu = 10$ against a two-sided alternative. If $\bar{X} = 11.4$, what is the p-value of the test?

    (a) 0.05

(b) 0.13
(c) 0.27
(d) 0.54 **
(e) 0.73

$Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} = \frac{11.4-10}{8/\sqrt{12}} = 0.606$. The p-value is double the area to the right of 0.606 under the standard normal curve. The area to the right of 0.61 is approximately 0.27, so the p-value is approximately 0.54.

11. Which one of the following statements about hypothesis testing is FALSE? (If A-D are all true, answer option E.)

   (a) If the null hypothesis is false, then we cannot make a Type I error. True. A Type I error is rejecting a true null hypothesis.
   (b) If the null hypothesis is true, then we cannot make a Type II error. True. A Type II error is not rejecting a false null hypothesis.
   (c) If we reject the null hypothesis at $\alpha = 0.05$, then we can be certain the null hypothesis is false. ** False. A small p-value gives strong evidence against the null hypothesis, but that does not mean (by any stretch of the imagination) that we can be certain the null hypothesis is false.
   (d) If the p-value of the test is large (0.999, say), then there is very strong evidence the null hypothesis is true.** False. A large p-value would simply mean we have no evidence whatsoever against the null hypothesis. It definitely does not mean we have strong evidence the null is true.
   (e) None of the above.

12. Which one of the following statements is FALSE? (Assume that the t distribution under discussion has finite degrees of freedom.)

   (a) The variance of the t distribution is greater than the variance of the standard normal distribution. True. This goes hand-in-hand with C.
   (b) The median of the t distribution is equal to the median of the standard normal distribution. True. Both have a median of 0.
   (c) The t distribution has more area in the tails and a lower peak than the standard normal distribution. True. This goes hand-in-hand with A.
   (d) As the degrees of freedom increase, the t distribution tends toward the standard normal distribution. True.
   (e) The t distribution is mathematically equivalent to the standard normal distribution if the degrees of freedom are at least 30. ** False. The distributions become similar if the t distribution has large degrees of freedom, but they are not identical (for any finite degrees of freedom).

13. Suppose we carry out a Z test of $H_0 : \mu = \mu_0$ against a two-sided alternative. Which one of the following statements is FALSE? (If A-D are all true, answer option E.)

   (a) A value of the test statistic that is far out in the tails of the standard normal distribution would give strong evidence against H_0. True.
   (b) A value of the test statistic that is far out in the tails of the standard normal distribution would give strong evidence in favour of H_a. True.

(c) A very small p-value would mean very strong evidence against H_0. True.

(d) A value of $\bar{X}$ that is close to μ_0 would mean there is strong evidence against H_0. False. A value of $\bar{X}$ that is close to the hypothesized value μ_0 would not give any evidence against H_0. (Depending on the precise definition of "close".)

(e) None of the above.

14. Bulut et al. (2014) investigated thickness of the soft tissue in the facial structure of Turkish men and women. In one part of the study, a sample of 32 Turkish males in their 30s had their upper lip thickness measured. The sample mean was found to be 12.42 mm, with an associated 95% confidence interval for μ of 11.75 mm to 13.09 mm.

    Of the following options, which one is the best interpretation of that confidence interval?

    (a) We can be 95% confident that the true mean upper lip thickness of the 32 Turkish males in the sample lies between 11.75 mm and 13.09 mm.

    (b) 95% of Turkish males in their 30s have an upper lip thickness that lies between 11.75 mm and 13.09 mm.

    (c) We can be 95% confident that the true mean upper lip thickness of all men lies between 11.75 mm and 13.09 mm.

    (d) We can be 95% confident that the true mean upper lip thickness of Turkish men in their 30s lies between 11.75 mm and 13.09 mm. **

    (e) We can be 95% confident that Turkish men in their 30s have a thick upper lip.

15. The procedure used to calculate the confidence interval in the previous question has certain assumptions (in other words, certain conditions that are required for the method to work effectively). Which one of the following statements best represents those assumptions?

