University of Guelph
Department of Mathematics and Statistics

STAT*2040
Statistics I

Test 2 Solutions (White version)

March 13 2015

Examiner: Jeremy Balka

This exam is 80 minutes in duration

Name:

ID:

Signature:

Please read the instructions:

1. Fill out your name and ID number above.
2. When the examination starts, make sure your question paper is complete. You should have 22 multiple choice questions, along with formula sheets and standard normal and t tables. The first question is just a bookkeeping question, and does not count for marks, but please fill it in to ensure your exam is properly graded.
3. Do all rough work on this paper.
4. You are allowed to bring in a calculator, and pens and pencils.
5. There is only **one** correct answer for each question. Fill in only one bubble for each question.
6. Fill out the computer answer sheet in pencil as you go. *There will be no extra time given at the end of the exam to fill in the sheet.*
7. The answers given in the exam are often rounded versions of the correct answer. Choose the closest value.

1. The colour of the first page of this examination booklet (the cover sheet) is:

   (a) White
   (b) Yellow

2. Suppose that green sea urchins are, to a reasonable approximation, randomly and independently distributed on a seabed at an average rate of 6 per 100 square metres. What is the probability that a randomly selected 20 square metre portion of the seabed has at least 2 green sea urchins? (Choose the closest value.)

   Copy & paste from Test 1 with changed numbers.

   The distribution of sea urchins over a given area is going to have (approximately) a Poisson distribution. (Given that they are distributed randomly and independently.) The mean number per 100 square metres is 6, so the mean number per 20 square metres is 1.20. If we let X represent the number of green sea urchins in a randomly selected 20 square metre area, then X has a Poisson distribution with $\lambda = 1.20$.

   $P(X \geq 2) = 1 - P(X \leq 1) = 1 - [P(X = 0) + P(X = 1)] = 1 - [\frac{1.20^0 e^{-1.20}}{0!} + \frac{1.20^1 e^{-1.20}}{1!}] = 0.33737.$

   (a) 0.22
   (b) 0.27
   (c) 0.34 **
   (d) 0.37
   (e) 0.41

3. Only one of the following is an appropriate set of hypotheses for a hypothesis test. Which one?

   We make hypotheses about the value of a parameter, and never about the value of a statistic.

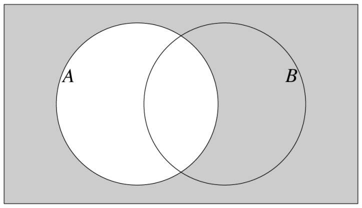
   (a) H_0: $\sigma^2 = 18$, H_a: $\sigma^2 \neq 18$ **
   (b) H_0: $\bar{X} = 18$, H_a: $\bar{X} \neq 18$
   (c) H_0: $s = 18$, H_a: $s \neq 18$
   (d) H_0: $\bar{X} = 18$, H_a: $\bar{X} > 18$
   (e) H_0: $s = 18$, H_a: $s > 18$

4. Suppose $P(A) = 0.20$, $P(B) = 0.60$, and $P(A \cap B) = 0.12$. What is $P(A^c \cup B)$?

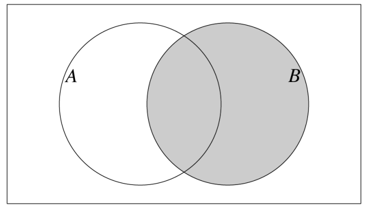
   Copy & paste from Test 1 with changed numbers.

   (a) 0.68
   (b) 0.72
   (c) 0.80
   (d) 0.88
   (e) 0.92 **

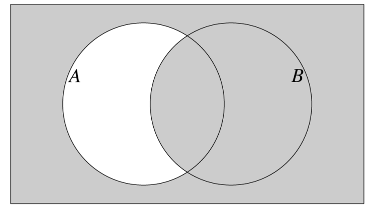
5. Dale (2010) studied birth statistics of elephants born in captivity. In one part of the study, it was found that a sample of 27 newborn male African elephants born in captivity had a mean weight of 112.0 kg and a standard deviation of 18.2 kg. Suppose it was previously claimed that newborn male African elephants born in captivity would have a mean weight of 100 kg, and we wish to carry out a hypothesis test of this claim. The following output from R summarizes the results of the test calculations.