    (a) The sample is a simple random sample of Turkish males in their 30s, and upper lip thickness is normally distributed. **

    (b) The sample is a simple random sample of Turkish males in their 30s, and upper lip thickness has a uniform distribution.

    (c) The sample is a simple random sample of males in their 30s, and $\alpha = 0.05$.

    (d) The sample is a clustered random sample of Turkish males in their 30s, and upper lip thickness has a right-skewed distribution.

    (e) No assumptions are required, since $n > 30$.

16. Bianchi et al. (2013) investigated the subjective-objective mismatch in sleep perception. In one part of the study, the total sleep time mismatch ($TST_{subjective} - TST_{objective}$, in minutes) was measured for 92 insomniacs. Each insomniac slept for a night, then upon awakening gave their (subjective) estimate of how much sleep they got. Their objective sleep time was measured by technology. A mismatch score of -30 indicates that the insomniac underestimated their total sleep time by 30 minutes.

    Suppose we wish to test whether the true mean mismatch score is 0, versus a two-sided alternative. We carry out the analysis in R, and observe the following results.

```
        One Sample t-test
data:  mismatch
t = -7.1672, df = 91, p-value = 1.953e-10
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -83.29225 -47.14253
sample estimates:
mean of x
-65.21739
```

Of the following options, which one is the most appropriate conclusion to this study?

(a) There is very strong evidence that, on average, insomniacs underestimate their total sleep time.** True. The tiny p-value gives very strong evidence against the null hypothesis. That small p-value, along with the negative value of the sample mean (and negative confidence interval for μ) are indicative that the true mean mismatch is negative. This suggests that insomniacs tend to underestimate their total sleep time.
(b) There is very strong evidence that, on average, insomniacs overestimate their total sleep time.
(c) There is very strong evidence that, on average, insomniacs tend to correctly estimate their total sleep time.
(d) There is no evidence that the true mean mismatch score for insomniacs differs from 0.
(e) No conclusion can be drawn, since no value of α was given.

17. Consider again the information in the previous question. Suppose we carried out further analysis on the same data. Which one of the following statements is FALSE? (If A-D are all true, answer option E.)

(a) If we carried out a test of H_0: $\mu = -50$ against a two-sided alternative, the p-value would be greater than 0.05. True, since -50 is contained in the 95% confidence interval for μ.
(b) If we calculated a 90% confidence interval for μ, it would be narrower than the 95% interval given in the output. True. Larger confidence levels lead to wider intervals.
(c) If we (incorrectly) used $z_{0.025}$ instead of $t_{0.025}$ to calculate the 95% interval, the interval based on z would be narrower than the one given in the output. True. $t_a > z_a$ for any positive a, leading to wider intervals when we use t. This can easily be verified by looking up the appropriate values in the tables.
(d) If we used an alternative hypothesis of H_a: $\mu < 0$ instead of the two-sided alternative, the p-value would be half the p-value that was reported in the output. True. The p-value given in the output is double the area to the left of -7.1672 under a t distribution with 91 degrees of freedom.
(e) None of the above. **

18. If Z is a standard normal random variable, what is $P(Z > 2|Z > 1)$? (Choose the closest value.)

(a) 0.023
(b) 0.073
(c) 0.143 **
(d) 0.183
(e) 0.213

$P(Z > 2|Z > 1) = \frac{P(Z>2 \cap Z>1)}{P(Z>1)} = \frac{P(Z>2)}{P(Z>1)} = \frac{0.0228}{0.1587}$.

19. Suppose we sample 18 values from a normally distributed population where μ is unknown, but σ is known to be 6. Which one of the following is the appropriate 72% margin of error when constructing a confidence interval for the population mean μ? (Choose the closest value.)

    (a) 0.82
    (b) 1.17
    (c) 1.32
    (d) 1.53 **
    (e) 1.69

    The margin of error is $z_{0.28/2} \frac{\sigma}{\sqrt{n}} = 1.08 \frac{6}{\sqrt{18}} = 1.527$.