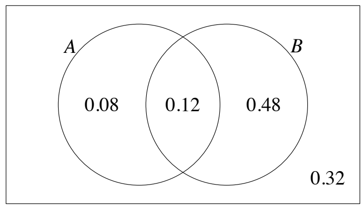
(a) A^c



(b) B



(c) $A^c \cup B$



(d) The probability of the shaded area is: $P(A^c \cup B) = 0.12 + 0.48 + 0.32 = 0.92$.

```
data:  elephants
t = 3.426, df = 26, p-value = 0.002046
alternative hypothesis: true mean is not equal to 100
95 percent confidence interval:
 104.8003 119.1997
sample estimates:
mean of x
      112
```

Assume for the purposes of this question that the sample of elephants can be thought of as a random sample of newborn male African elephants born in captivity, and assume that the weights are approximately normally distributed.

Of the following options, which one of the most appropriate conclusion to the hypothesis test?

The null hypothesis is that the true mean is 100 kg. The p-value is very small, indicating strong evidence against the null hypothesis, so there is very strong evidence that the true mean is not 100 kg. The value of the sample mean is positive, and so there is strong evidence that the true mean is in fact greater than 100 kg.

(a) There is very strong evidence that the true mean weight of newborn male African elephants born in captivity is greater than 100.0 kg. **

(b) There is very strong evidence that the sample mean weight of the 27 newborn male African elephants born in captivity is greater than 100.0 kg.

(c) There is not strong evidence that the sample mean weight of the 27 newborn male African elephants born in captivity differs from 100.0 kg.

(d) There is not strong evidence that the true mean weight of newborn male African elephants born in captivity differs from 100.0 kg.

(e) There is strong evidence that the true mean weight of newborn male African elephants born in captivity is equal to 100.0 kg.

6. Suppose that the body lengths of adult male crested porcupines in a certain region in Italy are approximately normally distributed with a mean of 73.1 cm and a standard deviation of 5.8 cm. (These values are based on a study by Mori et al. (2014).) If an adult male crested porcupine is randomly selected from this region, what is the probability that its body length lies between 70.0 and 75.0 cm? (Choose the closest value.)

   Let X represent the body length of the randomly selected porcupine. Then $X \sim N(73.1, 5.8^2)$.

$$\begin{aligned} P(70.0 \leq X \leq 75.0) &= P(\frac{70.0 - 73.1}{5.8} \leq X \leq \frac{75.0 - 73.1}{5.8}) \\ &= P(-0.5344 \leq Z \leq 0.3276) \\ &\approx 0.6293 - 0.2981 \qquad \text{(from the standard normal table)} \end{aligned}$$

   (a) 0.33 **
   (b) 0.43
   (c) 0.53
   (d) 0.63
   (e) 0.73

The next 3 questions refer to the following information.

Suppose that weights of adult female crested porcupines in a certain region in Italy are approximately normally distributed with a mean of 12.1 kg and a standard deviation of 1.9 kg. (These values are based on a study by Mori et al. (2014).)

7. If 10 adult female porcupines are randomly and independently sampled from this region, what is the standard deviation of their total weight?

   There was a very similar question on Test 1. The variance of the sum of independent random variables is the sum of their individual variances. $Var(X_1 + X_2 + \ldots + X_{10}) = Var(X_1) + Var(X_2) + \ldots Var(X_{10}) = 1.9^2 \cdot 10 = 36.1$. *The standard deviation is the square root of the variance:* $\sqrt{36.1} = 6.008$.

   (a) 0.6
   (b) 1.9
   (c) 4.4
   (d) 6.0 **
   (e) 19.0

8. If 2 adult female porcupines are randomly and independently sampled from this region, what is the probability that both porcupines weigh less than 12.5 kg? (In other words, what is the probability that porcupine #1 weighs less than 12.5 kg and porcupine #2 weighs less than 12.5 kg?)

   *If they are sampled randomly and independently, then to find the probability that *both* weigh less than 12.5 kg, we find the probability that each one weighs less than 12.5 kg and multiply the two probabilities together.*

   If we let X represent the weight of a randomly selected adult female porcupine from this region, *then* $X \sim N(12.1, 1.9^2)$.

$$\begin{aligned} P(X < 12.5) &= P(X < \frac{12.5 - 12.1}{1.9}) \\ &= P(Z < 0.21) \\ &= 0.583 \end{aligned}$$

This is the probability that an individual adult female porcupine weighs less than 12.5 kg. The probability that both of the randomly selected porcupines weigh less than 12.5 kg is $0.583^2 = 0.34$.

(a) 0.22
(b) 0.34 **
(c) 0.58
(d) 0.62
(e) 0.71

9. If 8 adult female porcupines are randomly and independently sampled from this region, what is the 95th percentile of the sampling distribution of their mean weight?

   $\bar{X}$ *is approximately normally distributed with a mean of 12.1 kg and a standard deviation of* $\frac{\sigma}{\sqrt{n}} = \frac{1.9}{\sqrt{8}}$.

   As worked through in a number of suggested exercises, we find the percentile of the sampling distribution of $\bar{X}$ *in two steps: 1) Find the 95th percentile of the standard normal distribution (it's 1.645). 2) Find the 95th percentile of the distribution of* $\bar{X}$ *by converting from* Z *to* $\bar{X}$. $Z = \frac{\bar{X} - mu}{\sigma/\sqrt{n}} \implies \bar{X} = \mu + \frac{\sigma}{\sqrt{n}} Z$, *which implies that the 95th percentile of the distribution of* $\bar{X}$ *is* $12.1 + \frac{1.9}{\sqrt{8}} \cdot 1.645 = 13.205$.

   (a) 13.2 **
   (b) 13.4
   (c) 13.7
   (d) 14.2
   (e) 15.2

10. Suppose we sample 8 observations from a normally distributed population where it is known that $\sigma = 10$, and we find a sample mean of 8.2. If we wish to calculate a 43.8% confidence interval for μ, what is the appropriate margin of error? (Choose the closest value.)

    The margin of error is $z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$. *Here we first need to find* $z_{\alpha/2}$, *where* $1 - \alpha = 0.438 \implies \alpha/2 = 0.281$. $z_{0.281} \approx 0.58$ *(from the standard normal table – I have a video where I work through examples if you find this confusing.)*

    The margin of error is $z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 0.58 \frac{10}{\sqrt{8}} = 2.05$.

    (a) 1.95
    (b) 2.05 **
    (c) 2.15
    (d) 2.25
    (e) 2.35

11. A study investigated the amount of estradiol (an estrogen) found in the blood of young males in the United States. In one part of the study, the mean estradiol concentration in 100 twenty-year-old males in the United States was found to be 38 pg/ml, with an associated confidence interval for μ of (37.1, 38.9).

    Consider the following statements:

    I. We can be 95% confident that the sample mean estradiol concentration in the blood of these 100 males lies between 37.1 pg/ml and 38.9 pg/ml. *This is wrong. Confidence interval interpretations *never* relate to the value of a statistic.*

    II. 95% of twenty-year-old males in the United States have a blood estradiol concentration that lies between 37.1 pg/ml and 38.9 pg/ml. *This is wrong. A confidence interval does not tell us anything about the percentage of a population that lies in the interval.*

    III. We can be 95% confident that if we measured the blood estradiol concentration of all twenty-year-old males in the United States, the mean would lie between 37.1 pg/ml and 38.9 pg/ml. *This is correct. The interpretation of a confidence interval always relates to the value of a parameter. In this case, the parameter is the population mean.*

    Which of these statements is an appropriate interpretation of the 95% confidence interval?

    (a) Just II.
    (b) Just III. **
    (c) I and II.
    (d) II and III.
    (e) All of them.

12. Suppose we are testing H_0: $\mu = 10$ against a two-sided alternative hypothesis at $\alpha = 0.05$. Which one of the following statements is false? (If A-D are all true, answer option E.)

    A Type I error is rejecting a true null hypothesis. A Type II error is not rejecting a false null hypothesis.

    (a) If $\mu = 10$, then we cannot make a Type II error. *True. Here the null hypothesis is true, so we cannot possibly make a Type II error.*
    (b) If $\mu = 5$, then we cannot make a Type I error. *True. Here the null hypothesis is false, so we cannot possibly make a Type I error.*
    (c) If $\mu = 15$, and the p-value is 0.01, then we will make a Type II error.** *False. Here we would be rejecting a false null hypothesis, which is the correct decision and not either error.*
    (d) If $\mu = 10$, and the p-value is 0.01, then we will make a Type I error. *True. Here we would be rejecting a true null hypothesis, and therefore we would make a Type I error.*
    (e) None of the above.

13. Which one of the following statements is true?

    (a) The variance of the t distribution is less than the variance of the standard normal distribution. *False. The variance of the t distribution is greater than the variance of the standard normal distribution.*
    (b) The mean of the t distribution is less than the mean of the standard normal distribution. *False. The t distribution and standard normal distribution both have a mean of 0.*

(c) The t distribution has less area in the tails and a higher peak than the standard normal distribution. *False. It's the other way around. The t distribution has greater area in the tails and a lower peak than the standard normal distribution.*
(d) As the degrees of freedom increase, the t distribution tends toward a normal distribution with a mean of 1. *False. As the degrees of freedom increase, the t distribution tends toward the standard normal distribution, which has a mean of 0.*
(e) None of the above. **

14. Approximately 39% of the Canadian population has blood type O^+. If Canadians are randomly and independently sampled, what is the probability that the first person with blood type O^+ occurs on or before the third person sampled? (Choose the closest value.)

A geometric probability problem. Let X represent the number of Canadians that need to be sampled until we get the first with blood type O^+.

$$\begin{aligned} P(X \leq 3) &= P(X = 1) + P(X = 2) + P(X = 3) \\ &= 0.61^0 \cdot 0.39 + 0.61^1 \cdot 0.39 + 0.61^2 \cdot 0.39 \\ &= 0.773 \end{aligned}$$

Alternatively, we could have used the logic that $X \leq 3$ is the complement of $X > 3$, and $P(X > 3) = 0.61^3$, implying $P(X \leq 3) = 1 - 0.61^3 = 0.773$.

(a) 0.15
(b) 0.63
(c) 0.71
(d) 0.77 **
(e) 0.86
(f) 0.89

15. A researcher is about to draw a random sample of 22 observations from a normally distributed population. The mean of the population is unknown, but the standard deviation of the population is known to be 6. The researcher wants to test the null hypothesis that the mean of the population is 11, against the alternative that it is different from 11. The researcher intends to use the test statistic:

$$\frac{\bar{X} - 11}{6/\sqrt{22}}$$

What is the sampling distribution of this test statistic if the null hypothesis is true?

This is the usual Z test statistic: $Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} = \frac{\bar{X}-11}{6/\sqrt{22}}$. If the null hypothesis is true, this test statistic has the standard normal distribution.

(a) The standard normal distribution. **
(b) A normal distribution with a mean of 11 and a standard deviation of 6.
(c) A t distribution with 22 degrees of freedom.
(d) A t distribution with 21 degrees of freedom.
(e) This statistic does not have a distribution, as it can only take on one possible value.

16. A researcher wants to test the null hypothesis that the mean of a certain population is 0, against the alternative hypothesis that the population mean differs from 0. He has taken a statistics course, but he forgets most of the material. He decides to take a random sample of size 50, calculate the Z statistic (the same one we have used in this course), and he will reject the null hypothesis if the Z statistic is greater than 1 or less than -1. If the null hypothesis is true, what is this researcher's probability of making a Type I error? (Choose the closest value.)

$$\begin{aligned} P(\text{Type I error}) &= P(\text{Reject } H_0|H_0 \text{ is true}) \\ &= P(Z < -1) + P(Z > 1) \quad \text{where } Z \text{ has the SND} \\ &= 0.1587 + 0.1587 \\ &= 0.3174 \end{aligned}$$

(a) 0.025
(b) 0.05
(c) 0.10
(d) 0.15
(e) 0.32 **

17. Suppose we sample 8 observations from a normally distributed population where it is known that $\sigma = 10$, and we find a sample mean of 8.2. If we test H_0: $\mu = 11$ against a two-sided alternative hypothesis, what is the p-value of the test? (Choose the closest value.)

The test statistic is $Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} = \frac{8.2-11}{10/\sqrt{8}} = -0.792$. *The alternative hypothesis is two-sided, so the p-value is double the area to the left of* -0.792 *under the standard normal curve. From the standard normal table,* $P(Z < -0.79)$ *is approximately 0.21, so the p-value is approximately 0.42.*

(a) 0.1
(b) 0.2
(c) 0.3
(d) 0.4 **
(e) 0.5

18. Suppose we sample 3 observations from a normally distributed population, and find that the values are:

$$8.1, 12.7, 24.2$$

Which one of the following is a 95% confidence interval for the population mean μ?

We are not given σ*, so we will need to estimate it from the observed data using* s*. The sample standard deviation of the given values is* $s = 8.292768$*. Since we are using an estimate of the standard deviation that is based on sample data, the confidence interval is* $\bar{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$*. There are 3 observations, so there are 2 degrees of freedom, leading to* $t_{.025} = 4.303$*. The interval is given by* $15 \pm 4.303\frac{8.292768}{\sqrt{3}}$.

(a) 15 ± 16.3
(b) 15 ± 18.4
(c) 15 ± 20.6 **
(d) 15 ± 21.8

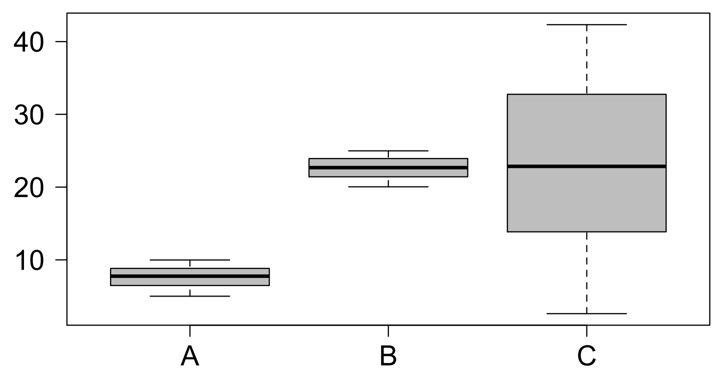
(e) 15 ± 22.3

19. Suppose we sample 62 values from a normally distributed population, and using methods learned in STAT*2040, we find a 95% confidence interval for μ of $(2.1, 3.1)$.

    Which one of the following statements is false?

    (a) 95% of the area under the sampling distribution of μ lies between 2.1 and 3.1. ** *False. μ is a parameter, not a random variable, and it does not have a sampling distribution.*
    (b) If we used the same data to construct a 90% confidence interval for μ, the interval would be narrower than the 95% interval.
    (c) If we used the same data to carry out a hypothesis test of H_0: $\mu = 2.7$ against a two-sided alternative, the p-value would be greater than 0.05.
    (d) If we used the same data to carry out a hypothesis test of H_0: $\mu = 5.7$ against a two-sided alternative, the p-value would be less than 0.05.
    (e) The sample mean was 2.6.

20. The following figure illustrates boxplots for 3 samples (labelled A, B, and C). Each sample has a sample size of 50.



    Suppose that for each sample, we carry out a hypothesis test of H_0: $\mu = 20$ against a two-sided alternative hypothesis. Which one of the 3 tests would have the smallest p-value, and which one would have the largest p-value?

    Sample A would have the smallest p-value of this test (the greatest evidence that its population mean differs from 20). Sample C would have the largest p-value of this test (the least evidence that its population mean differs from 20).

    (a) The test for Sample C would have the smallest p-value, and the test for Sample A would have the largest p-value.
    (b) The test for Sample A would have the smallest p-value, and the test for Sample C would have the largest p-value.**
    (c) The test for Sample A would have the smallest p-value, and the test for Sample B would have the largest p-value.
    (d) The test for Sample B would have the smallest p-value, and the test for Sample C would have the largest p-value.

(e) The test for Sample B would have the smallest p-value, and the test for Sample A would have the largest p-value.

21. Which one of the following statements is false? (If A-D are all true, answer option E.)

    (a) All other factors being equal, the power of a test increases as the sample size increases.
    (b) All other factors being equal, the power of a test increases as the significance level decreases.** *Power is the probability of rejecting the null hypothesis, conditioning on the null being false (conditioning on a specific value of μ that differs from the hypothesized value). The significance level is the probability of rejecting the null hypothesis, conditioning on the null being true. If we make the significance level smaller, then we are making it harder to reject the null hypothesis, resulting in lower power.*
    (c) If we are carrying out a hypothesis test at a fixed significance level α, we reject the null hypothesis if p-value $\leq \alpha$.
    (d) Suppose we are sampling from a normally distributed population and carrying out a t test on the population mean. If the null hypothesis is true, then on average the p-value will equal 0.5.
    (e) None of the above.

22. A researcher wants to estimate the average weight of adult skunks in a large town. They trap 16 adult skunks, measure their weights, and find a mean of 4.1 kg and a standard deviation of 1.2 kg. They then calculate a confidence interval using the formula $4.1 \pm 1.96\frac{1.2}{\sqrt{16}}$, which works out to (3.512, 4.688). They use this interval to say that they are 95% confident that the true mean weight of all adult skunks in this large town lies between 3.512 kg and 4.688 kg.

    Which of the following 3 statements represent possible problems that may affect the validity of this confidence interval?

    I. The population may not be normally distributed. *Non-normal populations can negatively impact the performance of the t procedures, and result in misleading conclusions. The sample size is only 16 here, so we cannot rely on the central limit theorem to save us.*

    II. The sampling method may be biased. *This might be a major problem. Perhaps skunks that are trapped tend to be larger or smaller than skunks in the general population. It's hard to say the direction of the bias here, but it could definitely be a problem that leads to misleading conclusions.*

    III. The researcher used a z value to calculate the interval, when they should have been using a t value. *This is a problem. The researcher found the standard deviation based on sample data, but then used a z value. This results in a margin of error that is too small, and an interval that is narrower than it should be.*

    (a) Just I.
    (b) Just III.
    (c) I and II.
    (d) I and III
    (e) I, II, and III. **